8.5 Volume of a Cone

Strand:
Measurement and Geometry

Strand:
Number Sense and Algebra

Student Text Pages
451–456

Suggested Timing
80 min

Tools
- empty cylindrical cans (or plastic models of cylinder and cone)
- construction paper
- scissors
- tape
- sand, rice, or other suitable materials

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- Microsoft® *Excel*
- computers

Related Resources
BLM A11 Group Work Assessment Recording Sheet
BLM 8.5.1 Practice: Volume of a Cone
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM T3 Microsoft® *Excel*

Mathematical Processes Expectations Emphasis
- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Solving Problems Involving Perimeter, Area, Surface Area, and Volume

MG2.02 solve problems using the Pythagorean theorem, as required in applications (e.g., calculate the height of a cone, given the radius and the slant height, in order to determine the volume of the cone);

MG2.04 develop, through investigation (e.g., using concrete materials), the formulas for the volume of a pyramid, a cone, and a sphere (e.g., use three-dimensional figures to show that the volume of a pyramid [or cone] is the volume of a prism [or cylinder] with the same base and height, and therefore that $V_{\text{pyramid}} = \dfrac{V_{\text{prism}}}{3}$ or $V_{\text{pyramid}} = \dfrac{(\text{area of base})(\text{height})}{3}$;

MG2.06 solve problems involving the surface areas and volumes of prisms, pyramids, cylinders, cones, and spheres, including composite figures.

Operating with Exponents

NA1.01 substitute into and evaluate algebraic expressions involving exponents (i.e., evaluate expressions involving natural-number exponents with rational-number bases [e.g., evaluate $\left(\frac{3}{2}\right)^3$ by hand and 9.8^3 by using a calculator]);

Link to Get Ready

The Get Ready sections Apply Area Formulas and Calculate Surface Area and Volume provide the needed skills for this section. You may wish to have students complete the questions involving circles and cylinders before starting this section. Assign questions 4b), 6b), and 7, if they have not been completed yet.

Warm-Up

1. Determine the volume of each figure:
   a) square-based prism 5 cm by 5 cm by 18 cm
   b) square-based pyramid with a base length of 5 cm and a height of 18 cm
2. How do the volumes of the prism and pyramid in question 1 compare?
3. Determine the volume of the following:
   a) a cylinder with radius 6 cm and height 20 cm
   b) a cylinder with height of 4 m and diameter of 2 m

Warm-Up Answers

1. **a)** 450 cm^3 **b)** 150 cm^3
2. The volume of the pyramid is one-third the volume of the prism.
3. **a)** 2261.9 cm^3 **b)** 12.6 m^3

Common Errors

- Some students may experience trouble using the volume formula at first because of the fraction involved.

R_x Ensure that students realize that $V = \frac{1}{3}\pi r^2 h$ is the same as $V = \frac{\pi r^2 h}{3}$, is the same as $V = \frac{\pi}{3} r^2 h$. They may need help using their calculators at first. It may be helpful to use an overhead graphing calculator to demonstrate the different ways the values can be entered.

Ongoing Assessment

- Question 10, the Chapter Problem question, can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.

Accommodations

Gifted and Enrichment—Challenge students to create a cone shape and use it to determine the different geometrical shapes, such as a circle or an ellipse, that can be made by cutting the cone in different ways.

Visual—Let students work with a partner when using *The Geometer's Sketchpad®*.

Memory—Review with the students the Pythagorean theorem and the steps required to use a graphing calculator to do a series of calculations.

Teaching Suggestions

- Have students bring in empty cans from home to use for the Investigate. Have students work with a partner or in small groups. (10–15 min)
- You may wish to use **BLM A11 Group Work Assessment Recording Sheet** to assist you in assessing your students.
- Alternatively, you could demonstrate the Investigate for the class or use models of a cylinder and cone with the same base and height. Fill the models with water to demonstrate the volume relationship. (5–10 min)
- Review Examples 1 and 2. (5–10 min)
- Discuss the Communicate Your Understanding questions. (5 min)
- You may wish to use **BLM 8.5.1 Practice: Volume of a Cone** for remediation or extra practice.

Investigate Answers (page 451)

Answers will vary. Sample solution:

1. Radius of can is 10 cm and height is 20 cm.

2. a) $(\text{Slant height})^2 = 20^2 + 10^2$
Slant height = 22.36 cm

3. about 3

4. a) If a cone and a cylinder have the same height and the radius of their bases is also equal, then:
Volume of the cone = $\frac{1}{3}$ × volume of the cylinder

b) $V = \frac{1}{3}\pi r^2 h$

Communicate Your Understanding Responses (page 454)

C1. The volume of the cylinder is three times the volume of the cone. Create a paper cone and a paper cylinder with the same height and radius. Start filling the cone with rice and pouring it into the cylinder. Approximately three cones filled with rice, when poured into the cylinder, should fill the cylinder up.

C2. Doubling the height of a cone will double the volume of the cone.

C3. Doubling the radius of a cone quadruples the volume of the cone.

Practise

Practise questions 1, 2, and 4 are similar to the Examples, so students should have few problems completing these on their own.

Question 3 requires the use of the Pythagorean theorem to determine the height of the funnel given the slant height and radius.

Connect and Apply

In question 5, students should be reminded that when the cone "just fits" inside the cylinder, this means that they can assume the cylinder and cone have congruent bases. This question stresses the relationship between the volumes of a cylinder and a cone. Point out to students that it is not necessary to know the dimensions of the cylinder in a case like this. The volume of the cone will simply be $\frac{1}{3}$(volume of the cylinder) or $\frac{1}{3}(300)$, which is 100 cm^3.

Question 6 allows for some student originality.

Question 8 requires students to solve for the height given the volume and base radius. By now, most students should be comfortable with rearranging the equation to solve for the unknown. Students with weaker algebraic skills may need assistance. Once the equation is set up, encourage students to multiply by 3 to remove the fraction from the equation.

Question 9 will help students realize the different effects that the radius and height have on the volume. In question 9a), some students may predict that the volumes of the cones will be the same. After calculating the volumes in question 9b), students should understand the different roles of *r* and *h* in the volume formula.

Question 10, the Chapter Problem, is a revisit of the Chapter Problem from Section 8.4 (question 11). In this question, students are calculating the volume of the concrete in the same fountain that they calculated the surface area for earlier.

In question 11, students may need reminding that 1 L = 1000 cm^3. Question 11a) has students rearrange the formula before substituting, and then in question 11b), solve for the height, as in question 8.

Question 12 is similar to questions 8 and 11, but this time, students will solve for the radius. This requires a higher level of algebraic skills, since students must take the square root in order to determine the radius.

Extend

Question 13 examines the ratio of the volume of a cone as compared to the volume of a cube surrounding it. Stronger students will enjoy estimating this ratio and as an extension could determine this ratio in general.

$$\begin{aligned}\frac{V_{cone}}{V_{cube}} &= \frac{\frac{1}{3}\pi\left(\frac{s}{2}\right)^2 s}{s^3} \\ &= \frac{\frac{1}{12}\ \pi s^3}{s^3} \\ &= \frac{\pi}{12}\end{aligned}$$

Question 14 will extend the students' algebra skills. Have students represent the height with $2r$ in the volume formula and solve for r.

Question 15 has students using a graphing calculator, *The Geometer's Sketchpad*®, or spreadsheet software. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3**, **BLM T5 *The Geometer's Sketchpad*® 4**, or **BLM T3 Microsoft® *Excel*** to support this activity.

In question 16, when students substitute the height of 20 cm, the volume formula becomes $V = \frac{20}{3}\pi r^2$. You may wish to have students use a graphing calculator, consider the radius to be the variable x, and graph $y = \left(\frac{20}{3}\right)\pi x^2$. Discuss the fact that the quadratic relation that results is an example of a non-linear relation.

Exercise Guide

Category	Question Number
Minimum (essential questions for all students to cover the expectations)	1–4
Typical	1–9, 11, 12
Extension	13–17