9.6 Minimize the Surface Area of a Cylinder

Strand:
Measurement and Geometry

Student Text Pages
510 to 515

Suggested Timing
80 min

Tools
- construction paper
- rulers
- scissors
- tape

Technology Tools
- Corel® *Quattro Pro*®
- Microsoft® *Excel*
- computers

Related Resources
BLM T1 Corel® *Quattro Pro*® 8
BLM T2 Corel® *Quattro Pro*® 10
BLM T3 Microsoft® *Excel*
BLM 9.6.1 Practice: Minimize the Surface Area of a Cylinder
BLM 9.6.2 Achievement Check Rubric
BLM A23 News Report Checklist
BLM A4 Presentation Checklist

Mathematical Process Expectations Emphasis

- ☑ Problem Solving
- ☑ Reasoning and Proving
- ☑ Reflecting
- ☑ Selecting Tools and Computational Strategies
- ☑ Connecting
- ☑ Representing
- ☑ Communicating

Specific Expectations

Investigating the Optimal Value of Measurements

MG1.03 identify, through investigation with a variety of tools (e.g. concrete materials, computer software), the effect of varying the dimensions on the surface area [or volume] of square-based prisms and cylinders, given a fixed volume [or surface area];

MG1.04 explain the significance of optimal area, surface area, or volume in various applications (e.g., the minimum amount of packaging material; the relationship between surface area and heat loss);

MG1.05 pose and solve problems involving maximization and minimization of measurements of geometric shapes and figures (e.g., determine the dimensions of the rectangular field with the maximum area that can be enclosed by a fixed amount of fencing, if the fencing is required on only three sides).

Link to Get Ready

The Get Ready questions involving cylinders provide the needed skills for this section. Have students complete Get Ready questions 4 and 6 before starting this section.

Warm-Up

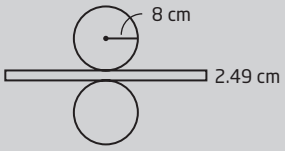
1. Determine the height of a cylinder with a volume of 500 cm^3 and a radius of 8 cm.

2. Draw a net for the cylinder in question 1.

Warm-Up Answers

1. 2.49 cm

2.



Teaching Suggestions

- Have students work in small groups for the Investigate. (20–25 min)
- Alternatively, have students work with a partner within groups of six students, using a different radius to form one cylinder with a volume of 500 cm^3. Then, have the three partner groups record their results in a table as a group. (10 min)
- Students who have used spreadsheets several times through this chapter will be comfortable with Method 2. You may want to go through the algebraic manipulation necessary to develop the formulas with them. You may wish to use **BLM T1 Corel® *Quattro Pro*® 8**, **BLM T2 Corel® *Quattro Pro*® 10**, or **BLM T3 Microsoft® *Excel*** to support this activity.
- Discuss the Example. (10 min) Students should realize from the Investigate that a cylinder with a height equal to its diameter is the optimal shape for a given volume.

Common Errors

- Some students may continue to struggle with the algebraic skills necessary for this section.

R_x Encourage students to use spreadsheets to investigate the optimal shape for the cylinder. It is important that they recognize this optimal shape, even if they have trouble performing the algebraic skills.

Ongoing Assessment

- Use Achievement Check question 10 to monitor student success. See Achievement Check Answers and **BLM 9.6.2 Achievement Check Rubric**.
- Chapter Problem question 8 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' Communication skills.

- Assign and discuss the Communicate Your Understanding questions. (5 min)
- Assign Practise questions 1 to 3.
- You may wish to assign **BLM 9.6.2 Practice: Minimize the Surface Area of a Cylinder** for remediation or extra practice.

Investigate Answers (page 510)

Method 1

1. Answers will vary. Possible answer: radius = 4.3 cm, to the nearest tenth, area = 58.09 cm^2

2. Answers will vary. Possible answer: height = 8.6 cm, to the nearest tenth

3. Answers will vary. Possible answer: circumference = 27.02 cm, to the nearest hundredth

5. a) Answers will vary. Possible answer: area of rectangle = 232.42 cm^2, to the nearest hundredth

b) Answers will vary. Possible answer: 348.67 cm^2, to the nearest hundredth

6.

Cylinder	Radius (cm)	Base Area (cm^2)	Height (cm)	Surface Area (cm^2)
1	4.3	58.088	8.608	348.734

7.

Cylinder	Radius (cm)	Base Area (cm^2)	Height (cm)	Surface Area (cm^2)
2	2.0	12.566	39.789	525.133
3	2.5	19.635	25.465	439.270
4	3.0	28.274	17.684	389.882
5	3.5	38.485	12.992	362.683
6	4.0	50.265	9.947	350.531
7	4.5	63.617	7.860	349.457
8	5.0	78.540	6.366	357.080

8. Cylinder 1 has the least surface area. Height = diameter.

9. For a given volume, the cylinder whose height is equal to the diameter has the least surface area.

Method 2

1.

Cylinder	Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height (cm)	Surface Area (cm^2)
1	1	3.142	500	159.155	1006.283
2	2	12.566	500	39.789	525.133
3	3	28.274	500	17.684	389.882
4	4	50.265	500	9.947	350.531
5	5	78.540	500	6.366	357.080
6	6	113.097	500	4.421	392.861
7	7	153.938	500	3.248	450.733

Accommodations

Gifted and Enrichment—Challenge students to investigate the food products contained in cylinders and rectangular prisms.

Perceptual—Allow students to work in pairs when completing the questions in this section.

Memory—Remind students to use colour-coding to add "like terms" such as $SA = 2\pi r^2 + 4\pi r^2$
$= 6\pi r^2$
and to use small sequential steps when solving equations.

ESL—Encourage students to work with a partner when working through the questions in this section.

Student Success

Assign the concepts of Sections 9.3 through 9.6 to groups. Each group investigates real-life applications, then shares with the other groups using a **carousel** strategy.

2. radius = 4 cm (whole-numbered radius).

Cylinder	Radius (cm)	Base Area (cm^2)	Volume (cm^3)	Height(cm)	Surface Area(cm^2)
1	3.5	38.485	500	12.992	362.683
2	3.6	40.715	500	12.280	359.208
3	3.7	43.008	500	11.626	356.287
4	3.8	45.365	500	11.022	353.887
5	3.9	47.784	500	10.464	351.978
6	4.0	50.265	500	9.947	350.531
7	4.1	52.810	500	9.468	349.523
8	4.2	55.418	500	9.022	348.931
9	4.3	58.088	500	8.608	348.734
10	4.4	60.821	500	8.221	348.915

3. radius = 4.3 cm and the height = 8.6 cm, or 2 × radius

4. radius = 5.3 cm and the height = 10.65 cm ≈ 2 × radius

5. radius = 6 cm and the height = 12.02 cm ≈ 2 × radius

6. The cylinder whose height is equal to the twice the radius has the minimum surface area.

Communicate Your Understanding Responses (page 513)

C1. Answers will vary. Sample answer: constructing a cylindrical container for a quantity of liquid.

C2. Cylinder B has the least surface area because its height is equal to its diameter.

Practise

Practise questions 1 to 3 are similar to the Example. Students should not have many problems completing these questions.

Connect and Apply

For questions 4 and 5, remind students that 1 L = 1000 cm^3. For question 6, have students use their spreadsheets from the previous sections to justify their answers.

Conduct a class discussion for question 7.

The Achievement Check question 10 is a variation of question 4 from Section 9.5. Use **BLM 9.6.2 Achievement Check Rubric** to assist you in assessing your students.

Achievement Check Answers (page 515)

10. a) Note that 600 L = 600 000 cm^3.
If the radius is 20 cm, the area of the end is
$\pi(20)^2 \doteq 1256.6$
Then, the height of the cylinder is $\frac{600\,000}{1256.6}$ cm, or about 477.5 cm.
If the radius is 30 cm, the area of the end is
$\pi(30)^2 \doteq 2827.4$
Then, the height of the cylinder is $\frac{600\,000}{2827.4}$ cm, or about 212.2 cm.

b) For an optimal design, the height must equal the diameter of the cylinder.

$600\ 000 = 2\pi r^3$

$r = \sqrt[3]{\frac{600\ 000}{2\pi}}$

$r \doteq 45.7$

The radius of the cylinder should be 45.7 cm and the height 91.4 cm.

c) The optimal rectangular shape is a cube. The side length would have to be $\sqrt[3]{600\ 000}$ cm, or about 84.3 cm. Both shapes have the same capacity. The surface area of the cube is 46 638 cm^2. The surface area of the cylinder is 26 244 cm^2. The cylinder shape requires less material because its surface area is less. The cylinder may also be preferred since a curved shape is more aerodynamic.

Extend

Question 11 involves a cylinder without a lid. Students will find this question easier if they have completed the lidless cylinder questions 6 and 9 from Section 9.5.

Question 12 will be easier for students who have already completed question 8 from Section 9.5.

The Math Contest questions 13 to 16 extend and consolidate the concepts of this section and Section 9.5.

Literacy Connections

News Report

Assign the following activity. Look at the picture at the beginning of Section 9.6. Imagine that the headline reads: "Acme Company Cans Food in All Sizes!" Write a news report about the photo and the given headline. Remember to have a link between the photo and the headline. Try to answer the questions who, what, where, when, why, and how in your news report. Also, remember to write in the third person.

Review students' news reports, and ensure that students have answered the questions who, what, where, when, why, and how, and written in the third person. You may wish to use **BLM A23 News Report Checklist** to assess your students. You may also wish to provide an opportunity for students to present their news reports to the class. Use **BLM A4 Presentation Checklist** to assess students' presentations.

Exercise Guide

Category	Question Number
Minimum	1, 2, 3, 4
Typical	1–9
Extension	11–16