5.4 Factor Quadratic Expressions of the Form $x^2 + bx + c$

Student Text Pages
236–241

Suggested Timing
70 min

Tools
- algebra tiles

Related Resources
- BLM 5–7 Section 5.4 Practice Master
- A–10 Observation General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

Investigate

- In **Investigate A**, students benefit from the manipulative and visual nature of algebra tiles. If a polynomial can be rearranged into a rectangle, it is factorable. Negatives are very difficult to work with using algebra tiles, so it is recommended that only positives be modelled. They are meant to be a bridge to algebraic thinking. (10 min)
- In **Investigate B**, students will benefit from the algebraic and patterning approach, even if algebra tiles were used initially. This activity shows why the factoring procedure of sum and product works. (20 min)
- You could break up the polynomials containing positives and negatives, or allow the students to discover them together. This type of exploration requires a high level of communication skills.

Examples

- Discuss **Examples 1** and **2**. Provide sufficient examples so students can understand the differences between binomials containing both positives and negatives. Include an application to support real-world use of quadratics. (20 min)
- Suggest to students that they list the factors of the product first, and select the pair that satisfies the sum. Encourage students to come up with their own memory technique, such as # + # = b and # × # = c.

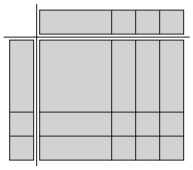
Communicate Your Understanding

- Review the vocabulary term (quadratic expression) before moving on to the questions in this section.
- Have students discuss these questions in pairs before having a class discussion.
- Question C3 is the best indicator of student understanding. (10 min)
- Use **A–10 Observation General Scoring Rubric** at any point during this section to assist you in assessing students.
- Use **BLM 5–7 Section 5.4 Practice Master** for remediation or extra practice.

Investigate Answers (pages 236–237)

A

1. a)



b) $x^2 + 5x + 6 = (x + 2)(x + 3)$

2. a) $x^2 + 6x + 5 = (x + 1)(x + 5)$ **b)** $x^2 + 3x + 2 = (x + 1)(x + 2)$
c) $x^2 + 4x + 3 = (x + 1)(x + 3)$ **d)** $x^2 + 6x + 8 = (x + 2)(x + 4)$

3. The coefficient b is the sum of the last terms of the binomial factors; the coefficient c is the product of the last terms of the binomial factors.

4. a) $x^2 + 7x + 6 = (x + 1)(x + 6)$ **b)** $x^2 + 8x + 12 = (x + 2)(x + 6)$

5. Answers will vary. For example: Arrange algebra tiles representing the quadratic expression into a rectangle. The dimensions of the rectangle are the factors of the quadratic expression.

B

Part A

1. a) $(x + 4)(x + 3) = x^2 + 7x + 12$ **b)** $(x + 1)(x + 5) = x^2 + 6x + 5$
c) $(x + 7)(x + 8) = x^2 + 15x + 56$

2. The coefficient b is the sum of the values of r and s in the binomial factors; the coefficient c is the product of the values of r and s in the binomial factors.

3. a) $x^2 + 6x + 8 = (x + 2)(x + 4)$ **b)** $x^2 + 7x + 10 = (x + 2)(x + 5)$
c) $x^2 + 9x + 20 = (x + 4)(x + 5)$ **d)** $x^2 + 10x + 21 = (x + 3)(x + 7)$

4. Answers will vary. For example: Find two integers, r and s, whose sum is b and whose product is c.

Part B

1. a) $(x - 3)(x - 2) = x^2 - 2x - 3x + 6$
$= x^2 - 5x + 6$

b) $(x - 1)(x - 5) = x^2 - 5x - x + 5$
$= x^2 - 6x + 5$

c) $(x - 1)(x + 5) = x^2 + 5x - x - 5$
$= x^2 + 4x - 5$

d) $(x + 3)(x - 8) = x^2 - 8x + 3x - 24$
$= x^2 - 5x - 24$

2. a) When the values of r and s were both negative, the sign of the value b was negative, and the sign of the value c was positive.

b) When only one of the values of r and s was negative, the sign of the value b was negative when the greater number of b and c was negative and the sign of the value b was positive when the greater number of b and c was positive, and the sign of the value c was negative.

3. a) $x^2 - 7x + 10 = (x - 2)(x - 5)$ **b)** $x^2 + 4x - 5 = (x + 5)(x - 1)$
c) $x^2 - 4x - 5 = (x - 5)(x + 1)$ **d)** $x^2 - 3x - 10 = (x - 5)(x + 2)$

4. Answers will vary.

Communicate Your Understanding Responses (page 240)

C1. Rearrange the tiles until they form a rectangle. The dimensions of the rectangle will be the factors.

C2. Look for the factors of –40 that have a sum of –6. These are the two numbers added to x in each factor. The numbers are –10 and +4. The factors are $(x - 10)(x + 4)$.

C3. None of the integer factors of –9 give a sum of +5.

Common Errors

- Some students may have difficulty with negative products and not be able to determine the appropriate pair that gives the indicated sum.

R_x Have students consider that they are looking for two numbers that are b apart. For example, if the product is −10 and the sum is 3, find the factor pair of 10 that are 3 apart (2 and 5). Then, the numbers will have opposite signs. Since the sum is positive, the "bigger" number will be positive, i.e., 5 and −2.

- Some students may assign a negative to the wrong factor.

R_x Remind students that the sign of the sum goes on the greater factor.

Accommodations

Visual—Provide students with visual or verbal clues to remember the steps involved when factoring trinomials of the form $x^2 + bx + c$.

Perceptual—Let students use graphing calculators to check that they have factored the trinomial expressions in the Practise exercise correctly.

Motor—Give students photocopies of the charts needed to determine the factors of a number that will equal a given sum and a given product.

Memory—Write the steps required to factor a quadratic expression that can be factored in two steps, including common factoring and factoring a trinomial, on separate cue cards. Have the students arrange the cue cards in order.

Student Success

Have each student create a simple quadratic expression. Then use Think-Pair-Share to have partners attempt to factor each other's expression.

Use an inside/outside circle to have students describe their method for factoring simple quadratic expressions.

Refer to the introduction of this Teacher's Resource for more information about how to use Think-Pair-Share and inside/outside circle strategies.

Practise

- Assign a variety from **questions 1** through **5**. Include a mixture of examples of positives and negatives. Allow students to use algebra tiles if they feel it is necessary to support understanding. However, negatives are very difficult to model with algebra tiles.
- **Questions 8**, **9**, and **12** are all communication questions that can indicate a student's level of understanding.
- **Question 13** refers to parabolas, an important connection to Chapter 4.
- **Questions 15** and **16** extend the concepts to trinomials of degree 4. It may help to replace x^2 with y and x^4 with y^2.

Literacy Connections

This section uses the term "quadratic expression." This means that the greatest degree of any of its terms is two, that is, a square. Have students investigate the idea of a square versus the idea that a "quad" means four, as in "quadrilateral."

The second item in the Key Concepts states that not all quadratic expressions can be factored over the integers. Discuss the term "integral values" so students know that when they write a quadratic expression in the form $x^2 + bx + c$, the coefficients b and c must be integers.

Add "quadratic expression" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	n/a
Reasoning and Proving	8, 9, 11, 12, 15–17
Reflecting	10
Selecting Tools and Computational Strategies	n/a
Connecting	6, 13–15, 17
Representing	1, 13
Communicating	11, 12, 14, 15

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.