9.2 Perimeter and Area Relationships of a Rectangle

Strand:
Measurement and Geometry

Student Text Pages
484 to 490

Suggested Timing
80 minutes

Tools
- toothpicks
- grid paper
- geoboard
- elastics

Technology Tools
- graphing calculators
- *The Geometer's Sketchpad®*
- Corel® *Quattro Pro®*
- Microsoft® *Excel*
- computers

Related Resources
BLM G10 Grid Paper
BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle
BLM T4 *The Geometer's Sketchpad®* 3
BLM T5 *The Geometer's Sketchpad®* 4
BLM T1 Corel® *Quattro Pro®* 8
BLM T2 Corel® *Quattro Pro®* 10
BLM T3 Microsoft® *Excel*
BLM 9.2.2 Achievement Check Rubric

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.01 determine the maximum area of a rectangle with a given perimeter by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, toothpicks, a pre-made dynamic geometry sketch), and by examining various values of the area as the side lengths change and the perimeter remains constant;

MG1.02 determine the minimum perimeter of a rectangle with a given area by constructing a variety of rectangles, using a variety of tools (e.g., geoboards, graph paper, a premade dynamic geometry sketch), and by examining various values of the side lengths and the perimeter as the area stays constant;

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

This section involves perimeter and area of rectangles. Assign Get Ready question 1 before starting this section.

Warm-Up

Have students construct a rectangle with a perimeter of 24 units, using grid paper, a geoboard, and elastics or toothpicks. Use **BLM G10 Grid Paper** for this activity. Have students determine the area of each rectangle they construct.

Teaching Suggestions

- Review graphing using the lists on a graphing calculator and by entering a function. Use a linear example that would review skills in previous chapters, or a non-linear example that will be similar to the quadratic results in the Investigate.
- Have students work with a partner in or small groups on the Investigate A: Method 1. (5–10 min)
- In Method 2, students enter the width and the area of the rectangles into L1 and L2 on a graphing calculator. The resulting graph of area versus width is a quadratic relationship. (10 min)
- Investigate B involves a rectangle enclosed on just three sides. Have students work with a partner. This Investigate illustrates that a square will not necessarily enclose the maximum area for a given perimeter. (10 min)
- Follow up with a class discussion.
- Complete the Communicate Your Understanding questions as a class.
- You may wish to use **BLM 9.2.1 Practice: Perimeter and Area Relationships of a Rectangle** for remediation or extra practice.

Common Errors

- Some students may think that a square will always produce the maximum area for a given perimeter.

R_x It is important that all students carry out investigations where the shape is not enclosed on all sides. Investigate B should be done by all students for this reason. It should be pointed out that the optimal shape depends on how many sides are enclosed.

Ongoing Assessment

- Use Achievement Check question 12 to monitor student success. See the suggested solutions and **BLM 9.2.2 Achievement Check Rubric**.
- Chapter Problem question 7 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

Investigate Answers (page 484)

A: Method 1

1. See measurements in table in Q.2.

2.

Rectangle	Width (m)	Length (m)	Perimeter (m)	Area (m^2)
1	1	15	32	15
2	2	14	32	28
3	3	13	32	39
4	4	12	32	48
5	5	11	32	55
6	6	10	32	60
7	7	9	32	63
8	8	8	32	64

3. a) 8 m × 8 m **b)** 64 m^2 **c)** square

4. a) Answers will vary. Possible answer: 10 m × 10 m
c) Answers will vary. Possible answer: The result matches the prediction.

5. Answers will vary. Divide the perimeter by 4 to get the length of each side.

$$\frac{\text{Perimeter}}{4} = \text{side length}$$

6. a) $\frac{60}{4}$, or 15 m

b) $\frac{30}{4} = 7.5$. In this case, we have a decimal number. To create a rectangle with this side length, two toothpicks will have to be broken into halves.

A: Method 2

3. a) $P = 2 \times (\ell + w) = 32$ m
$(\ell + w) = 16$
$\ell = 16 - w$
Let $w = x$
$\ell = (16 - x)$

b) $A = w \times \ell$
$A = x \times (16 - x)$
$A = 16x - x^2$

4. The graph of $x(16 - x)$ overlaps the scatter plot graph created in question 1. However, in this graph, x also assumes negative values and passes through origin.

5. a) $x = 8$, $y = 64$

b) width is 8 m. This point is at the top or the peak of the curve.

6. a) length is now $(20 - x)$. The point of maximum area: (10, 100)

b) Square; Yes; 10 m × 10 m

7. a) Answers will vary. 11.25 m × 11.25 m

b) Answers will vary. Yes.

8. Divide the perimeter by 4 to get the length of each side and square the number to get the area.

$$\frac{\text{Perimeter}}{4} = \text{side length}$$

$$\text{Area} = (\text{side length})^2$$

B:

1. a) Answers will vary. He will be able to enclose a greater area.

b) Answers will vary. The shape will be a rectangle.

c) Answers will vary. The dimension of the side opposite the hedge will be twice as long as the other two sides.

Accommodations

Gifted and Enrichment—Challenge students to create extra Math Contest questions and to extend question 15 to find the dimensions of other shapes of maximum area that can be inscribed in a circle of radius 10 cm.

Visual—Let students use technology to complete the questions in this section.

Perceptual—Encourage students to draw a diagram when solving the questions in this section.

Memory—Review with the students the steps required to create a scatter plot using a graphing calculator.

Student Success

Have students do an **Internet search** for real-life examples of where the concepts of this section are/could be used. Students can then share their information using the **timed retell** strategy.

2.

Rectangle	Width (m)	Length (m)	Sum of Length of Three Sides (m)	Area (m^2)
1	1	30	32	30
2	2	28	32	56
3	3	26	32	78
4	4	24	32	96
5	5	22	32	110
6	6	20	32	120
7	7	18	32	126
8	8	16	32	128
9	9	14	32	126
10	10	12	32	120
11	11	10	32	110
12	12	8	32	96
13	13	6	32	78
14	14	4	32	56
15	15	2	32	30

3. a) 8 m × 16 m

b) Answers will vary. Sample solution: The result is consistent with the hypothesis.

c) The hedge will allow Brandon to enclose twice as much space.

4. $\ell = 2w$

5. a) Answers will vary. Sample solution: 10 m × 20 m

b) Answers will vary.

6. Divide the sum by 4 to get the dimension of the width; the length is twice the width.

$$\text{Width} = \frac{\text{sum of the three sides}}{4}$$

$$\text{Length} = 2 \times \text{width}$$

Communicate Your Understanding Responses (page 487)

C1. Create a square field. Use a hedge or a wall to fence one side of the field.

C2. a) When all four edges need to be built/fenced.

b) When three or fewer sides need to be fenced (i.e. one or more sides are already fenced/built).

C3. $\ell = 2w$

Connect and Apply

Question 2 is similar to the Investigate. Point out to students that the amount of light entering the room is dependent on the area of the window.

In question 5, have students focus on the number of pieces of fencing. Use toothpicks to model the pieces. Once the students determine that a square is the best shape, and they should calculate the actual length of each side using the fact that each piece is 2.8 m.

Question 6 is similar to Investigate B since the corral is fenced on only three sides. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3 or BLM T5 *The Geometer's Sketchpad*® 4** to support question 6.

The Chapter Problem (question 7) involves enclosing only two sides of a rectangle.

Question 8 gives students the opportunity to consolidate the results of the various investigations they have carried out (enclosing a rectangle on four, three, and two sides).

Question 9 involves using a table or a spreadsheet to record results. This question is similar to question 6 and Investigate B. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.

The Achievement Check question will be easier for students who have already completed Chapter Problem question 7. You may wish to use **BLM 9.2.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 490)

12. a) The maximum area is 9 m^2. The optimal dimensions are 3 m by 3 m, a square.

b) Again, complete a table from diagrams on paper or using manipulatives:

Rectangle	Width (m)	Length (m)	Fence Used (m)	Area (m^2)
1	1	10	12	10
2	2	8	12	16
3	3	6	12	18
4	4	4	12	16
5	5	2	12	10
6	6	0	12	0

The maximum area is 18 m^2 and this occurs when the dimensions are 3 m by 6 m.

c) The maximum area is 36 m^2 and this occurs when the dimensions are 6 m by 6 m.

Extend

Questions 13 and 14 involve minimizing the perimeter for a fixed area.

The Geometer's Sketchpad® could be used for the investigation in question 15. You may wish to use **BLM T4 *The Geometer's Sketchpad*® 3** or **BLM T5 *The Geometer's Sketchpad*® 4** to support this activity.

Math Contest question 16 requires the application of the Pythagorean theorem. The diagonal of the inscribed square is the diameter of the circle. Solving for the length of the side of the square will require good algebraic skills.

Students who have completed question 7 in Section 9.1 will be more likely to think about a circle as a possibility in question 17. If students have not been assigned this question from the previous section, they may need to be encouraged to think beyond rectangular shapes. Suggest they consider other polygons, for example a hexagon or a dodecagon. This should lead them to realize that a circle is possible.

Exercise Guide

Category	Question Number
Minimum	1, 2, 4, 5, 10
Typical	1–6, 8–11
Extension	13–18