9.3 Minimize the Surface Area of a Square-Based Prism

Strand:
Measurement and Geometry

Student Text Pages
491 to 497

Suggested Timing
80–160 min

Tools
- interlocking cubes
- isometric dot paper

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM 9.3.1 Square-Based Prism Data Recording Table
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.3.2 Practice: Minimize the Surface Area of a Square-Based Prism
BLM G8 Isometric Dot Paper
BLM A5 Problem Solving Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

Get Ready questions 3 and 4 review the surface area and volume of prisms that are necessary skills for this section.

Warm-Up

Have students use interlocking cubes to form a prism with a volume of 16 cubic units. Students could share their models with the class to illustrate the different shapes that are possible. Ask students which of these prisms are "square-based." (This section only deals with square-based prisms and students need to get accustomed to this.) Have students predict which shape would have the least surface area. (Surface area calculations are not necessary at this point. These calculations can be left to the Investigate.) This will provide an excellent lead-in to the Investigate, where they determine the shape with the least surface area.

Teaching Suggestions

- Have students work with a partner or in small groups to complete the Investigate. You may wish to use **BLM 9.3.1 Square-Based Prism Data Recording Table** for the Investigate.
- Method 1 uses interlocking cubes and records the results in a table. If interlocking cubes are not available, use cube- or rectangular prism-shaped wrapped candy, such as Jolly Rancher, Starburst, or caramels. Be sure to check for students' allergies before introducing any food into the classroom. (10–15 min)
- Method 2 is similar, but uses a spreadsheet with formulas. If computers are available, this is a good opportunity to have students use a spreadsheet. The technology is advantageous because it allows students to investigate dimensions other than whole numbers. Instruct students to save their spreadsheet for future use in the exercise questions. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity. (20 min)
- Review Examples 1 and 2. (10–15 min) Students may need considerable

Common Errors

- Some students may need assistance in visualizing prisms involved in this section.

R_x Give students many opportunities to work with models for the prisms. Interlocking cubes and empty boxes, such as cereal boxes, juice boxes, tissue boxes, and other cardboard cartons, are very useful for modelling the prisms. Be sure to check for students' allergies before introducing any food containers into the classroom. Students will have more success in understanding the concepts of surface area and volume with these models available. It will also help them understand the practical limitations involved in packaging. (Would a cube-shaped 2-L milk carton be easy to hold?) The models also can be used to examine the extra material required for flaps.

- Some students may have trouble drawing three-dimensional sketches.

R_x Use isometric dot paper, such as **BLM G8 Isometric Dot Paper** to make it easier for students to draw in three dimensions.

Ongoing Assessment

- Chapter Problem question 6 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

help in finding the cube root of a number as required in Example 1. Students who completed some of the more challenging questions from the last chapter (for example, determine the radius of a sphere given its volume) will have better skills in this area.

- Discuss the Communicate Your Understanding questions. Question C2 emphasizes that minimizing surface area is not always practical. (For example, a cube-shaped cereal box would be difficult for a young child to handle.) (5–10 min)
- Assign Practise questions 1 to 4. (10–15 min)
- You may wish to use **BLM 9.3.1 Practice: Minimize the Surface Area of a Square-Based Prism** for remediation or extra practice.

Investigate Answers (page 491)

Method 1

1.

Length	Width	Height	Volume	Surface Area
1	1	16	16	66
2	2	4	16	40
4	4	1	16	48

2. Volume of the prism is 16 cubic units. If the prism has a base with three cubes on each side, the area of the base will be 9 square units. In this case, for the volume to be 16 cubic units, the height of this prism will have to 1.78 units (a fraction). Since, interlocking cubes are whole and cannot be split into fractions, it is not possible to build a prism with this volume that has a square base with three cubes on each side. Below is a set of equation which explains the problem:

Volume = length × width × height

$16 = 3 \times 3 \times \text{height}$

$\frac{16}{9} = \text{height}$

$1.78 = \text{height}$ (which is not possible)

3. The figure with dimensions $2 \times 2 \times 4$ has the minimum surface area. Its shape is the closest to a cube.

4. a) Answers will vary. 4 cubes × 4 cubes × 4 cubes

b)

Length	Width	Height	Volume	Surface Area
1	1	64	64	258
2	2	16	64	136
4	4	4	64	96
8	8	1	64	160

5. 27 cubes are used: 3 cubes × 3 cubes × 3 cubes, 125 cubes are used: 5 cubes × 5 cubes × 5 cubes

6. For a given volume, the square based prism with the minimum surface area is formed when length = width = height.

Method 2

2. Area of square base = $(\text{side length of square base})^2$

3. Consider the following steps:

Volume = length × width × height

length = width (because the base is square)

Volume = $(\text{length})^2 \times \text{height}$

Also,

$(\text{length})^2 = (\text{area of the square base})$

$\text{Volume} = (\text{area of the square base}) \times \text{height}$

$64 = (\text{area of the square base}) \times \text{height}$

$\frac{64}{(\text{area of the square base})} = \text{height}$

4. Surface Area of any rectangular prism =
$2 \times (\text{length} \times \text{width} + \text{width} \times \text{height} + \text{height} \times \text{length})$

$SA = 2 \times [(\ell \times w) + (w \times h) + (h \times \ell)]$
$\ell = w$ (because the base is square)
$SA = 2 \times [(\ell^2) + (\ell \times h) + (h \times \ell)]$
$SA = 2 \times (\ell^2) + (4 \times h \times \ell)$
$SA = (2 \times \text{area of the base}) + (4 \times \text{height} \times \text{side length})$
So, $SA = 2*(\text{column B}) + 4*(\text{column A})*(\text{column C})$

5.

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.064	64	.0258
2	4	.016	64	.0136
3	9	7.111	64	103.332
4	16	.04	64	.096
5	25	2.56	64	101.2
6	36	1.778	64	114.67
7	49	1.306	64	134.57
8	64	.01	64	.016

6. The 4th prism; its dimensions are $4 \times 4 \times 4$. The shape is a cube.

7. a) 5 cm × 5 cm × 5 cm

b)

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.0125	125	.0502
2	4	31.25	125	.0258
3	9	13.889	125	184.667
4	16	7.813	125	.0157
5	25	.05	125	.0150
6	36	3.472	125	155.333
7	49	2.551	125	169.429
8	64	1.953	125	190.5
9	81	1.543	125	217.556
10	100	1.25	125	.0250

8. a) Answers will vary. Possible answer: 6.7 cm

b)

Side Length of Square Base (cm)	Area of Square Base (cm^2)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	1	.0300	300	.01202
2	4	.075	300	.0608
3	9	33.333	300	.0418
4	16	18.75	300	.0332
5	25	.012	300	.0290
6	36	8.333	300	.0272
7	49	6.122	300	269.429
8	64	4.689	300	.0278
9	81	3.704	300	295.333
10	100	.03	300	.0320

7 cm × 7 cm × 6.122 cm

c) 6.7 cm × 6.7 cm × 6.7 cm. The shape is a cube.

9. To get the minimum surface area of a square-based prism with a given volume, equate the length, width, and height.

Accommodations

Gifted and Enrichment—Challenge students to calculate third roots, fourth roots, and fifth roots of different numbers using a calculator.

Perceptual—Allow students to use manipulatives or a spreadsheet when working through the questions in this section.

Language—Provide verbal clues for the students as they work through the questions in this section so that they can visualize and record the shapes for the questions in this section.

Memory—Encourage students to use a calculator to determine cube roots.

Student Success

Have students use technology extensively in Sections 9.3 through 9.6.

Communicate Your Understanding Responses (page 495)

C1. Answers will vary. The surface area must be minimized when designing packages and containers to save on heat loss and minimize packaging costs.

C2. Answers will vary. The products may not be cube shaped, they may be long rectangles, and a cube box would be inefficient. Sometimes, a cube-shaped container does not look attractive. Hence, not all boxes are cube-shaped. For example, when creating laptop packaging boxes.

Practise

If the Investigate and Examples 1 and 2 have been covered in class, students should not have difficulty with Practise questions 1–4.

Question 4 involves minimizing heat loss similar to Example 2.

Connect and Apply

In both questions 5 and 9, explain to students that a cube may not always be a practical shape for a container, because it is more difficult for us to handle (as a laundry detergent container and as a juice box). Questions 5 to 7 involve finding the dimensions of a cube with a given volume. In questions 6 and 7, students must determine the surface area of their optimal shape.

Question 8 is an extension of question 7, where the box has no lid.

Question 9 c) provides an opportunity for a Literacy Connection.

Question 10 allows for some student originality. It could be used as an assessment that is somewhat open-ended. You may wish to use **BLM A5 Problem Solving Checklist** to assist you in assessing your students.

Extend

Question 12 requires students to design a carton to hold 24 tissue boxes. Students may need help realizing that the tissue boxes should be packed in a "cube-like" configuration. A cube-like configuration is possible here if the boxes are stacked in a 48 cm cube.

Question 13 is similar to the case of the lidless box in question 8.

Question 14 provides a good reminder that extra cardboard is required for the flaps of a box. Students should realize that in most of the previous questions, flaps have been ignored.

Question 15 looks at the flaps of the box in further detail.

Literacy Connections

The Cube

The maximum volume occurs when we have a cube. Have students write a dialogue between two students who are working on this section together, one who thinks this is an obvious concept, and one of who thinks it is very challenging.

Exercise Guide

Category	Question Number
Minimum	1–5
Typical	1–5, 7, 9, 10
Extension	8, 11–15