4.4 Graph $y = a(x - h)^2 + k$

Student Text Pages
180–188

Suggested Timing
70–140 min, depending on the level of understanding of the class

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- G–3 Coordinate Grids
- BLM 4–9 Section 4.4 Practice Master
- BLM 4–10 Section 4.4 Achievement Check Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- This section consolidates the learning from the previous section. Students graph parabolas with more than one transformation from $y = x^2$. It is important that the students learn the concepts by working through the investigation, which is visual and hands-on and requires minimal input from you. Key concepts to come out of this investigation are: vertex (h, k), axis of symmetry $x = h$, direction of opening (sign of a), and vertical stretch or compression factor of a.

Investigate

- Have students work through the steps of this section. Encourage them to summarize using numbers, words, algebra, and a graph. (30 min)
- It might be a good idea to show students how to make the graph of the base relation, $y = x^2$, appear as a thicker line. That is, arrow back to the left of **Y1** and press **ENTER**. This graph then appears thicker than the others.
- The **Investigate** can also be done using the function features of the TI-83 Plus or TI-84 Plus graphing calculator. For example, enter x^2 into **Y1**. Then, in **Y2** enter 2Y1 + 5 for $2x^2 + 5$. Similarly, in **Y3** enter Y1$(x - 4) + 1$ for $(x - 4)^2 + 1$, or in **Y4** enter 2Y1$(x - 4) + 1$ for $2(x - 4)^2 + 1$.
- Another option is to use sliders in either *Fathom*™ or *The Geometer's Sketchpad*®. In *Fathom*™, graph $y = ax^2 + k$, $y = (x - h)^2 + k$, and so on, where students have created sliders, or teachers have pre-made files, as is common with *The Geometer's Sketchpad*®.

Examples

- Present **Examples 1** and **2**, or similar examples that consolidate learning. Applications similar to **Example 3** are important to give context to quadratics. (30 min)

Key Concepts

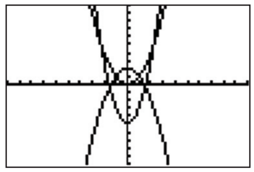
- Have students copy the table from the **Key Concepts** onto a cue card for reminder and studying purposes.
- Students can play a game called "I have, who has … ," in which different representations of a relation (equation, graph, description) are distributed on cards around the class. A person displays their card, and then whoever has the matching equation, graph, or description holds it up and reads the equation. Note: TI-Navigator™ can send a graph to students while in the Activity Centre. All students then go through a process to construct the equation via experimentation and send in their final equation. This is very effective because students are not able to see the equation—all they have to go on is the graph sent.
- Another game is Concentration, where students turn over cards and determine whether they have a match.

Communicate Your Understanding

- As a class, discuss the questions in this section. Question C2 needs a full explanation. Students should understand that it is a vertical stretch because the multiplication occurs after the exponent has been applied. In other words, the *y*-values have been doubled. (5 min)
- For initial assessment, it is important that all of the questions be assigned and taken up as a class.
- Use **BLM 4–9 Section 4.4 Practice Master** for remediation or extra practice.

Investigate Answers (pages 180–181)

4. a)

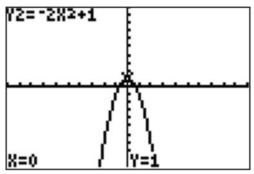


b) Answers will vary for second point coordinates.

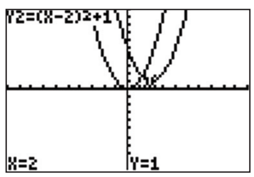
Equation	Vertex	Second Point
$y = x^2$	(0, 0)	(2, 4)
$y = 2x^2 - 5$	(0, –5)	(2, 3)
$y = -x^2 + 2$	(0, 2)	(2, –2)

c) The graph of $y = 2x^2 - 5$ is the graph of $y = x^2$ stretched vertically by a factor of 2 and translated 5 units downward. The graph of $y = -x^2 + 2$ is the graph of $y = x^2$ reflected in the *x*-axis and translated 2 units upward.

d)



6. a)

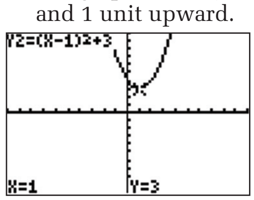


b), **c)** Answers will vary for second point coordinates.

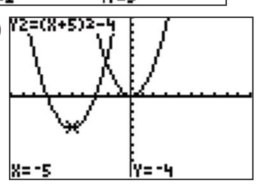
Equation	Vertex	Second Point	Axis of Symmetry
$y = x^2$	(0, 0)	(2, 4)	$x = 0$
$y = (x - 2)^2 + 1$	(2, 1)	(3, 2)	$x = 2$

d) The graph of $y = (x - 2)^2 + 1$ is the graph of $y = x^2$ translated 2 units to the right and 1 unit upward.

7.



8. a)

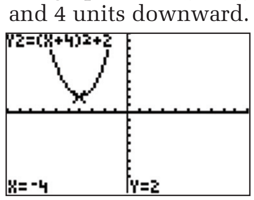


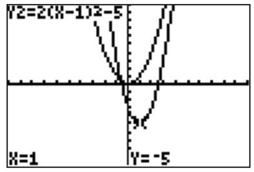
Answers will vary for second point coordinates.

Equation	Vertex	Second Point	Axis of Symmetry
$y = x^2$	(0, 0)	(2, 4)	$x = 0$
$y = (x + 5)^2 - 4$	(–5, –4)	(–2, 5)	$x = -5$

The graph of $y = (x + 5)^2 - 4$ is the graph of $y = x^2$ translated 5 units to the left and 4 units downward.

b)

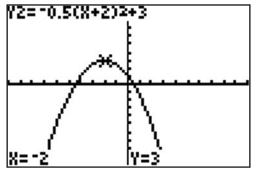


9. a) 

Answers will vary for second point coordinates.

Equation	Vertex	Second Point	Axis of Symmetry
$y = x^2$	(0, 0)	(2, 4)	$x = 0$
$y = 2(x - 1)^2 - 5$	(1, –5)	(2, –3)	$x = 1$

The graph of $y = 2(x - 1)^2 - 5$ is the graph of $y = x^2$ vertically stretched by a factor of 2 and translated 1 unit to the right and 5 units downward.

b) 

10. To sketch the graph of $y = a(x - h)^2 + k$, transform the parabola for $y = x^2$ such that it is stretched vertically or compressed vertically by a factor of a, then translate the parabola horizontally h units to the right or left and vertically k units upward or downward. The coordinates of the vertex are (h, k); the equation of the axis of symmetry is $x = h$; x may take on any real value; and y may take on any real value greater than or equal to k if a is positive or any real value less than or equal to k if a is negative.

Communicate Your Understanding Responses (page 186)

C1. It is called the axis of symmetry because this line will make the left side of the parabola reflect onto the right side of the parabola, and vice versa.

C2. Answers will vary. For example: Vertical stretches are used in descriptions because the location of a is outside of the squaring function, and so it is done after squaring and affects the y-values directly.

C3. C is correct. Since the parabola opens upward, a must be positive. In the graph of $y = x^2$, when x is two units from the vertex, the y-value is four units above the vertex. In this graph, when x is two units from the vertex, the y-value is only two units above the vertex, which means that the parabola was compressed vertically by $\frac{1}{2}$.

Practise

- **Question 1** is an ideal question to summarize the features of quadratic relations and their graphs and to consolidate understanding. This question will identify any difficulties students are having.
- **Questions 6** and **7** are good indicators of whether students can turn the process around and develop an equation from a graph.
- **Question 11** is a good communication question. Students need to practise justifying their responses.
- **Questions 12** through **15** are all good applications. Assign a selection of these.
- **Question 16** refers to the Chapter Problem. Assign this question now or wait until the Chapter Problem Wrap-Up. It involves the use of a graphing calculator as a tool, but requires students to do the thinking behind the graphing process.
- **Question 17** is a good example of where students need to understand the restrictions on the variables.
- **Question 18** is an Achievement Check and can be used as an assessment tool.
- **Question 19** is intended for students working at level 3 or 4. It requires understanding of transformation terminology.
- **Question 20** is an interesting extension to transformations of circles. It deals with the concept that the transformations are all applied before the exponent is applied, so h and k must be subtracted from x and y, respectively.

Common Errors

- Some students may apply the horizontal translation in reverse.

R_x Have students revisit Part C of the Investigate in Section 4.3, which is about graphing $y = (x - h)^2$, and summarize with words and graphs. Students may also make tables of values for $y = x^2$ and $y = (x - 2)^2$ and compare the resulting y-values.

Accommodations

Gifted and Enrichment—Challenge the students to create questions for their classmates where parabolas model real-life situations.

Visual—Use colour-coding to relate the y-value of the vertex to the maximum or minimum value of the relation.

Perceptual—Allow students to use a graphing calculator when completing the questions in this section.

Motor—Provide students with photocopies of the questions in this section and have them complete the questions using large sheets of grid paper.

Language—Allow students to answer the questions in this section orally.

ESL—Encourage students to use their dictionaries or translators to complete the questions in this section.

Achievement Check Sample Solution, question 18, page 187

Provide students with **BLM 4–10 Section 4.4 Achievement Check Rubric** to help them understand what is expected.

18. a) The maximum height reached is at the vertex of the parabola, 80 m.

b) The graph shows that the rocket reached its maximum height of 80 m after 4 s.

c) The rocket was fired at $t = 0$. From the graph, the height of the rocket above the water was 1.6 m.

d) The vertex is (4, 80), so $h = 4$ and $k = 80$. Substitute these values in the equation $y = a(x - h)^2 + k$.

$y = a(x - 4)^2 + 80$

The parabola passes through the point (0, 1.6). Substitute $x = 0$ and $y = 1.6$ and solve for a.

$$\begin{aligned} 1.6 &= a(0 - 4)^2 + 80 \\ 1.6 &= a(-4)^2 + 80 \\ 1.6 &= 16a + 80 \\ -78.4 &= 16a \\ a &= \frac{-78.4}{16} \\ a &= -4.9 \end{aligned}$$

An equation for the parabola is $h = -4.9(t - 4)^2 + 80$.

e) Estimated answer: The equation of the axis of symmetry of the graph representing the path of the rocket is $t = 4$. The rocket will be at the starting height of 1.6 m after 8 s and will hit the water just after that, between 8.0 s and 8.1 s.

Exact Answer: When the rocket hits the water $h = 0$. Substitute the value for h and solve for t.

$$\begin{aligned} 0 &= -4.9(t - 4)^2 + 80 \\ 4.9(t - 4)^2 &= 80 \\ (t - 4)^2 &= \frac{80}{4.9} \\ (t - 4)^2 &\doteq 16.3265 \\ t - 4 &\doteq \sqrt{16.325} \\ t - 4 &\doteq 4.04 \\ t &\doteq 8.04 \end{aligned}$$

The rocket falls into the water after approximately 8.04 s.

Literacy Connections

Add any words from this section that students have difficulty with to the Word Wall.

As suggested earlier, games such as Concentration and the "I have, who has … " matching activity fit nicely in this section. Students can also make up their own games and share them with the class.

Student Success

Have each student write a quadratic relation. Then use Think-Pair-Share to have partners list the important information from the equation.

Use a placemat activity to summarize information from a given quadratic relation. Use **G–2 Placemat** to support this activity.

Refer to the introduction of this Teacher's Resource for more information about how to use Think-Pair-Share and placemat strategies.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	21, 22
Reasoning and Proving	11, 18
Reflecting	n/a
Selecting Tools and Computational Strategies	10, 21
Connecting	12–15, 17, 18, 20–22
Representing	2–6, 8–10, 12, 13, 15, 17–19
Communicating	4, 11, 12, 14

Ongoing Assessment

- Use Achievement Check question 18 to monitor student success. See Achievement Check Answers and **BLM 4–10 Section 4.4 Achievement Check Rubric**.
- Chapter Problem question 16 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.