5.5 Factor Quadratic Expressions of the Form $ax^2 + bx + c$

Student Text Pages
242–247

Suggested Timing
70–140 min

Tools
- algebra tiles

Technology Tools
- TI-89 calculator

Related Resources
- G–2 Placemat
- BLM 5–8 Section 5.5 Practice Master
- BLM 5–9 Section 5.5 Achievement Check Rubric
- A–6 Knowledge/Understanding General Scoring Rubric
- A–9 Communication General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

Investigate

- Allow students to explore factoring with algebra tiles. Remind them that if a polynomial can be remodelled as a rectangle, it is factorable. The key is to see how the product comes from the product of a and c and the sum remains b. If there are insufficient tiles in individual kits, combine sets of algebra tiles and have students work in pairs. (20 min)

Examples

- Present **Examples 1**, **2**, and **3**. Go through numerous examples, showing how the product comes from $a \times c$. Show examples with both positives and negatives. However, negative sums may be best left for the second day. (30 min)
- Another option is to use a trial-and-error method. Have students list the factors of a in one column and the factors of c in another column. They can use trial and error to see which pairs work together to provide the appropriate sum. For example:

Factor $6x^2 + 19x + 10$.

Factors of 6	Factors of 10
6, 1 3, 2	10, 1 5, 2

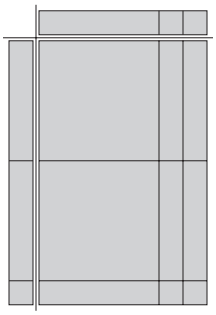
$6 \times 10 + 1 \times 1 = 61$
$6 \times 1 + 1 \times 10 = 16$
$6 \times 5 + 1 \times 2 = 32$
$6 \times 2 + 1 \times 5 = 17$
$3 \times 10 + 2 \times 1 = 32$
$3 \times 1 + 2 \times 10 = 23$
$3 \times 5 + 2 \times 2 = 19$
$6x^2 + 19x + 10 = (3x + 2)(2x + 5)$

Communicate Your Understanding

- Assign the questions in this section. Encourage students to work with a partner before discussing as a class. Question C1 is the best question to assess student understanding. (10 min)
- Use **BLM 5–8 Section 5.5 Practice Master** for remediation or extra practice.

Investigate Answers (page 242)

1.



2. Length of the rectangle: $(2x + 3)$; width of the rectangle: $(x + 1)$.

3. a) Length of the rectangle: $(3x + 2)$; width of the rectangle: $(x + 1)$.

b) Length of the rectangle: $(2x + 3)$; width of the rectangle: $(2x + 1)$.

4. a) Diagrams may vary. Length of the rectangle: $(2x + 1)$; width of the rectangle: $(x + 2)$.

b) Diagrams may vary. Length of the rectangle: $(5x + 3)$; width of the rectangle: $(x + 1)$.

5. Answers will vary. For example: Arrange algebra tiles representing the quadratic expression into a rectangle. The dimensions of the rectangle are the factors of the quadratic expression.

6. Answers will vary.

Communicate Your Understanding Responses (page 245)

C1. When a trinomial is factored the factored form contains two expressions. These represent the length and width of a rectangle, with the area equal to the original trinomial.

C2. Yes, because multiplication is commutative.

$$(2x + 3)(x + 3) = 2x^2 + 6x + 3x + 9 = 2x^2 + 9x + 9$$

$$(x + 3)(2x + 3) = 2x^2 + 3x + 6x + 9 = 2x^2 + 9x + 9$$

C3. Find the numbers that give a product of 45 and a sum of 18. Use these two numbers to rewrite $18x$. Then factor by grouping.

$5x^2 + 18x + 9 = (5x + 3)(x + 3)$

Practise

Common Errors

- Some students may ignore the value of a when determining the product.

R$_x$ Have students go back to the modelling activity to see how the product and sum are developed. They can use algebra tiles, *The Geometer's Sketchpad*®, or draw a rectangle. You can also have them verify by expanding their answers to check whether their factors are correct.

- For **questions 1** through **6**, assign a variety to provide practice for the students. Allow students to use algebra tiles if necessary; however, negatives are very difficult to model with algebra tiles.
- **Questions 8** and **9** require students to understand the reasons behind the product and sum. Remind them to consider how to come up with the product and sum. Use **A–6 Knowledge/Understanding General Scoring Rubric** when assessing students.
- **Questions 10** and **11** are communication questions and can be used to assess a student's understanding of the concepts. **Use A–9 Communication General Scoring Rubric** when assessing students.
- In **question 13**, remind students to let the height equal 0. Refer to Chapter 4 for how to find the intercepts of a parabola.
- **Question 14** asks students to factor a trinomial with decimal coefficients using a computer algebra system.
- **Question 16** is an Achievement Check. It is a good question for formative assessment.
- **Question 17** extends the concepts to trinomials of degree 4. It may help to replace x^2 with y and x^4 with y^2.
- In **question 19**, remind students that they will need to factor first.

Accommodations

Perceptual—Provide students with alternate solutions for factoring expressions of the form $ax^2 + bx + c$.

Motor—Let students work with an educational assistant who will act as a scribe and write the answers out for the questions.

Memory—Encourage students to work together to review and drill the pairs of factors for different integers. For example:

$12 = 12 \times 1$
$= 6 \times 2$
$= 4 \times 3$
$= (-12) \times (-1)$
$= (-6) \times (-2)$
$= (-4) \times (-3)$

$-8 = (-8) \times 1$
$= (-4) \times 2$
$= 8 \times (-1)$
$= 4 \times (-2)$

Student Success

Use graphic organizers to help students identify appropriate factor pairs.

Have groups of students use a placemat activity to factor quadratic expressions. Use **G–2 Placemat** to support this activity.

Refer to the introduction of this Teacher's Resource for more information about how to use a placemat strategy.

Achievement Check Sample Solution, question 16, page 247

Provide students with **BLM 5–9 Section 5.5 Achievement Check Rubric** to help them understand what is expected.

16. a) $5x^2 + 11x + 2$
$= 5x^2 + 10x + 1x + 2$
$= 5x(x + 2) + 1(x + 2)$
$= (x + 2)(5x + 1)$

b) Answers will vary. For example: $x^2 + x + 3$; there are not two numbers whose product is 3 and whose sum is 1.

c) $81 - 72x + 16x^2$ is a perfect square trinomial. Its factored form is $(9 - 4x)^2$.
The side length for the square is $9 - 4x$.
For a small perimeter, the side length must be small (but positive).
The smallest side length is 1, which occurs when $x = 2$.
The least possible measure for the perimeter is 4 units.

Literacy Connections

Discuss the Literacy Connections on page 243 with students. Have students think about other expressions that might mean the same as "break up" and "decompose," such as "pull apart."

The third item in the Key Concepts states that not all quadratic expressions can be factored over the integers. Again, discuss the term "integral values" so students know that when they write a quadratic expression in the form $ax^2 + bx + c$, the coefficients a, b, and c must be integers.

Add "decomposition" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	16
Reasoning and Proving	8–11, 16–18
Reflecting	7
Selecting Tools and Computational Strategies	n/a
Connecting	12–16, 19
Representing	1, 15
Communicating	7, 10, 11, 14, 16

Ongoing Assessment

- Use Achievement Check question 18 to monitor student success. See Achievement Check Answers and **BLM 5–9 Section 5.5 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.