8.4 Surface Area of a Cone

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
444–450

Suggested Timing
80–160 min

Tools
- models of cones
- construction paper
- scissors
- rulers
- compasses
- tape

Technology Tools
- *The Geometer's Sketchpad®*
- computers

Related Resources
BLM A18 My Progress as a Problem Solver
BLM 8.4.1 Practice: Surface Area of a Cone
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating With Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

Link to Get Ready

The Get Ready sections Calculate Perimeter and Circumference and Apply Area Formulas, specifically for circles and surface area of cylinders, will review skills necessary for this section. Assign questions 2, 4b), and 6b) before starting this section, if they have not been completed earlier.

Warm-Up

Present students with models of cones (plastic or wood models, as well as some constructed from construction paper). Discuss what the lateral surface would look like if flattened into a net.

Have students construct different sectors of a circle. For example, using a circle with a radius of 10 cm, have them construct a $\frac{3}{4}$ sector of the circle, a semi-circle, and a $\frac{1}{4}$ sector of the circle. Then, have the students tape the radius edges together to form various cones.

Discuss the slant height of the cone. Point out to students that the radius of their sector has become the slant height of the cone formed. Compare the heights of the various cones and the radii of their bases. No actual measurements are necessary here, just a discussion of their relative sizes.

Use the Warm-Up as a lead-in to the Investigate. The cones created from the sectors of the circle with radius 10 cm are the same ones that will be constructed in the Investigate. This will lessen the time needed for the Investigate, and provide students with the opportunity to have some hands-on experience in forming cones.

Teaching Suggestions

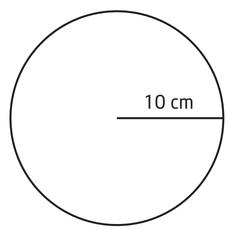
- Assign the Investigate. If the Warm-Up is used, less time may be needed. (15–20 min)
- Follow up on the Investigate with a class discussion. Using the class results from the Investigate, discuss with students that the lateral surface of the cone is proportional to the circumference of the cone. When less lateral surface area was used to form their cone, the circumference of the cone was smaller. For example, if a $\frac{3}{4}$ sector of the circle was used,

the circumference of the cone was $\frac{3}{4}$ the circumference of the original circle. If a semi-circle was used, the circumference of the cone was $\frac{1}{2}$ the circumference of the original circle. When a $\frac{1}{4}$ sector of the circle was used, the circumference of the cone that resulted was $\frac{1}{4}$ the circumference of the original circle.

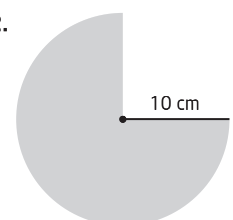
- With this experience with different cones, students are more likely to accept and understand the ratio concepts and the algebra shown in the discussion. Explain to students that they would not be expected to present this proportional reasoning on their own, but they should be able to follow the reasoning as you discuss it with them. The end result of the Investigate is the lateral surface area of a cone, πrs, where r is the radius of the cone and s is the slant height of the cone.
- Assign the Example. (5 min)
- Discuss the Communicate Your Understanding. (5–10 min)
- Assign the Practise questions.
- You may wish to use **BLM A18 My Progress as a Problem Solver** as a student self-assessment.
- You may wish to use **BLM 8.4.1 Practice: Surface Area of a Cone** for remediation or extra practice.

Investigate Answers (pages 444–445)

1.



2.



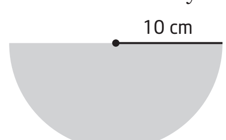
3. Answers will vary. The height should be about 6.61 cm and the radius should be about 7.5 cm.

4. $s = 10$ cm. The length of the third side is equal to the radius of the original circle.

5. 47.1 cm; $\frac{3}{4}$

6. $\frac{3}{4}$

7. a) Answers will vary.



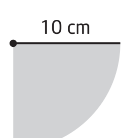
b) Step 3: Answers will vary. The height should be about 8.66 cm and the radius should be about 5 cm.

Step 4: The length of the third side, s, is equal to the radius of the original circle.

Step 5: 31.4 cm; $\frac{1}{2}$

Step 6: $\frac{1}{2}$

8. a) Answers will vary.



b) Step 3: Answers will vary. Height should be about 9.68 cm and the radius should be about 2.5 cm.

Step 4: The length of the third side, s, is equal to the radius of the original circle.

Common Errors

- Some students may continue to struggle with the concepts of slant height and height as they did with pyramids.

R_x By doing repeated calculations involving the slant height and height of the cone, students will realize that the slant height is the hypotenuse of the right triangle and the radius and height are the legs of the triangle. Students should begin to pick up on their own errors, recognizing that the slant height should always be longer than the height of the cone.

- Some students may have trouble deciding when the surface area of the cone should include the base and when the base should not be considered.

R_x When working on questions where the cone is presented without a context, students should include the base in the surface area calculation. However there may be cases with real-life scenarios where the base is not part of the cone, for example, a conical drinking cup or a conical pile of sand.

Ongoing Assessment

- Question 11, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

Accommodations

Visual—Give students a mathematical model of a cone to help them understand the concept of slant height.

Spatial—Encourage students to use paper to create mathematical models of the shapes in this section.

Language—Allow students to work with a partner when using technology to complete the questions in this section. Have one student be responsible for reading the instructions for using *The Geometer's Sketchpad®* aloud to their partner who will be using the software.

Memory—Let students use cue cards to memorize the formulas in this section.

Step 5: 15.7 cm; $\frac{1}{4}$
Step 6: $\frac{1}{4}$

9. Reflect: The ratio of the areas is the same as the ratio of the circumferences.

$$\frac{\text{Lateral area of cone}}{\text{Area of circle}} = \frac{\text{Circumference of cone}}{\text{Circumference of circle}}$$

Communicate Your Understanding Responses (page 447)

C1. Difference: The cone formed from the larger sector has the greater radius. The cone formed by the smaller sector has the greater height.
Similarity: Both cones have the same slant height.

C2. Difference: The cone formed from the sector of the larger circle has the greater radius, height, slant height, and circumference.
Similarity: The two cones are in proportion.

$$\frac{\text{Radius of cone 1}}{\text{Radius of cone 2}} = \frac{\text{slant height of cone 1}}{\text{slant height of cone 2}}$$

$$\frac{\text{Height of cone 1}}{\text{Height of cone 2}} = \frac{\text{circumference of cone 1}}{\text{circumference of cone 2}}$$

C3. No. When the slant height is doubled, the radius and the height must be increased proportionally by the Pythagorean theorem: $s^2 = r^2 + h^2$. So when s is doubled, $(2s)^2 = (2r)^2 + (2h)^2$. Then,

$$\begin{aligned} SA &= \pi(2r)^2 + \pi(2r)(2s) \\ &= 4\pi r^2 + 4\pi rs \\ &= 4(\pi r^2 + \pi rs) \end{aligned}$$

The surface area is actually quadrupled.

Practise

Students should be comfortable with the Practise questions after working through the Example together.

Connect and Apply

Students will quickly realize that the Pythagorean theorem is used in almost every question. Remind students that the surface area of a cone may or may not include the base of the cone. They must decide in each question if the base should be included, and they must learn to make their decision based on the context of the question.

In question 5, the lateral surface area and the radius are given, and students are required to solve for the slant height. This will require rearrangement of the formula. It may be helpful to do an example with the class to review the necessary algebra skills.

Questions similar to questions 6 and 7 have appeared in earlier sections. Many students will now be comfortable with the algebraic manipulation required when looking at the general case. Weaker students should be encouraged to try a particular cone and determine if the surface area doubles or not.

Questions 8 and 9 involve cones that just fit inside a cube or a cylinder. Explain to students that it will be assumed that the inside dimensions of the cube and cylinder are given. (For example, they may use 10 cm as the diameter and height of the cone. The outside dimensions of the cube would be slightly larger than 10 cm.)

Question 10 involving the frustum of a cone will be easier for students who completed question 16 involving the frustum of a pyramid in Section 8.3. In this case, the surface area calculation can be completed by subtracting the lateral surface area of the small cone that has been removed from the lateral surface area of the original large cone. Note that the surface area should include the top of the frustum (the base of the small cone that has been removed), and the base of the frustum (the base of the large cone).

Question 11, the Chapter Problem question, involves a frustum as well. Part a) lets the student decide whether the base of the frustum should be painted with the glaze. Some may argue that it should be to make the base weatherproof, but others may argue that a special glaze is not necessary on the bottom of the base of a birdbath. Either answer is acceptable.

Extend

Question 13 is an extension to question 8.

Question 14 is similar to question 5, but the re-arrangement of the lateral surface area formula is done in part a) before the known values have been substituted.

In question 15, the surface area of the volcano does not include the base.

If computers are available, students would benefit from completing question 16, which is a *The Geometer's Sketchpad*® activity. Remember that students have access to the Student Edition of *The Geometer's Sketchpad*®, so this activity could be completed at home by some students. You may wish to use **BLM *T4 The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Literacy Connections

Word Origins IV

Ask students, *Are there other situations in life in which you use the word* sector*?* Have students write a paragraph to explain one of the situations.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4
Typical	1–9, 12
Extension	13–16