9.4 Maximize the Volume of a Square-Based Prism

Strand:
Measurement and Geometry

Student Text Pages
498 to 503

Suggested Timing
80–160 min

Tools
- grid paper

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- *The Geometer's Sketchpad*®
- computers

Related Resources
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method
BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism
BLM G10 Grid Paper
BLM 9.4.3 Achievement Check Rubric
BLM A21 Opinion Piece Checklist
BLM A4 Presentation Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

Assign Get Ready questions 3 and 5 before starting this section, if they have not been completed earlier.

Warm-Up

1. Calculate the surface area and volume of a prism with each set of dimensions.
a) 2 cm by 2 cm by 11 cm **b)** 3 cm by 3 cm by 6.5 cm

2. Compare your answers in question 1. Which shape would be the better choice for a package? Explain your reasoning. Do you think there is a shape that would be even better than the two shapes described? If so, describe the shape.

Warm-Up Answers

1. a) SA = 96 cm^2; V = 44 cm^3 **b)** SA = 96 cm^2; V = 58.5 cm^3

2. Package b) is the better choice as it has greater volume for the same surface area.

Teaching Suggestions

- Assign the Warm-Up questions.
- Review the Warm-Up Answers. Both of these prisms have a surface area of 96 cm^2. Students should determine from their calculations that the 3 cm by 3 cm by 6.5 cm prism has greater volume. Some students may realize that a cube with the same surface area would have even greater volume, but it is not necessary for students to realize this at this point.
- Assign the Investigate. (10–15 min) If the Warm-Up was assigned, less time may be needed.
- Method 2 uses a spreadsheet, if computers are available. (10–15 min) Remind students to save their spreadsheets for future use. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- You may wish to have students use *The Geometer's Sketchpad*® for this activity. If so, use **BLM 9.4.1 Investigate: *The Geometer's Sketchpad*® Method** to support this activity.
- Do Example 1 as a class. (5 min)

Common Errors

- Some students using algebraic methods may not realize that in some questions, the material to be used to form a package comes in a particular shape and so the optimal shape for the package may not be possible.

R_x Give students a sheet of 8.5" by 11" paper (about 21.5 cm by 28 cm) or grid paper and have them cut and tape the paper to form the largest square-based prism possible. You may wish to use **BLM G10 Grid Paper**. This hands-on approach will illustrate that a cube is not always possible if the faces of the prism are made from a single piece of paper. This activity could be also be used for a summative assessment.

Ongoing Assessment

Use Achievement Check question 9 to monitor student success. See the Achievement Check Answers and **BLM 9.4.3 Achievement Check Rubric**.

Chapter Problem question 8 can also be used as an assessment tool.

Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Assign the Communicate Your Understanding questions to students as a think-pair-share activity. (5 min)
- Assign the Practise questions.
- Use **BLM 9.4.2 Practice: Maximize the Volume of a Square-Based Prism** for extra practice or remediation.

Investigate Answers (page 498)

Method 1

1.

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	24	5.5	5.5
2	2	4	24	2.0	8.0
3	3	9	24	0.5	4.5

2. Prism 2 has the maximum volume. Its shape is a cube.

3. a) Answers will vary. Possible answer: Prism 3. Dimensions: 3 cm × 3 cm × 3 cm

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	54	13.0	13
2	2	4	54	5.75	23
3	3	9	54	3.0	27
4	4	16	54	1.375	22
5	5	25	54	0.2	5

4.

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	2	4	96	11.0	44.0
2	3	9	96	6.5	58.5
3	4	16	96	4.0	64.0
4	5	25	96	2.3	57.5
5	6	36	96	1.0	36.0

5. The maximum volume of a square based prism with a given surface area is achieved when a cube is formed.
length = width = height

Method 2

2. The formula for height is found by rearranging the formula for surface area.

$$2 \times (\text{area of base}) + (4 \times s \times h) = 24$$

$$4 \times s \times h = 24 - [2 \times (\text{area of base})]$$

$$h = \frac{24 - [2(\text{area of base})]}{4(\text{side length of base})}$$

3. 2 cm × 2 cm × 2 cm. This cube has the greatest volume at 8 cm^3.

4. a) Answers will vary. Possible answer: Cube with 3 cm sides.

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	54	13.0	13
2	2	4	54	5.75	23
3	3	9	54	3.0	27
4	4	16	54	1.375	22
5	5	25	54	0.2	5

Accommodations

Gifted and Enrichment—Challenge students to determine why some companies do not use rectangular prism shapes for packaging that are not the most economical.

Visual—Work with groups of students to help them to solve equations sequentially using steps.

Spatial—Encourage students to draw scale diagrams to represent the shapes in this question.

Motor—Encourage students to work in groups when completing the questions in this section.

Memory—Encourage students to create more "formula cue-cards" to record the new formulas that they have learned in this Chapter.

5. a) Answers will vary. Possible answer: Cube with 4 cm sides.

b)

Prism Number	Side Length of Base (cm)	Area of Base (cm^2)	Surface Area (cm^2)	Height (cm)	Volume (cm^3)
1	1	1	96	23.5	23.5
2	2	4	96	11.0	44.0
3	3	9	96	6.5	58.5
4	4	16	96	4.0	64.0
5	5	25	96	2.3	57.5
6	6	36	96	1.0	36.0

6. The maximum volume of a square-based prism with a given surface area is achieved when a cube is formed.

length = width = height

Communicate Your Understanding Responses (page 501)

C1. Answers will vary. Possible answer: Packaging companies need to minimize the materials used on packaging products by maximizing the volume.

C2. We know that the maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube. Box C is a cube and hence has the greatest volume.

Practise

Question 1 is similar to Communicate Your Understanding question C2.

Questions 2 and 3 are similar to the Example.

Connect and Apply

Have students use their spreadsheets from Investigate: Method 2 to complete Connect and Apply questions 4 to 9. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity. These questions also could be done using algebraic techniques like Example 1. Note to students that extra material required for seams and flaps in the containers will be ignored for all calculations in all the questions.

The Chapter Problem question 8 involves shipping an item in the largest box that can be made from 2500 cm^2 of cardboard. The box will be a cube, and students are required to calculate the empty space in the box.

The Achievement Check question 9 involves making a box with and without a lid. This will require students to alter the algebra in their solutions, dividing the given surface area by 6 for the box with a lid, but by 5 for the box without a lid. The answers in this case will require rounding.

Achievement Check Answers (page 503)

9. a) 1500 cm^2

b) Using the fact that the maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube, the area of each square face is $\frac{1500}{6}$ cm^2 or 250 cm^2.
Therefore, the dimensions of each face are 15.8 cm by 15.8 cm.
$(\sqrt{250} \doteq 15.811\ldots)$

c) Set up a table to explore this relationship:

Box	Width (cm)	Length (cm)	Height (cm)	Area (cm^2)	Volume (cm^3)
1	.010	.010	.035	1500	.03500
2	.020	.020	13.8	1500	.05520
3	.030	.030	.05	1500	.04500
4	.025	.025	8.8	1500	.05500
5	.021	.021	12.6	1500	.05557
6	.022	.022	11.5	1500	.05588
7	.023	.023	10.6	1500	.05583
8	22.5	22.5	11.0	1500	.05590
9	22.6	22.6	10.9	1500	5589.2
10	**22.4**	**22.4**	**11.1**	**1500**	**5590.14**
11	22.3	22.3	11.2	1500	5590.10

The maximum volume is reached by making the base 22.4 cm on a side.

d) Answers may vary. For example: ignore material for seams or overlap flaps.

Extend

In Question 10 there is one sheet of plywood to be used. It is not possible to use all the plywood and make the box a cube with a lid with each face made from a single piece of wood. If each face is 60 cm by 60 cm, there will be 60 cm by 120 cm, or 7200 cm^2, of plywood wasted. Some students may decide to piece the faces together to form a bigger box. Discuss how the pieces of wood might be joined. This question is designed to be open-ended in this way. Ensure that students identify any assumptions they are making in their solutions.

Question 11 is similar to question 10. If the box is made with a lid, it can be formed by using six 10 cm by 10 cm faces and all the stained glass will be used. If the box has no lid, the only way to create a bigger box than a 10 cm cube is to piece the faces together. In this case, the box is being made out of stained glass, so students may argue that it is reasonable to assume that pieces of glass can be joined together to form a face for the box. It is possible to get a bigger box without a lid if this is done. Any method is acceptable as long as students realize the assumptions that are being made.

You may wish to use **BLM G10 Grid Paper** for questions 10 and 11.

Literacy Connections

Opinion Piece

Assign the following activity. Write an opinion piece, a series of paragraphs expressing your opinion, on the topic below. Develop your main idea with supporting details. Be sure to write your opinion piece with your audience in mind, an adult who is interested in your opinion.
Topic: The cylinder is the best shape for a beverage container.

Review students' opinion pieces, and ensure that students have included three paragraphs: an introduction, the body, and a conclusion. Their opinion should be clearly stated and students should provide details with reasons, examples, and facts to support their opinions. You may wish to use **BLM A21 Opinion Piece Checklist** to assess the students. You may also wish to provide an opportunity for students to present their opinion pieces to the class. Use **BLM A4 Presentation Checklist** to assess students' presentations.

Exercise Guide

Category	Question Number
Minimum	1, 2, 3, 5
Typical	1–7
Extension	10, 11