7.5 Midpoints and Diagonals in Quadrilaterals

Strand:
Measurement and Geometry

Student Text Pages
401 to 407

Suggested Timing
80 min

Tools
- grid paper
- rulers
- protractors

Technology Tools
- *The Geometer's Sketchpad*®
- computers
- *Cabri*® *Jr.*
- graphing calculators

Related Resources
BLM G10 Grid Paper
BLM G4 Protractor
BLM T4 *The Geometer's Sketchpad*® 3
BLM T5 *The Geometer's Sketchpad*® 4
BLM 7.5.1 Constructing Parallelograms
BLM 7.5.2 Practice: Midpoints and Diagonals in Quadrilaterals
BLM 7.5.3 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating and Applying Geometric Relationships

MG3.02 determine, through investigation using a variety of tools (e.g., dynamic geometry software, paper folding), and describe some properties of polygons (e.g., the figure that results from joining the midpoints of the sides of a quadrilateral is a parallelogram; the diagonals of a rectangle bisect each other; the line segment joining the midpoints of two sides of a triangle is half the length of the third side), and apply the results in problem solving (e.g., given the width of the base of an A-frame tree house, determine the length of a horizontal support beam that is attached half way up the sloping sides);

MG3.03 pose questions about geometric relationships, investigate them, and present their findings, using a variety of mathematical forms (e.g., written explanations, diagrams, dynamic sketches, formulas, tables);

MG3.04 illustrate a statement about a geometric property by demonstrating the statement with multiple examples, or deny the statement on the basis of a counter-example, with or without the use of dynamic geometry software.

Teaching Suggestions

- The skills developed now will help students who go on to take the grade 12 geometry and discrete mathematics course.
- As in the rest of the chapter, begin by creating groups and deciding what method, or combination of methods, students will use. If all methods are available, consult with students on their preference.
- Outline what students will be doing in the Investigate, and ask them if they have any idea of what might happen.
- Have the groups work through the Investigate. You may wish to use **BLM G10 Grid Paper**, **BLM G4 Protractor**, and/or **BLM T4 *The Geometer's Sketchpad*® 3**, or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. (10–15 min)
- Prepared sketches for *The Geometer's Sketchpad*® and *Cabri*® *Jr.* are available on the McGraw Hill Ryerson Web site. Go to http://www.mcgrawhill.ca/books/principles9.
- Follow up with a summary of students' findings and a discussion of the Reflect question. As in the earlier sections in this chapter, if students have used different methods, provide an opportunity for them to compare results. If a student asks "Does this always happen?" point out that asking and investigating such questions is the attitude of a mathematician! (5–10 min)
- Assign the Examples. Stress that making conjectures, then testing it to see if it is true, or finding a counter-example are fundamental to establishing properties of geometric figures. (10–15 min for each)
- Example 1 and several of the exercises deal with parallelograms. If your students are using technology, you may wish to use **BLM 7.5.1 Constructing Parallelograms**, which gives detailed instructions for *The Geometer's Sketchpad*® and *Cabri*® *Jr.*
- Discuss the Communicate Your Understanding questions. Following question 2, you could have students create parallelograms from paper and confirm their responses. Alternatively, you could have them use a MIRA.
- You may wish to use **BLM 7.5.2 Practice: Midpoints and Diagonals in Quadrilaterals** for remediation or extra practice.

Common Errors

- Some students may encounter a problem with interpreting the questions, either due to reading difficulties or a lack of understanding.

R_x Prepare some comments about each question to assist students in their understanding.

- As in previous sections, some students may tend to draw regular figures.

R_x Be sure to stress in all examples that a figure is only regular if that information is provided.

Ongoing Assessment

- Use Achievement Check question 12 to monitor student success. See Achievement Check Answers and **BLM 7.5.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Gifted and Enrichment—Challenge students to research Euclidean proofs at the library or on the Internet.

Investigate Answers (pages 401—402)

Method 1

2. EFGH is a parallelogram. Students may produce a rhombus, rectangle, or square if they start with some type of parallelogram.

3. HE = GF and EF = HG

5. Opposite sides are parallel.

6. Answers will vary. Sample solution: Opposite sides are parallel and equal so, the quadrilateral is a parallelogram.

8. Answers will vary. Sample solution: Since the class produced parallelograms with many different quadrilaterals, it is likely that joining the midpoints of the sides of any quadrilateral produces a parallelogram.

Method 2

3. EFGH is a parallelogram. Students may produce a rhombus, rectangle, or square if they start with some type of parallelogram.

4. Opposite sides are equal in length.

5. No. The relationships do not change if you change the location of the vertices.

7. Opposite sides are parallel. Moving a vertex of the original quadrilateral does not change the angle sums.

8. Answers will vary. Sample solution: Opposite sides are parallel and equal so the quadrilateral is a parallelogram. Since EFGH remains a parallelogram regardless of the location of the vertices of ABCD, joining the midpoints of the sides of any quadrilateral always produces a parallelogram.

Communicate Your Understanding Responses (page 404)

C1. Answers will vary. Sample solution: If the co-interior angles between two sides are supplementary, the sides are parallel.

C2. Answers will vary. Sample solution: Fold the parallelogram so that one pair of opposite vertices touch, and then, fold it so that the other pair of opposite vertices touch. The intersection of the two fold lines is at the intersection of the diagonals. Therefore, the intersection of the diagonals is an equal distance from opposite vertices.

Practise

Questions 1 to 3 are straightforward.

Connect and Apply

Question 4 is not difficult, but students may not know what a jack is or how it is used. Be prepared to explain this tool to them.

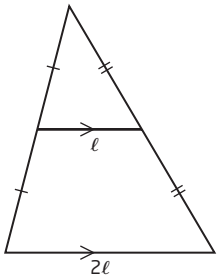
If possible, suggest students approach question 6 using *The Geometer's Sketchpad*®, so that they can easily test their conjecture. You may wish to use **T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

For question 9, students may not know what a kite-shaped quadrilateral is. Be aware that investigating any properties of a kite may be new to some students.

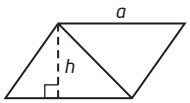
For question 12, the Achievement Check, you may wish to use **BLM 7.5.3 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 407)

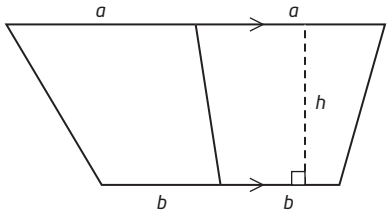
12. a) False. Any triangle can be used as a counter-example. The line segment joining the two midpoints divides the triangle into a smaller triangle and a trapezoid with equal heights. Of these two sections, the trapezoid clearly has the larger area. In the Communicate Your Understanding question C1 in Section 7.4, students show that the area of the smaller triangle is one quarter that of the larger triangle. Therefore, a line segment joining the midpoints of two sides of a triangle does not bisect the area of the triangle.



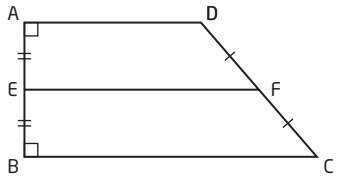
b) True. The two triangles formed by the diagonal have equal bases and equal altitudes. Therefore, the areas of the two triangles are equal. Students may use *The Geometer's Sketchpad*® to show that the areas are equal regardless of the shape of the parallelogram.



c) True. The line segment divides the original trapezoid into two smaller trapezoids. Both of the smaller trapezoids have the same height as the original trapezoid, and both have parallel sides half the length of those in the original trapezoid. Since the area of a trapezoid is $\frac{(a + b)\ h}{2}$, the two smaller trapezoids have equal areas. Students may use *The Geometer's Sketchpad*® to show that the areas are equal regardless of the shape of the trapezoid.



d) False. Many counter-examples are possible. Here is one of the simplest:
In this quadrilateral AD is much shorter than BC, so the area of AEFD is clearly less than the area of EBCF.
Students may use *The Geometer's Sketchpad*® to show that a line segment joining the midpoints of opposite sides of a quadrilateral often forms two smaller quadrilaterals with unequal areas.



Extend

These questions may pose difficulties for some students. Be prepared to discuss them in detail if used. To show that a quadrilateral is a parallelogram, students should establish that adjacent angles are supplementary.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 5, 7, 8
Typical	1–10
Extension	13–17