8.2 Perimeter and Area of Composite Figures

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
426 to 435

Suggested Timing
80 min

Tools
- tangrams, pattern blocks, or geoboards

Related Resources
BLM 8.2.1 Practice: Perimeter and Area of Composite Figures
BLM G5 Tangram
BLM 8.2.2 Achievement Check Rubric
BLM A9 Communication General Scoring Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☐ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume
MG2.03 solve problems involving the areas and perimeters of composite two-dimensional shapes (i.e., combinations of rectangles, triangles, parallelograms, trapezoids, and circles)

Operating With Exponents
NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

The Get Ready segments Calculate Perimeter and Circumference and Apply Area Formulas provide the needed skills for this section. Students who experience trouble with composite shapes may benefit from completing Get Ready questions 1 to 5 before starting this section. You may wish to use **BLM 8.2.1 Practice: Perimeter and Area of Composite Figures** as remediation or extra practice.

Warm-Up

Make copies of **BLM G5 Tangram** for each student. Tangrams are seven-piece puzzles of familiar composite shapes that originated in ancient China. The area of the finished tangram is the sum of the areas of the separate pieces. Have students use tangram sets, if available.

Discuss with students how the areas of the pieces are related to each other. For example, ask students, *If the area of the smaller triangle is one square unit, what are the areas of each of the other tangram pieces?* Have students create composite shapes with the tangram pieces, and calculate the areas of these figures. Have students share shapes using overhead tangram pieces, if they are available.

There are many Internet sites with tangram puzzles. You may wish to have students do an Internet search for such sites.

Alternatively, you may wish to use pattern blocks to generate the same type of discussion. One pattern block is assigned an area value, and then the areas of the remaining blocks are determined relative to it. Have students create composite shapes with the pattern blocks and calculate the areas of these figures.

Geoboards can also be used to introduce composite shapes. Construct a shape on an overhead geoboard, and ask students to explain how they would calculate the area of the figure.

Alternatively, you could ask students to create a composite shape with a given area (e.g., 12 square units) on the geoboard. Have students share their solutions with the class.

Common Errors

- Some students may include interior dimensions in their calculations when calculating the perimeter of composite shapes.

R_x Stress to students that the perimeter of a figure consists of outside measurements only. Usually these inner dimensions are shown as dotted lines on diagrams for this reason.

Ongoing Assessment

- Use Achievement Check question 13 to monitor student success. See Achievement Check Answers and **BLM 8.2.2 Achievement Check Rubric**.
- Question 8, the Chapter Problem question, can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students.

Teaching Suggestions

- Begin the Investigate by leading the class through some estimation techniques. Stress that they are not making detailed calculations at this point, but trying only to get a rough estimate. When estimating, students will begin to see the steps involved in completing the rest of the Investigate. Suggest that students pick their estimate of the cost of the patio from a range of values. For example, ask, *Will the cost be under $2000, $2000–$3000, $3000–$4000, $4000–$5000, or over $5000?*
- Assign the Investigate, and have students work with a partner or in small groups. (10–15 min) The main focus here is to apply their skills for determining perimeter and area to calculate the costs involved in building the patio. Follow up with a class discussion. Ask students, *Is it necessary to add 10% for waste?* Students may draw on their own experiences with similar projects at home.
- Assign Examples 1 and 2 and have students work with a partner. (5–10 min)
- Some students may use the trapezoid formula in Example 1, especially if Get Ready question 5b) involving a trapezoid has already been assigned. Encourage students to try both strategies. Either solution to Example 1 is acceptable.
- Discuss the Communicate Your Understanding questions. You may wish to use **BLM A9 Communication General Scoring Rubric** to assist you in assessing your students. (5–10 min)
- Assign Practise questions 1 and 2. (5–10 min)
- You may wish to use **BLM 8.2.1 Practice: Perimeter and Area of Composite Figures** for remediation or extra practice.

Investigate Answers (pages 426–427)

1. Answers will vary. $4000

2. a) The perimeter of the semicircle can be calculated using $\frac{1}{2} \times$ circumference of a circle. $\frac{1}{2} \times 2\pi r = \pi r$
The length of the two unlabelled sides of the triangles can be found using the Pythagorean theorem.
b) The perimeter of the round part is 9.4 m. The hypotenuse of the smaller triangle is 5.0 m. The hypotenuse of the larger triangle is 6.4 m.
c) The total perimeter is 34.8 m. The perimeter plus 10% is 38.3 m.

3. a) two triangles, one rectangle, and one semicircle
b) The dimensions of each of the simple shapes are already known. Use the area formula for each shape.
c) total area $= 54.1$ m^2, area plus 10% $= 59.5$ m^2

4. a) $3744.39
b) $4306.05
c) Answers will vary. $4000 is fairly close to $4306.05.

5. I can use the formulas for simple shapes to easily calculate the missing dimension and area of each shape. Then, these can be added together to find the perimeter and area of the complex figure.

Accommodations

Gifted and Enrichment—Challenge students to think of a business and design a logo for their business using computer design software. Have students research Leonardo of Pisa and Fibonacci numbers on the Internet.

Perceptual—Encourage students to colour-code the different shapes in the composite figures to the formulas that they are using to calculate a specific composite area.

Language—Let students use words instead of formulas when working with the formulas in this section.

Memory—Review with the students the steps required to calculate percents.

Student Success

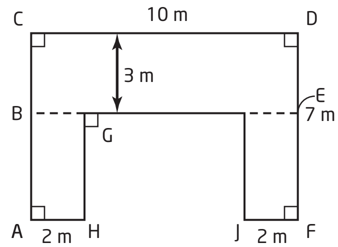
Have students design a room in their dream house, and write a report. Their report should include a fully labelled diagram, and all calculations involving construction and costs. Alternatively, students can construct a three-dimensional model of the room.

Communicate Your Understanding Responses (page 431)

C1. The area of the patio can be determined by adding the area of a trapezoid (formed by the two triangles and the rectangle) and a semicircle.

C2. a) Use known lengths and subtraction to find the lengths of the unknown sides.

b)



As shown in the figure, the yard can be divided into three rectangles. Add the areas of the three rectangles to find the area of the yard.

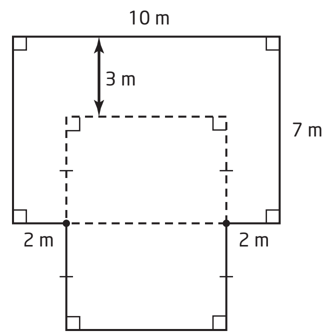
c) Subtract the area of rectangle HGIJ from the area of rectangle ACDF.

C3. a) Some of the sides of the rectangles do not belong to the perimeter of the yard.

b) The perimeter of this yard is greater than the perimeter of a yard that is 10 m by 7 m.

C4. a) Perimeters are the same.

b) Divide the yard into two rectangles and add their areas to determine the area of this yard.



c) The area of this yard is greater than the area of the yard in question C2.

Practise

Use the visuals provided for each of these questions to make it easier to discuss strategies with students and for students to work together to develop a strategy.

Questions 1c), d), and e) require the Pythagorean theorem to determine missing dimensions so that the perimeter can be determined. For example, in question 1c) students need to determine the length of the hypotenuse first and then calculate the perimeter. Encourage students to do this in two distinct steps.

When calculating areas, for example in question 2b), have students draw the two shapes separately (the rectangle and the isosceles triangle) that make up the composite shape. Then, they can calculate the area of each shape showing the appropriate formulas, and then calculate the sum of the areas, as shown in the examples. More students will be successful if they handle the solution in separate steps, so encourage this technique. If students choose to consider the sum of the areas from the start, ensure that they use proper mathematical form throughout their solutions.

Connect and Apply

For question 4, remind students that the arrow is painted on the front side of the sign only.

Question 11 is a revisit of the concepts of section 8.1. Students will need to use the given area to determine the length of the sides of the square.

In question 12, some students will determine the area of the frame by calculating the area of the four trapezoids. Another, more elegant, solution would be to subtract the area of the window from the area of the window and the frame together.

Achievement Check Answers (page 434)

13. a) Each section of the roof must be calculated separately:

$$A = 18 \times 9 + 2\left(\frac{(20 + 18)4.5}{2}\right) + 2\left(\frac{(11 + 9)4.5}{2}\right)$$
$$= 423$$

The area of the roof is 423 m^2.

b) If there were no waste, Susan would need 43 packages.

c) Answers will vary. One approach would be to add 10% (e.g., four packages). Another approach would be to treat each section as a rectangle to accommodate waste and round up to the nearest package for each.

Extend

For question 14, many students will solve the problem by applying the Pythagorean theorem to determine the lengths of the sides of the inner square. Encourage students to realize that the area of the inner square is half the area of the outer square. Drawing the diagonals of the inner square will divide the entire area into eight congruent triangles and help them see this relationship. Students who have completed question 14 will have more insight to help them with question 18.

For question 18, point out to students that the figure on the inside of the rectangle is a rhombus. The area of the rhombus is equal to the area of the shaded area. Students will be able to visualize this if the diagonals of the rhombus are constructed to create four congruent triangles. With this insight, an easy way to calculate the shaded area is to take half of the area of the rectangle. $\frac{1}{2}(80\ \text{cm}^2) = 40\ \text{cm}^2$

Literacy Connections

Word Origins II

Have students look up the meaning of and history behind the term *composite*. Ask, *Are there any other terms in this section for which you can find a historical reference?* Have students write a paragraph to compare the origins of any such terms.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–3, 11, 12
Typical	1–6, 10–12
Extension	14–18