8.6 Surface Area of a Sphere

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
457–461

Suggested Timing
80 min

Tools
- oranges
- string
- rulers
- centimetre grid paper
- plastic balls

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G9 Centimetre Grid Paper
BLM 8.6.1 Practice: Surface Area of a Sphere
BLM A1 Assessment Recording Sheet
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Operating with Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \frac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \frac{(\text{area of base})(\text{height})}{3}$;

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Link to Get Ready

The Get Ready sections Calculate the Perimeter and Circumference and Calculate the Surface Area and Volume review skills needed for this section. In particular, the questions involving circles and cylinders, such as questions 2 and 6b) should be completed before starting this section.

Warm-Up

1. Find the area of the following:
a) a circle with radius 6 cm
b) a circle with diameter 3 m

2. Find the surface area of each figure:
a) a cylinder with radius 6 cm and height 8 cm
b) a cone with radius 6 cm and height 8 cm

3. Find the radius of the following, correct to two decimal places:
a) a circle with circumference 31.4 cm
b) a circle with circumference 1 m

Warm-Up Answers

1. a) 113.1 cm^2 **b)** 7.1 m^2

2. a) 527.8 cm^2 **b)** 301.6 cm^2

3. a) 5.00 cm **b)** 0.16 m

Common Errors

- Some students may continue to struggle with the proper units for surface area. Since they are dealing with a three-dimensional object, some will have trouble visualizing the surface area of the sphere.

R_x The Investigate will help reinforce the concepts of surface area as the students flatten the orange peel to cover the centimetre grid paper. A plastic ball cut and flattened out will help students visualize the surface area of the sphere as well.

Ongoing Assessment

- Question 7, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Accommodations

Perceptual—Provide students with three-dimensional shapes to help them understand the concept of a shape such as a cube inscribed in a sphere.

Spatial—Encourage students to use technology to solve the questions in this section.

Teaching Suggestions

- Assign the Investigate. Have students work in small groups. You may wish to use **BLM G9 Centimetre Grid Paper** to support this activity. (15–25 min)
- An orange is needed for each group. Oranges that peel easily, such as Clementine oranges, will be easier and less messy to work with. Ensure that you check student allergies before introducing any food to the classroom. While the students work with the oranges, you could cut apart an inexpensive plastic ball (the type where the plastic does not stretch when inflated), about 30 cm in diameter. Such balls are available from a dollar store. Have groups share their results with the class. Class discussion should lead the students to the formula $SA = 4\pi r^2$.
- Discuss Examples 1 and 2. Example 2 requires finding the radius of a sphere given its surface area. (10 min)
- Discuss the Communicate Your Understanding questions.
- Assign the Practise questions.
- You may wish to use **BLM 8.6.1 Practice: Surface Area of a Sphere** for remediation or extra practice.

Investigate Answers (page 457)

Answers will vary. Sample answers are provided below.

1. An estimate of the surface area of an orange might be between 140 and 170 cm.

2. a) Circumference of the orange is 22 cm.
b) Radius of the orange is 3.5 cm.

3. almost 155 cm^2

4. a) 38.5 cm^2
b) approximately 4:1
c) A possible formula for the surface area of a sphere is four times the area of a circle with the same radius.

5. $SA = 4\pi r^2$

Communicate Your Understanding Responses (page 459)

C1. Step 1: Use a string to go around the surface of the softball.
Step 2: Use the length of this string and the formula for the circumference of a circle to find the radius of the softball.
Step 3: To determine the surface area, substitute the value of the radius into the formula for the surface area of a sphere, $(SA = 4\pi r^2)$.

C2. No, doubling the radius will quadruple the surface area of a sphere. The radius is squared in the formula for the surface area of a sphere.

$SA_{old} = 4\pi r^2$

$SA_{new} = 4\pi(2r)^2$

$SA_{new} = 4(4\pi r^2)$

$SA_{new} = 4 \times SA_{old}$

Practise

Practise questions 1 and 2 are similar to Example 1.

Question 3 is similar to Example 2.

Connect and Apply

Questions 4 to 7 are typical application questions.

In question 5, students should recognize that they are assuming that the surface of Earth is smooth and that it is spherical.

In question 8, students predict how the surface area increases when the radius is increased.

Question 9 has students examine the quadratic nature of the surface area function and use the features of the graphing calculator to determine the surface area given the radius and vice versa. You may wish to use **BLM A1 Assessment Recording Sheet** to assist you in assessing students' use of technology.

Extend

Question 10 requires the algebra skills necessary to rearrange the surface area formula. Students may need assistance with the rearranging. Example 2 and question 3 have already involved rearranging with substituted values, so this is the same process for the general case. The graphing calculator is used to graph this square root function.

Question 11 is an extension of question 8. It looks at the factor by which the surface area increases when the diameter is tripled.

Question 13 is a *The Geometer's Sketchpad*® activity that looks at the ratio of the surface areas of a cube and a sphere that just fits inside the cube. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity. An extension of this activity would be to consider the algebraic model for this problem:

$$\begin{aligned}\frac{SA \text{ of cube}}{SA \text{ of sphere}} &= \frac{6(2r)^2}{4\pi r^2} \\ &= \frac{24r^2}{4\pi r^2} \\ &= \frac{6}{\pi}\end{aligned}$$

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–6, 8, 9
Extension	10–13