9.5 Maximize the Volume of a Cylinder

Strand:
Measurement and Geometry

Student Text Pages
504 to 509

Suggested Timing
80 min

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM 9.5.1 Cylinder Data Recording Table
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM A18 My Progress as a Problem Solver
BLM 9.5.2 Practice: Maximize the Volume of a Cylinder

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

The Get Ready questions involving cylinders provide the needed skills for this section. Have students complete Get Ready questions 4 and 6 before starting this section.

Warm-Up

1. Determine the volume and surface area of a cylinder with radius 6 cm and height of 20 cm.

2. Determine the height of a cylinder with a surface area of 375 cm^2 and a radius of

a) 7 cm **b)** 5 cm

Warm-Up Answers

1. $V = 2261.9\ cm^2$; $SA = 980.18\ cm^2$

2. a) 1.53 cm **b)** 6.94 cm

Teaching Suggestions

- The Investigate Method 1 requires algebraic skills to isolate the height in the formula for surface area of a cylinder. Work through the first step of the Investigate as a class. (Rearranging formulas was done previously in Section 4.4.)
- Review the necessary algebra skills. (5 min)
- Assign the rest of the Investigate. (15–20 min)
- Students can use copies of **BLM 9.5.1 Cylinder Data Recording Table** to record their results.
- Alternatively, once students understand the rearranging of the formula, have them use a spreadsheet and complete Method 2. If students are familiar with spreadsheets, this method will be advantageous. It will take less time and allow students to complete the Investigate for different surface areas. Remind students to save their spreadsheets for future use. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- Follow up the Investigate (Method 1 or 2) with a class discussion to ensure that students realize that a cylinder with a height equal to its diameter has the largest volume for a given surface area.

Common Errors

- Some students may struggle with the algebraic skills required in this section.

R_x Once you have led your students through the algebraic skills necessary for the Investigate, you may want to keep the focus on the results. It is important that students understand that the optimal shape for the cylinder with a given surface area is one with a height equal to its radius. Again, once you have led your students through the algebraic technique in the Example, ensure that they have the key concept $SA = 6\pi r^2$. Many of your students will have no trouble understanding that this becomes $SA = 5\pi r^2$ when the cylinder has no lid since the one circular lid (with area πr^2) has been removed.

Ongoing Assessment

- Chapter Problem question 5 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Review Example 1 as a class. (5–10 min)
- Discuss the Communicate Your Understanding questions as a class. (5 min)
- You may wish to use **BLM A18 My Progress as a Problem Solver** as a self-assessment for students as they work through the Investigate.
- Use **BLM 9.5.2 Practice: Maximize the Volume of a Cylinder** for extra practice or remediation.

Investigate Answers (page 504)

Method 1

1.

Step	Explanation
$SA = 2\pi r^2 + 2\pi rh$	
$375 = 2\pi r^2 + 2\pi rh$	Substitute $SA = 375$
$375 - 2\pi r^2 = 2\pi r^2 + 2\pi rh - 2\pi r^2$	Subtract $2\pi r^2$ from both sides.
$375 - 2\pi r^2 = 2\pi rh$	Simplify.
$\frac{375 - 2\pi r^2}{2\pi r} = \frac{\cancel{2\pi r}h}{\cancel{2\pi r}}$	Divide both sides by $2\pi r$.
$h = \frac{375 - 2\pi r^2}{2\pi r}$	Simplify to isolate h.

2. a) $h = \frac{375 - 2\pi}{2\pi} = \frac{375}{2\pi} - 1 \approx 58.68$ cm

b) $V = 184.36$ cm^3

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	58.68	184.36	375

3.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
2	27.84	349.87	375
3	16.89	477.68	375
4	10.92	548.94	375
5	6.94	544.80	375
6	3.95	446.42	375
7	1.53	234.93	375

4. Maximum volume = 548.94 cm^3, height = 10.92 cm, radius = 4 cm

5. Not necessarily. Extend the investigation to use decimal values for the radius.

Method 2

2.

Radius (cm)	Height (cm)	Volume (cm^3)	Surface Area (cm^2)
1	58.68	184.36	375
2	27.84	349.87	375
3	16.89	477.68	375
4	10.92	548.94	375
5	6.94	544.80	375
6	3.95	446.42	375
7	1.53	234.93	375

When the radius is 8 cm, the surface area of the top and bottom is 402.12 cm^3, which is greater than the entire surface area of the cylinder. Therefore, to compensate, the height becomes negative.

3. Maximum volume at 4 cm radius. Yes. Using decimals to the nearest tenths, the greatest volume is 557.47 cm^3 when the radius is 4.5 cm.

4. Volume = 557.539 cm^3, radius = 4.46 cm

5. a) radius = 4.46 cm, height = 8.92 cm

b) $r = \frac{h}{2}$ **c)** diameter = height

6. Volume = 858.36 cm^3, radius = 5.15 cm, height = 10.30 cm. The radius is half the height.

7. Volume = 2427.88 cm^3, radius = 7.28 cm, height = 14.58 cm. The radius is half the height.

8. a) $r = \frac{h}{2}$

b) In a square based prism, the maximum volume for a fixed surface area occurs when all the dimensions of are equal (length = width = height). However, in a cylinder, the maximum volume for a fixed surface area occurs when the radius of the base equals half the height or the diameter of the base equals the height.

Communicate Your Understanding Responses (page 508)

C1. Answers will vary. Sample solution: It will be necessary to maximize the volume of a cylinder, given its surface area when trying to find the dimensions for oxygen cylinders to be kept in rockets going into the space.

C2. Cylinder B has the greatest volume because it is the one whose height is the closest to its diameter.

C3. Answers will vary. Glasses with the maximum volume are not very practical because they are hard to hold. Also, an important consideration is how the glasses appear to the customer. The customer would want the glasses to look elegant and appear to hold more liquid.

Accommodations

Perceptual—Encourage students to colour-code "like terms" when simplifying the formula for surface area.

Motor—Let students work together and use a spreadsheet to complete the questions requiring technology in this section.

Language—Allow students to use a metric converter to change the units from cm to mm, etc.

Memory—Review with the students the steps required to find a square root on a calculator.

Practise

Practise questions 1 and 2 are similar to the Example, so students should have few problems completing these on their own.

Connect and Apply

If Method 2 was used for the Investigate, have students use their spreadsheets to complete some of the questions. Ensure that they can also solve the questions algebraically.

The Chapter Problem question 5 requires students to determine the best height for the cylinder, and then determine how many CDs can be stacked in a cylinder of this height. Students might describe assumptions about the amount of empty space allowed in the cylinder.

Question 6 is an extension to the previous questions with an open-topped cylinder. Advanced students will be able to handle the implications this makes on the algebra involved, but other students may require assistance with this.

Extend

Question 7 compares the volume of a square-based prism and a cylinder made with the same amount of sheet metal. Again, students who have created a spreadsheet for Investigate could use these spreadsheets to determine the volume of each container. Ensure that students hypothesize their answer before performing the calculations.

Question 8 extends the possibilities to include a sphere. This is a great extension question, especially for advanced students. Remind students about the formulas for the surface area and volume of a sphere (Making Connections, page 509).

Question 9 is an appropriate extension for those students who have created the spreadsheet for a cylinder already. In this question, students would alter the formulas in their spreadsheet for a cylinder with no lid.

Exercise Guide

Category	Question Number
Minimum	1–3
Typical	1–4, 6
Extension	7–10