8.3 Surface Area and Volume of Prisms and Pyramids

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
436 to 443

Suggested Timing
80–160 min

Tools
- empty 250-mL milk cartons
- construction paper
- scissors
- tape
- sand, rice, or other suitable materials
- Bristol board
- pyramid models
- interlocking cubes

Related Resources
BLM 8.3.1 Net for a Pyramid
BLM 8.3.2 Net for a Prism
BLM A5 Problem Solving Checklist
BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids
BLM 8.3.4 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.05 determine, through investigation, the relationship for calculating the surface area of a pyramid (e.g., use the net of a square based pyramid to determine that the surface area is the area of the square base plus the areas of the four congruent triangles);

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating With Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.83 by using a calculator]);

Link to Get Ready

The Get Ready segment Calculate Surface Area and Volume provides the needed skills for this section. Have students complete Get Ready questions 6 to 8 before starting this section.

Warm-Up

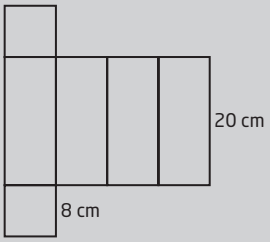
Construct a net for the following shapes:

1. a) Square-based prism with dimensions 8 cm by 8 cm by 20 cm

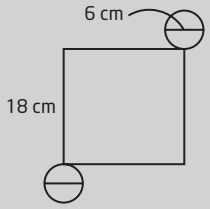
b) Cylinder with a height of 18 cm and a diameter of 6 cm

Warm-Up Answers

1. a) Square-based prism with dimensions 8 cm by 8 cm by 20 cm



b) Cylinder with a height of 18 cm and a diameter of 6 cm



Teaching Suggestions

- Assign the Investigate, and have students work in small groups.
- Have students discuss Part A question 1 in groups, then discuss question 2 as a class. Use **BLM 8.3.1 Net for a Pyramid** for this activity. Its dimensions and height are the same as a 250 mL milk carton, so it can be used in the investigation of the volume of a pyramid in part B.

Common Errors

- Some students may have difficulty distinguishing between the height of a pyramid and the slant height of its lateral sides.

R_x The three-dimensional visuals provided in the student book show the right triangle for most pyramids, and this will help students see the Pythagorean relationship that exists between the height and the slant height. Use three-dimensional models of pyramids wherever possible to help clarify these dimensions for students.

- Some students may use inappropriate units on answers to surface area and volume problems.

R_x Use interlocking cubes to model the volume of prisms. These manipulatives will allow students to see the interlocking cubes as units of volume and the surface area as the sum of the areas of the faces. When students can properly conceptualize surface area and volume, they will be less likely to use inappropriate units.

Ongoing Assessment

- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 8.3.4 Achievement Check Rubric**.
- Question 12, the Chapter Problem question, can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

- As an alternative to Part B, demonstrate for the class, using a milk carton with the top cut off in advance. (Discard the top; it is not used in the rest of the Investigate.) The object is to create a pyramid that has the same base and height as this remaining prism. Discuss with the class what dimensions are known (the length of the square base, and the height) and what dimensions will need to be determined to construct the net for the pyramid. They will need to calculate the slant height, which will be the height of the isosceles triangles in the net for the pyramid. You may want to have students measure the slant height of the pyramid formed from the net in Part A to verify these dimensions.
- You may wish to use **BLM 8.3.2 Net for a Prism** for Part B and have students construct the rectangular prism by using this net rather than an actual milk carton.
- To save time, you may want to construct the pyramid in advance to demonstrate with the class. If you use a heavy grade of paper such as Bristol board, the model will stand up better when filling it with sand or rice.
- Alternatively, three-dimensional plastic models of a prism and pyramid with identical square bases and heights could be used to demonstrate the relationship between their volumes. If such models are available, the investigation will be much faster, but will lack the hands-on approach of the Investigate. Depending on which approach is used, the Investigate could take anywhere from 10 minutes to a full hour.
- Review Examples 1 to 3. (10–20 min)
- You may wish to omit Example 1 if students complete the Investigate, however the pyramid at the Louvre is a wonderful example and may be worth a brief review.
- Spend extra time on Example 2, stressing the difference between the height of the pyramid and its slant height. Some students may have difficulty with this concept. Refer to pyramidal models in the classroom when discussing problems with pyramids.
- You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students.
- You may wish to use **BLM 8.3.3 Practice: Surface Area and Volume of Prisms and Pyramids** for remediation or extra practice.

Investigate Answers (pages 436–437)

Part A

1. a) Square; $(\text{side length})^2$

b) Triangle; $\frac{1}{2} \times \text{base} \times \text{height of triangle}$

c) Surface Area $= (\text{side length})^2 + 4\left(\frac{1}{2} \times \text{base} \times \text{height of triangle}\right)$
Surface Area $= s^2 + 2sh$

2. Surface area of a hexagon-based pyramid:
Area of the hexagon + 6 × Area of one lateral surface
Surface area of an octagon-based pyramid:
Area of the octagon + 8 × Area of one lateral surface
To find the surface area of any pyramid:
Step 1: Find the area of the base.
Step 2: Find the area of one lateral surface and multiply it by the number of sides.
Step 3: Add the areas from steps 1 and 2 together to obtain the total surface area of the pyramid.

Part B

1. Answers will vary.

2. Answers will vary. Sample answer: 3:1.

Accommodations

Gifted and Enrichment—Challenge students to prepare a research essay on the history and construction of the pyramids in Egypt.

Perceptual—Encourage students to draw diagrams when completing the questions in this section.

Spatial—Let students change three-dimensional shapes to two-dimensional nets by creating models of the shapes and unfolding them.

Memory—Review with the students the steps for multiplying or dividing by powers of 10. For instance when multiplying by 100, move the decimal two places to the right, and when dividing by 100, move the decimal two places to the left.

ESL—Allow students to use their dictionaries or translators to understand the meanings of the new words in this section.

Student Success

- Have students build an ongoing **journal** of sketches and formulae for three-dimensional shapes. Students can include worked examples of volume and surface area for each shape.
- Ask students to collect a **portfolio** of three-dimensional items and compute their volumes and surface areas (see the photo on the first page of Section 8.3 for a starting point). This portfolio should continue until the end of Section 8.7, and then be presented to the class.

3. a) Answers will vary.
b) approximately 3
c) Volume of the pyramid is about $\frac{1}{3}$ the volume of the prism.

4. Given the same base and height, the volume of the pyramid is $\frac{1}{3}$ the volume of the prism.

Communicate Your Understanding Responses (page 440)

C1. Answers will vary.
Alike: All three shapes have flat bases with sides of equal lengths, and equal heights. All the surfaces are flat.
Different: A and B are prisms but C is a pyramid.

C2. For A and B, multiply the area of the base by the height. For C, multiply the area of the base by the height and divide the result by 3.

C3. A has the greatest volume. The triangular prism's base has only half the area of the rectangular prism, and their heights are the same, so, the triangular prism's volume will be half of the rectangular prism's. The pyramid has the same base area and height as the rectangular prism, so we know the pyramid's volume will be one-third that of the prism.

C4. In each figure, find the area of each surface and add.

C5. For B: You need to find the length of the hypotenuse of the triangular base. Use the Pythagorean theorem to calculate this.
For C: You need to find the slant height of any one lateral surface. The slant height will be the hypotenuse of the right triangle where the other two sides are the perpendicular distance from the vertex of the pyramid to the middle of the base, and half the length of the base.

Practise

If you have covered the Investigate and Examples 1 to 3 in class, students should not have difficulty with questions 1 to 5.

Connect and Apply

Questions 6, 8, and 10 involve determining one of the dimensions of a prism or pyramid given the volume and another dimension. Students may need assistance with these at first. Remind students to rearrange the formulas to solve for the unknown. This will require algebra skills from previous chapters.

Achievement Check Answers (page 443)

14. Answers will vary
a) The box could have dimensions 20 cm by 20 cm by 20 cm (found by trial and error—three numbers whose product is 8000).
For the cylinder, start with a radius of 10 cm (this makes a diameter of 20 cm which is similar to the box).
The area of the base is 314.159…
Dividing the volume by the area of the base gives the height of about 25.46 cm.
b) The surface area of the prism is 6(20)(20) or 2400 cm^2.
The surface area for the cylinder is
$2(314.16) + 25.46(2)(\pi)(10)$ or 227 cm^3.
c) Answers will vary.
The cylinder would seem to be the better choice since it requires less material. Other choices for dimensions could lead to different conclusions.
Another consideration might be aesthetics, such as approximating the Golden ratio for one face's dimensions (i.e., side view). Boxes may be more efficient to pack in shipping containers since there are no gaps between boxes. Consumer research may indicate that one shape is preferred over the other.

Extend

In question 16, to calculate the surface area of the frustum, students may need some guidance. The easiest way to calculate the surface area is to find the areas of the four trapezoidal faces and then add the areas of the square top and bottom. Note that in question 16b), the bottom of the frustum is not painted and so this base should be excluded in the surface area calculation.

Literacy Connections

Word Origins III

Ask students, *Can you think of other situations in which we use the word* lateral*?* Have students find three words that have Latin origins, and three words that have Greek origins. Encourage them to find ways to use these words in their everyday conversations this week.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–5, 7
Typical	1–7, 9–11, 13
Extension	15–18