8

Measurement Relationships

Vocabulary

hypotenuse
Pythagorean theorem
surface area
volume
pyramid
lateral faces
cone
sphere

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site, at http://www.mcgrawhill.ca/books/ principles9. You will need your password to access this material.

Curriculum Expectations
Mathematical Process Expectations
Throughout this course, students will:

PROBLEM SOLVING
MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING
MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING
MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES
MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING
MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING
MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING
MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Overall Expectations
By the end of this course, students will:
NAV.01 demonstrate an understanding of the exponent rules of multiplication and division, and apply them to simplify expressions;
NAV.02 manipulate numerical and polynomial expressions, and solve first-degree equations.

Strand:
Number Sense and Algebra

Specific Expectations

Operating With Exponents

By the end of this chapter, students will:

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]).

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.02 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.03 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

By the end of this chapter, students will:

MG2.01 relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$;

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles);

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \dfrac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \dfrac{(\text{area of base})(\text{height})}{3}$;

MG2.05 determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Chapter Problem

The Chapter Problem deals with measurement concepts that might be encountered in a landscape design business. Have students explore their understanding of the topic by asking them to suggest measurement skills that a landscape designer might find useful. You may wish to have students complete the Chapter Problem revisits that occur throughout the chapter. These questions incorporate new skills in the sections where they are introduced, and are designed to help students move toward the Chapter Problem Wrap-Up on page 473.

Alternatively, you may wish to assign only the Chapter Problem Wrap-Up when students have completed the chapter. The Chapter Problem Wrap-Up is a summative assessment. It is open-ended in nature to allow students originality in their solutions.

Chapter 8 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter Opener • 15 min	412–413			
Get Ready • 80-160 min	414–417	• BLM 8.GR.2 Practice: Get Ready • BLM T4 *The Geometer's Sketchpad®* 3 • BLM T5 *The Geometer's Sketchpad®* 4	• BLM 8.GR.2 Get Ready Self-Assessment Checklist	**Technology Tools** • *The Geometer's Sketchpad®* • computers
8.1 Apply the Pythagorean Theorem • 80 min	418–425	• BLM 8.1.1 Practice: Apply the Pythagorean Theorem • BLM G10 Grid Paper	• BLM A8 Application General Scoring Rubric • BLM A21 Opinion Piece Checklist	**Tools** • grid paper • rulers • an assortment of cardboard boxes **Technology Tools** • *The Geometer's Sketchpad®* • computers
8.2 Perimeter and Area of Composite Figures • 80 min	426–435	• BLM 8.2.1 Practice: Perimeter and Area of Composite Figures • BLM G5 Tangram	• BLM A9 Communication General Scoring Rubric • BLM 8.2.2 Achievement Check Rubric	**Tools** • tangrams, pattern blocks, or geoboards
8.3 Surface Area and Volume of Prisms and Pyramids • 80–160 min	436–443	• BLM 8.3.1 Net for a Pyramid • BLM 8.3.2 Net for a Prism • BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids	• BLM A5 Problem Solving Checklist • BLM 8.3.4 Achievement Check Rubric	**Tools** • empty 250-mL milk cartons • construction paper • scissors • tape • sand, rice, or another suitable materials • Bristol board • pyramid models • interlocking cubes
8.4 Surface Area of a Cone • 80–160 min	444–450	• BLM 8.4.1 Practice: Surface Area of a Cone • BML T4 *The Geometer's Sketchpad®* 3 • BML T5 *The Geometer's Sketchpad®* 4	• BLM A18 My Progress as a Problem Solver	**Tools** • models of cones • construction paper • scissors • rulers • compasses • tape **Technology Tools** • *The Geometer's Sketchpad®* • computers
8.5 Volume of a Cone • 80 min	451–456	• BLM 8.5.1 Practice: Volume of a Cone • BML T4 *The Geometer's Sketchpad®* 3 • BML T5 *The Geometer's Sketchpad®* 4 • BLM T3 Microsoft® *Excel*	• BLM A11 Group Work Assessment Recording Sheet	**Tools** • empty cylindrical cans • construction paper • scissors • tape • sand, rice or other suitable materials **Technology Tools** • graphing calculators • *The Geometer's Sketchpad®* • Microsoft® *Excel* • computers
8.6 Surface Area of a Sphere • 80 min	457–461	• BLM G9 Centimetre Grid Paper • BLM 8.6.1 Practice: Surface Area of a Sphere • BML T4 *The Geometer's Sketchpad®* 3 • BML T5 *The Geometer's Sketchpad®* 4	• BLM A1 Assessment Recording Sheet	**Tools** • oranges • string • rulers • centimetre grid paper • plastic balls **Technology Tools** • *The Geometer's Sketchpad®* • computers • graphing calculators

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
8.7 Volume of a Sphere • 80 min	462–469	• BLM 8.7.1 Practice: Volume of a Sphere • BML T4 *The Geometer's Sketchpad®* 3 • BML T5 *The Geometer's Sketchpad®* 4	• BLM A6 Knowledge/ Understanding General Scoring Rubric • BLM 8.7.2 Achievement Check Rubric	**Tools** • cylindrical containers that each just hold three tennis balls • three tennis balls for each group of students • water • containers to catch the overflow water • identical small spheres (e.g., marbles, tennis balls, or table tennis balls) **Technology Tools** • graphing calculators • *The Geometer's Sketchpad®* • computers
Chapter 8 Review • 80 min	470–471	• BLM 8.CR.1 Chapter 8 Review		
Chapter 8 Practice Test • 50–70 min	472–473		• BLM 8.PT.1 Chapter 8 Practice Test • BLM 8.CT.1 Chapter 8 Test	
Chapter 8 Problem Wrap-Up • 30–60 min	473	• BLM G10 Grid Paper • BLM G9 Centimetre Grid Paper	• BLM 8.CP.1 Chapter Problem Wrap-Up Rubric	**Tools** • compasses • grid paper • centimetre grid paper **Technology Tools** • graphing calculators

Chapter 8 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 8.GR.1	Practice: Get Ready	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM 8.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
8.1: Apply the Pythagorean Theorem			
	BLM A8	Application General Scoring Rubric	Assessment
	BLM 8.1.1	Practice: Apply the Pythagorean Theorem	Practice
	BLM G10	Grid Paper	Student Support
	BLM A21	Opinion Piece Checklist	Assessment
8.2: Perimeter and Area of Composite Figures			
	BLM 8.2.1	Practice: Perimeter and Area of Composite Figures	Practice
	BLM 8.2.2	Achievement Check Rubric	Assessment
	BLM G5	Tangram	Student Support
	BLM A9	Communication General Scoring Rubric	Assessment
8.3: Surface Area and Volume of Prisms and Pyramids			
	BLM 8.3.1	Net for a Pyramid	Student Support
	BLM 8.3.2	Net for a Prism	Student Support
	BLM A5	Problem Solving Checklist	Assessment
	BLM 8.3.3	Practice: Surface Area and Volume of Prisms and Pyramids	Practice
	BLM 8.3.4	Achievement Check Rubric	Assessment
8.4: Surface Area of a Cone			
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM 8.4.1	Practice: Surface Area of a Cone	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
8.5: Volume of a Cone			
	BLM A11	Group Work Assessment Recording Sheet	Assessment Group Work
	BLM 8.5.1	Practice: Volume of a Cone	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T3	Microsoft® *Excel*	Technology

	BLM	Title	Purpose
8.6: Surface Area of a Sphere			
	BLM G9	Centimetre Grid Paper	Student Support
	BLM 8.6.1	Practice: Surface Area of a Sphere	Practice
	BLM A1	Assessment Recording Sheet	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
8.7: Volume of a Sphere			
	BLM 8.7.1	Practice: Volume of a Sphere	Practice
	BLM A6	Knowledge/Understanding General Scoring Rubric	Assessment
	BLM 8.7.2	Achievement Check Rubric	Assessment
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
Chapter 8 Review			
	BLM 8.CR.1	Chapter 8 Review	Practice
Chapter 8 Practice Test			
	BLM 8.PT.1	Chapter 8 Practice Test	Diagnostic Assessment
	BLM 8.CT.1	Chapter 8 Test	Summative Assessment
Chapter 8 Problem Wrap-Up			
	BLM G10	Grid Paper	Student Support
	BLM G9	Centimetre Grid Paper	Student Support
	BLM 8.CP.1	Chapter 8 Problem Wrap-Up Rubric	Summative Assessment

Get Ready

Common Errors

• Some students may not show all their work or use proper form in their solutions.

R_x Have students show the formula that they are using. In the next line, have them show the values they are substituting into the formula, and then show the rest of their solution following the proper order of operations. Encourage students to use equals signs appropriately and show their solution flowing vertically as they move from line to line.

• Some students may need to be reminded to use proper units.

R_x Ensure that students have a clear understanding of the difference between linear units (e.g., m and cm) and square units (e.g., m^2 and cm^2). Use a visual representation to clarify the distinction. Have students discuss possible measurement situations and the units that would be used for each (e.g., installing new baseboard trim in the classroom, putting in floor tiles, painting walls, etc.).

Accommodations

Perceptual—Have students use diagrams and different forms of a formula when calculating dimensions.

Spatial—Encourage students to draw nets of the three-dimensional shapes to help them understand the formulas for surface area and volume.

Teaching Suggestions

• Have students work with a partner or in small groups.
• You may wish to assign the Calculate Perimeter and Circumference and the Apply Area Formulas sections as homework, since most students will recall these skills fairly readily.
• More class time may be necessary for the Calculate Surface Area and Volume section.
• You may wish to use **BLM 8.GR.1 Practice: Get Ready** as remediation or extra practice.
• Computer access is necessary for the Use *The Geometer's Sketchpad®* section. You may wish to use **BLM T4 *The Geometer's Sketchpad® 3*** or **BLM T5 *The Geometer's Sketchpad® 4*** to support this activity. If students are already familiar with *The Geometer's Sketchpad®* and have made significant use of it earlier in the course, less time will be necessary here. Alternatively, have students use the Student Edition of *The Geometer's Sketchpad®* and complete some of this section as homework.
• The OSAPAC (Ontario Software Acquisition Program Advisory Committee) has licensed the student edition of *The Geometer's Sketchpad®* for use at home by students. Make students aware of this opportunity.
• It may be useful to use a computer with a projection display to demonstrate the features of *The Geometer's Sketchpad®* used in this chapter that students may not be familiar with, such as the **Measure** menu.

Assessment

Assess student readiness to proceed by informal observation as students work on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 curriculum for this chapter. Student self-assessment is also an effective technique; you may wish to give students **BLM 8.GR.2 Get Ready Self-Assessment Checklist**, and have them place a checkmark beside topics in which they feel confident with the necessary skills. Remedial action can be taken in small groups or with a whole class skill review.