CHAPTER

9

Optimizing Measurements

Vocabulary

optimization
maximum
minimum

Curriculum Expectations

Mathematical Process Expectations

Throughout this course, students will:

PROBLEM SOLVING

MPS.01 develop, select, apply, and compare a variety of problem-solving strategies as they pose and solve problems and conduct investigations, to help deepen their mathematical understanding;

REASONING AND PROVING

MPS.02 develop and apply reasoning skills (e.g., recognition of relationships, generalization through inductive reasoning, use of counter-examples) to make mathematical conjectures, assess conjectures, and justify conclusions, and plan and construct organized mathematical arguments;

REFLECTING

MPS.03 demonstrate that they are reflecting on and monitoring their thinking to help clarify their understanding as they complete an investigation or solve a problem (e.g., by assessing the effectiveness of strategies and processes used, by proposing alternative approaches, by judging the reasonableness of results, by verifying solutions);

SELECTING TOOLS AND COMPUTATIONAL STRATEGIES

MPS.04 select and use a variety of concrete, visual, and electronic learning tools and appropriate computational strategies to investigate mathematical ideas and to solve problems;

CONNECTING

MPS.05 make connections among mathematical concepts and procedures, and relate mathematical ideas to situations or phenomena drawn from other contexts (e.g., other curriculum areas, daily life, current events, art and culture, sports);

REPRESENTING

MPS.06 create a variety of representations of mathematical ideas (e.g., numeric, geometric, algebraic, graphical, pictorial representations; onscreen dynamic representations), connect and compare them, and select and apply the appropriate representations to solve problems;

COMMUNICATING

MPS.07 communicate mathematical thinking orally, visually, and in writing, using mathematical vocabulary and a variety of appropriate representations, and observing mathematical conventions.

Additional information and teaching materials for this chapter are available on the McGraw-Hill Ryerson web site at http://www.mcgrawhill.ca/books/principles9. You will need your password to access this material.

Strand:
Measurement and Geometry

Overall Expectations

By the end of this course, students will:

MGV.01 determine, through investigation, the optimal values of various measurements;

MGV.02 solve problems involving the measurements of two-dimensional shapes and the surface areas and volumes of three-dimensional figures;

MGV.03 verify, through investigation facilitated by dynamic geometry software, geometric properties and relationships involving two-dimensional shapes, and apply the results to solving problems.

Specific Expectations

Investigating the Optimal Values of Measurements

By the end of this chapter, students will:

MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;

MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;

MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Chapter Problem

The Chapter Problem focusses on packaging computer supplies for shipment to customers. Have students discuss their understanding of the topic, encouraging them to suggest factors that need to be considered when designing the best package for an item. The supplies need to be packaged so that they are not likely to be damaged during the shipping process. However, we also want to avoid "over-packaging" and use recyclable materials as much as possible to protect our environment.

You may wish to have students complete the Chapter Problem questions that occur throughout the chapter. These questions incorporate acquired skills as they are introduced in the sections where they occur. They are designed to help students move toward the Chapter Problem Wrap-Up on page 519.

Alternatively, you may wish to assign the Chapter Problem Wrap-Up when students have completed the chapter. The Chapter Problem Wrap-Up may be used as a summative assessment and is open-ended in nature to allow students originality in their solutions.

Chapter 9 Planning Chart

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 9 Opener • 15 min	474–475			
Get Ready • 80 min	476–477	• BLM 9.GR.1 Practice: Ready	• BLM 9 GR.2 Get Ready Self-Assessment Checklist	
9.1 Investigate Measurement Concepts • 80–160 min	478–483	• BLM 9.1.1 Rectangle Data Recording Table • BLM G10 Grid Paper • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.1.2 Practice: Investigate Measurement Concepts	• BLM A11 Group Work Assessment Recording Sheet • BLM A17 Teamwork Self Assessment • BLM A3 Portfolio Checklist	**Tools** • geoboards • toothpicks • elastic bands • grid paper **Technology Tools** • *The Geometer's Sketchpad*® • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.2 Perimeter and Area Relationships of a Rectangle • 80 min	484–490	• BLM G10 Grid Paper • BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle • BLM T4 *The Geometer's Sketchpad*® 3 • BLM T5 *The Geometer's Sketchpad*® 4 • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel*	• BLM 9.2.2 Achievement Check Rubric	**Tools** • toothpicks • geoboard • elastics • grid paper **Technology Tools** • graphing calculators • *The Geometer's Sketchpad*® • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.3 Minimize the Surface Area of a Square-Based Prism • 80–160 min	491–497	• BLM 9.3.1 Square-Based Prism Data Recording Table • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.3.2 Practice: Minimize the Surface Area of a Square-Based Prims • BLM G8 Isometric Dot Paper	• BLM A5 Problem Solving Checklist	**Tools** • interlocking cubes • isometric dot paper **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.4 Maximize the Volume of a Square-Based Prism • 80–160 min	498–503	• BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method • BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism • BLM G10 Grid Paper	• BLM 9.4.3 Achievement Check Rubric • BLM A21 Opinion Piece Checklist • BLM A4 Presentation Checklist	**Tools** • grid paper **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • *The Geometer's Sketchpad*® • computers
9.5 Maximize the Volume of a Cylinder • 80 min	504–509	• BLM 9.5.1 Cylinder Data Recording Table • BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.5.2 Practice: Maximize the Volume of a Cylinder	• BLM A18 My Progress as a Problem Solver	**Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
9.6 Minimize the Surface Area of a Cylinder • 80 min	510–515	• BLM T1 Corel® *Quattro Pro*® 8 • BLM T2 Corel® *Quattro Pro*® 10 • BLM T3 Microsoft® *Excel* • BLM 9.6.1 Practice: Minimize the Surface Area of a Cylinder	• BLM 9.6.2 Achievement Check Rubric • BLM A23 News Report Checklist • BLM A4 Presentation Checklist	**Tools** • construction paper • rulers • scissors • tape **Technology Tools** • Corel® *Quattro Pro*® • Microsoft® *Excel* • computers
Chapter 9 Review • 80 min	516–517	• BLM 9.CR.1 Chapter 9 Review		

Section Suggested Timing	Student Text Page (s)	Teacher's Resource Blackline Masters	Assessment	Tools
Chapter 9 Practice Test • 60 min	518–519		• BLM 9.PT.1 Chapter 9 Practice Test • BLM 9.CT.1 Chapter 9 Test	
Chapter 9 Problem Wrap-Up • 30–60 min	519		• BLM 9.CP.1 Chapter 9 Problem Wrap-Up Rubric	
Chapters 7 to 9 Review • 80 min	520–521		• BLM A14 Self-Assessment Recording Sheet • BLM A15 Self-Assessment Checklist	
Task: The Horse Barn • 20 min	522		• BLM 9.T1.1 Task: The Horse Barn Rubric	
Task: The Ice Rink • 20 min	523	• BLM G10 Grid Paper	• BLM 9.T2.1 Task The Ice Rink Rubric	**Tools** • grid paper
Task: Packing Compressed Air • 20 min	523		• BLM 9.T3.1 Task: Packing Compressed Air Rubric	

Chapter 9 Blackline Masters Checklist

	BLM	Title	Purpose
Get Ready			
	BLM 9.GR.1	Practice: Get Ready	Practice
	BLM 9.GR.2	Get Ready Self-Assessment Checklist	Student Self-Assessment
9.1 Investigate Measurement Concepts			
	BLM G10	Grid Paper	Student Support
	BLM 9.1.1	Rectangle Data Recording Table	Student Support
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.1.2	Practice: Investigate Measurement Concepts	Practice
	BLM A11	Group Work Assessment Recording Sheet	Assessment Group Work
	BLM A17	Teamwork Self Assessment	Assessment Group Work
	BLM A3	Portfolio Checklist	Assessment
9.2 Perimeter and Area Relationships of a Rectangle			
	BLM G10	Grid Paper	Student Support
	BLM 9.2.1	Practice: Perimeter and Area Relationships of a Rectangle	Practice
	BLM T4	*The Geometer's Sketchpad*® 3	Technology
	BLM T5	*The Geometer's Sketchpad*® 4	Technology
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.2.2	Achievement Check Rubric	Assessment
9.3 Minimize the Surface Area of a Square-Based Prism			
	BLM 9.3.1	Square-Based Prism Data Recording Table	Student Support
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.3.1	Practice: Minimize the Surface Area of a Square-Based Prism	Practice
	BLM G8	Isometric Dot Paper	Student Support
	BLM A5	Problem Solving Checkist	Assessment

	BLM	Title	Purpose
9.4 Maximize the Volume of a Square-Based Prism			
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.4.1	Investigate: *The Geometer's Sketchpad*® Method	Student Support Technology
	BLM 9.4.2	Practice: Maximize the Volume of a Square-Based Prism	Practice
	BLM G10	Grid Paper	Student Support
	BLM 9.4.3	Achievement Check Rubric	Assessment
	BLM A21	Opinion Piece Checklist	Assessment Literacy
	BLM A4	Presentation Checklist	Assessment
9.5 Maximize the Volume of a Cylinder			
	BLM 9.5.1	Cylinder Data Recording Sheet	Student Support
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM A18	My Progress as a Problem Solver	Student Self-Assessment
	BLM 9.5.2	Practice: Maximize the Volume of a Cylinder	Practice
9.6 Minimize the Surface Area of a Cylinder			
	BLM T1	Corel® *Quattro Pro*® 8	Technology
	BLM T2	Corel® *Quattro Pro*® 10	Technology
	BLM T3	Microsoft *Excel*®	Technology
	BLM 9.6.1	Practice: Minimize the Surface Area of a Cylinder	Practice
	BLM 9.6.2	Achievement Check Rubric	Assessment
	BLM A23	News Report Checklist	Assessment Literacy
	BLM A4	Presentation Checklist	Assessment
Chapter 9 Review			
	BLM 9.CR.1	Chapter 9 Review	Review
Chapter 9 Practice Test			
	BLM 9.PT.1	Chapter 9 Practice Test	Diagnostic Assessment
	BLM 9.CT.1	Chapter 9 Test	Summative Assessment
Chapter 9 Problem Wrap-Up			
	BLM 9.CP.1	Chapter 9 Problem Wrap-Up Rubric	Summative Assessment

	BLM	Title	Purpose
Chapters 7 to 9 Review			
	BLM A14	Self-Assessment Recording Sheet	Student Self-Assessment
	BLM A15	Student Self-Assessment Checklist	Student Self-Assessment
Task: The Horse Barn			
	BLM 9.T1.1	Task: The Horse Barn Rubric	Summative Assessment
Task: The Ice Rink			
	BLM G10	Grid Paper	Student Support
	BLM 9.T2.1	Task: The Ice Rink Rubric	Summative Assessment
Task: Packing Compressed Air			
	BLM 9.T3.1	Task: Packing Compressed Air Rubric	Summative Assessment

Get Ready

Student Text Pages
476 to 477

Suggested Timing
80 min

Related Resources
BLM 9.GR.1 Practice: Get Ready

BLM 9.GR.2 Get Ready Self-Assessment Checklist

Common Errors

- Some students may use improper units in their answers.

R_x Ensure that students have a clear understanding of the difference among linear units (m, cm) square units (m^2, cm^2), and cubic units (m^3, cm^3). Use a visual representation to clarify the difference in these units. For example, use interlocking cubes, and discuss the length of an edge, the area of a face, and the volume of a shape.

- Some students may need to be reminded to use proper form in their solutions.

R_x Have students show the formula they are using, in the next line show the values they are substituting into the formula, and then the rest of their solution following the proper order of operations. Encourage students to use equal signs appropriately and show their solution flowing vertically as they move from line to line. Concluding statements should be made, especially if there is a context provided in the question.

Accommodations

Visual—Provide visual clues for the students such as colour-coding the variables in the formulas to match the related dimensions on a diagram of the shape.

Perceptual—Allow students to use a scientific calculator when calculating answers.

Memory—Let students use the "formula cue-cards" that they have created while working their way through the textbook.

Teaching Suggestions

- Have students work with a partner or in a small group.
- The Get Ready segment Measurement Concepts: Perimeter, Circumference, Area, Surface Area, and Volume reviews the measurement skills developed in Chapter 8. This chapter incorporates these same skills in optimization problems, so it is important that skills from Chapter 8 have been properly developed.
- The questions in this section could be assigned on an individual basis as required. You may also wish to use **BLM 9.GR.1 Practice: Get Ready** for remediation or extra practice.
- If you are confident that your students have the necessary skills from Chapter 8, you may wish to assign the Compare Figures segment. This section provides a preview of the rest of this chapter. Students calculate the surface area and volume of two containers and make comparisons. After students have completed questions 5 and 6, ask students which container they would recommend and why. A discussion is a perfect lead-in to this chapter.
- All **BLMs** referred to throughout this chapter can be found in the Principles of Mathematics 9 Teacher's Resource CD-ROM.

Assessment

Assess student readiness to proceed by informal observation while students are working on the exercises. A formal test would be inappropriate since this material is not part of the grade 9 Curriculum for this chapter. Student self-assessment is also an effective technique. Using **BLM 9.GR.2 Get Ready Self-Assessment Checklist**, have students place a checkmark beside topics in the Get Ready in which they feel confident with the necessary skills. Take remedial action in small groups or with a class skill review.