8.1 Apply the Pythagorean Theorem

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
418 to 425

Suggested Timing
80 min

Tools
- grid paper
- rulers
- an assortment of cardboard boxes

Technology Tools
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM G10 Grid Paper
BLM A8 Application General Scoring Rubric
BLM 8.1.1 Practice: Apply the Pythagorean Theorem
BLM A21 Opinion Piece Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☐ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.01 relate the geometric representation of the Pythagorean theorem and the algebraic representation $a^2 + b^2 = c^2$;

Operating With Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

Discuss with students that a square is a special rectangle, and that its area formula can be expressed as $A = s^2$, where s is the length of the side of the square. Review the area of a rectangle in the Get Ready section Apply Area Formulas.

Warm-Up

1. Calculate the area of a square with sides of length:
a) 5 cm **b)** 20 cm **c)** 1.2 m **d)** 24 m **e)** $\sqrt{3}$ cm **f)** $\sqrt{24}$ m

2. Calculate the following square roots (round to two decimal places where necessary):
a) $\sqrt{36}$ **b)** $\sqrt{121}$ **c)** $\sqrt{20}$ **d)** $\sqrt{1000}$ **e)** $\sqrt{7}$ **f)** $\sqrt{5.2}$

Warm-Up Answers

1. a) 25 cm^2 **b)** 400 cm^2 **c)** 1.44 m^2 **d)** 576 m^2 **e)** 3 cm^2 **f)** 24 m^2

2. a) 6 **b)** 11 **c)** 4.47 **d)** 31.62 **e)** 2.65 **f)** 2.28

Teaching Suggestions

- Students will have used the Pythagorean theorem previously. Discuss the photo and Pythagoras. (5 min)
- Assign the Investigate, Method 1. (5–10 min)
- You may wish to use **BLM G10 Grid Paper** to support this activity. Many students will recall the formula $c^2 = a^2 + b^2$, but fewer will be able to explain and use the formula properly. Some students will take longer than others to complete this activity. Have students share their results. You may wish to have some students share their results with the class using the blackboard or an overhead.
- Alternatively, you may want to do Method 2. Have students work with a partner for this activity. Direct students to the McGraw Hill Ryerson web site for the interactive proof. Go to http://www.mcgrawhill.ca/links/principles9. (10–20 min)
- Another approach would be to begin with pencil and paper and then demonstrate using *The Geometer's Sketchpad®*. You may wish to conduct the demonstration yourself, or ask a gifted student. (5–10 min for the pencil and paper Investigate, 10 min for the demonstration)

Common Errors

- Some students may misuse the Pythagorean formula $c^2 = a^2 + b^2$, letting c be the unknown side, even when it is not the hypotenuse.

R$_x$ Stress to students that the c value is always the length of the hypotenuse. Always have students start with the equation in the form $(\text{hypotenuse})^2 = \ldots$ and rearrange the formula as necessary to solve for the unknown.

- Some students may think that $c^2 = a^2 + b^2$ implies that $c = a + b$.

R$_x$ Use a numerical example, such as $5^2 = 3^2 + 4^2$ and point out that $5 \neq 3 + 4$.

- Some students may rely on measurement formulas without having a clear understanding of what a question is asking. This will be especially true for problems that require multi-step solutions.

R$_x$ Encourage students to draw a diagram to represent a given problem. Most students will more readily understand what is required if they can picture the problem. Encourage students to develop a problem-solving plan when tackling a problem that requires several steps.

Ongoing Assessment

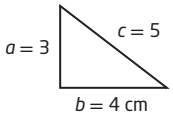
- Question 9, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

- Assign and discuss Examples 1 to 3.
- The OSAPAC (Ontario Software Acquisition Program Advisory Committee) has licensed the student edition of *The Geometer's Sketchpad*® for use at home by students. Make students aware of this opportunity.
- You may wish to use **BLM A8 Application General Scoring Rubric** to assist you in assessing your students.
- You may wish to use **BLM 8.1.1 Practice: Apply the Pythagorean Theorem** for remediation or extra practice.

Investigate Answers (pages 418–420)

Method 1

1. Answers will vary. Sample solution provided.



2. Answers will vary. 5 cm, 4 cm, and 3 cm

3. Answers will vary based on art from question 1.

a) $c^2 = 25$

b) $a^2 + b^2 = 3^2 + 4^2$
$= 25$

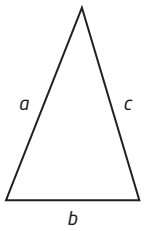
c) $a^2 + b^2 = c^2$

4. Answers will vary based on art from question 1.

a) 5 cm

b) This value is equal to the length of the hypotenuse.

5. No, the Pythagorean relationship does not hold with a non-right triangle.
No, the relationship does not hold true.



6. The Pythagorean theorem states that in a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides. In this activity, I constructed a right triangle and measured the lengths of the three sides. Next, I calculated the square of the length of the hypotenuse and the sum of the squares of the lengths of the two shorter sides and found these two values were equal. However, this relation did not hold true for a non-right triangle.

Method 2

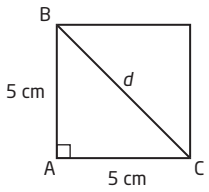
1–5. Answers will vary.

6. No, the Pythagorean relationship does not hold in a non-right triangle. No, the square of the longest side does not equal the sum of the squares of the other two sides.

7. Answers will vary. Sample solution: In this activity, I constructed right and non-right triangles, and saw through examples that the Pythagorean theorem holds in right triangles, but not in non-right triangles.

Communicate Your Understanding Responses (page 422)

C1.



Since BAC is a right triangle, I can apply the Pythagorean theorem to determine the length of the diagonal, or hypotenuse. Use $d^2 = AB^2 + AC^2$ with $AB = AC = 5$ cm to find the value of d.

Accommodations

Gifted and Enrichment—Challenge students to learn more about H.E. Dudeney, the Wheel of Theodorus, and Pythagoras, and to present their findings to the class. As an extra assignment, give students an opportunity to extend the Pythagorean triples (3, 4, 5; 5, 12, 13; 7, 24, 25, etc.) found in the Math Contest question in this section, and to look for as many patterns as they can find.

Visual—Encourage students to construct a triangle with sides 3 units, 4 units, and 5 units using paper and pencil, to draw unit squares on each of the sides of the triangle, and to count the number of unit squares on each side in order to understand the Pythagorean theorem.

Perceptual—Some students may have difficulty visualizing the three-dimensional shapes in the questions. Provide these students with diagrams or have them work together with a classmate to construct the diagrams.

Motor—Let students work with a partner or in small groups when they use technology to complete the Investigate, and allow them to use enlarged grid paper when working with paper and pencil to complete the questions in this section.

Student Success

Have students conduct an Internet search on the history of the Pythagorean theorem. Then, ask them to write a report answering the question, "Should the Pythagorean theorem be named for someone else?" You may wish to use **BLM A21 Opinion Piece Checklist** to assist you in assessing your students.

C2. Count the squares to find the base and the height of the right triangle formed by the dotted lines and line segment AB. Apply the Pythagorean theorem to find the length of AB.

$(AB)^2 = 4^2 + 3^2$

$(AB)^2 = 25$

$AB = 5$ units

C3.

Step 1: Apply the Pythagorean theorem to the triangle.

$5^2 + b^2 = 13^2$

Step 2: Solve the equation to find b.

$b^2 = 13^2 - 5^2$

$b = 12$ cm

Step 3: Substitute 12 cm for the base and 5 cm for the height into the formula for the area of a triangle.

$\text{Area} = \frac{1}{2} \times 12 \times 5$

$\text{Area} = 30 \text{ cm}^2$

Practise

Questions 1 to 3 are similar to Examples 1 to 3. Students should be able to model their solutions by reviewing the Examples.

Connect and Apply

For question 7, remind students that it is just the reverse of question 6, where they were given the side length and asked for the diagonal. Here they are given the diagonal and are asked for the side length. Some students might find their algebra skills useful here. Suggest to students that they label the unknown sides of the square with a single variable.

Extend

Some of these questions deal with three dimensions. Boxes may help students visualize three-dimensional problems. For example, in question 10, it is easier for students to demonstrate a space diagonal of a box using a model. Use a shoebox to model the room in question 11. Flatten the shoebox and use the net to show students the actual path that the spider crawls. (Note that the spider and the fly are on opposite end walls of the room. The visual might appear that the spider and fly are on adjacent walls.) Some students may be interested in researching more problems by H.E. Dudeney on the Internet. Direct students to http://thinks.com/puzzles/dudeney/dudeney.htm.

Literacy Connections

Word Origins

Have students choose two of the words in this section and research their word origins. Ask them to write a paragraph comparing their origins.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5
Typical	1–8
Extension	10–13