6.3 Graph Quadratics Using the *x*-Intercepts

Student Text Pages
282–291

Suggested Timing
70 min

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 6–7 Section 6.3 Practice Master
- A–8 Application General Scoring Rubric
- A–9 Communication General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- Students will find the leap from solving equations by factoring to the x-intercepts of a parabola quite smooth, provided the link is made graphically and that $y = 0$. Introduce the lesson by discussing this connection.

Investigate

- **Investigate A** makes the link mentioned above. Students may have difficulty with step 2c), finding the y-coordinate of the vertex. Remind students to substitute the known x-coordinate into the relation to solve for y. (15 min)
- Using a graphing calculator, **Investigate B** provides a graphical interpretation of the use of the intercepts, as well as the effect of multiplying by a constant. (15 min)

Examples

- Go through the three parts of **Example 1**, or similar examples. Include examples where the coefficient of x^2 is 1, greater than 1, and negative. Discuss the vocabulary. Specifically, mention that the solution to an equation is known as the root. A quadratic relation has x-intercepts, which are also known as the zeros. (15 min)
- Students must be able to determine the equation of a parabola, given two points other than the vertex. **Example 2** models the process. (5 min)
- Include examples that are contextual, such as **Example 3**. Arches and paths of projectiles are both valid applications. (5 min)

Communicate Your Understanding

- Encourage students to write their answers to the questions before discussing them as a class. This strategy promotes improved mathematical literacy. (10 min)
- Use **BLM 6–7 Section 6.3 Practice Master** for remediation or extra practice.

Investigate Answers (pages 282–284)

A

1. The form $y = a(x - h)^2 + k$ is easiest to use for finding the vertex; the form $y = (ax + b)(cx + d)$ is easiest to use for finding the x-intercepts; the form $y = ax^2 + bx + c$ is easiest to use for finding the y-intercept.

2. a) Let $y = 0$ and solve for x.

$y = (x - 2)(x + 4)$

$0 = (x - 2)(x + 4)$

Either $x - 2 = 0$ or $x + 4 = 0$

$x = 2$ $\qquad$ $x = -4$

b) The x-coordinate of the vertex is halfway between the x-intercepts. Answers will vary.

c) The x-coordinate of the vertex is $x = -1$.

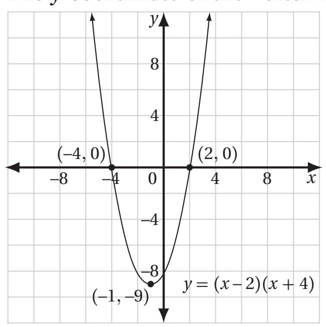
$y = (-1 - 2)(-1 + 4)$

$= (-3)(3)$

$= -9$

The y-coordinate of the vertex is -9.

d)



3. a) x-intercepts: -5, -1; axis of symmetry: $x = -3$; vertex: $(-3, -4)$

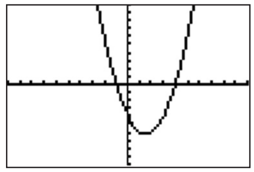
b) x-intercepts: -1.5, 0.5; axis of symmetry: $x = -0.5$; vertex: $(-0.5, -8)$

4. a) Answers may vary. For example:
Let $y = 0$ in the relation $y = x^2 + 3x - 10$. Solve the quadratic equation $x^2 + 3x - 10 = 0$ by factoring to find the x-intercepts -5 and 2. Determine the x-coordinate of the vertex by finding the x-value, $x = -1.5$, which is halfway between the x-intercepts -5 and 2. Substitute -1.5 for x in $y = x^2 + 3x - 10$ to find the y-coordinate of the vertex, $y = -12.25$. Graph the parabola using paper and pencil or a graphing calculator.

b) Answers may vary. For example:
Let $y = 0$ in the relation $y = 6x^2 - 17x + 5$. Solve the quadratic equation $6x^2 - 17x + 5 = 0$ by factoring to find the x-intercepts $\frac{1}{3}$ and $\frac{5}{2}$. Determine the x-coordinate of the vertex by finding the x-value, $x = \frac{17}{12}$, which is halfway between the x-intercepts $\frac{1}{3}$ and $\frac{5}{2}$. Substitute $\frac{17}{12}$ for x in $y = 6x^2 - 17x + 5$ to find the y-coordinate of the vertex, $y = -\frac{169}{24}$. Graph the parabola using paper and pencil or a graphing calculator.

5. Answers will vary.

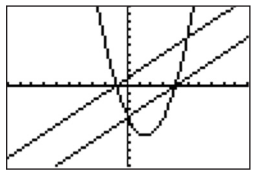
B

1. a) 

b) The zeros are –1 and 4. **c)** 0

2. a) points: (–1, 0), (4, 0); the x-intercepts are the roots

b) Answers may vary. For example:
Form two equations with x on the left side of the equation and the x-coordinates of each point on the right side of the equation. Write each equation with zero on the right side. Then, $x = -1$ is $x + 1 = 0$, and $x = 4$ is $x - 4 = 0$. The factors of the corresponding quadratic equation are $(x + 1)$ and $(x - 4)$.

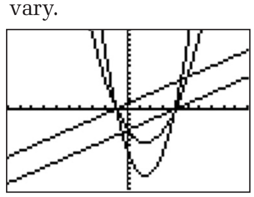
c) 

Answers may vary. For example:
The x-intercept of the line $y = x + 1$ is –1. This is the point (–1, 0) on the graph. The x-intercept of the line $y = x - 4$ is 4. This is the point (4, 0) on the graph. Therefore, the line $y = x + 1$ and the line $y = x - 4$ pass through the x-intercepts of the graph of $y = x^2 - 3x - 4$.

d) No. The graph $y = (x + 1)(x - 4)$ is the same as the graph of $y = x^2 - 3x - 4$.

3. a) At $x = 1$, the values of the factors are 2 and –3. The product of the values of the factors is $2 \times (-3)$, or –6. The point (1, –6) is a point on the graph of $y = x^2 - 3x - 4$.

b) At $x = -1$, the values of the factors are 0 and –5. The product of the values of the factors is $0 \times (-5)$, or 0. The point (–1, 0) is a point on the graph of $y = x^2 - 3x - 4$.
At $x = 4$, the values of the factors are 5 and 0. The product of the values of the factors is 5×0, or 0. The point (4, 0) is a point on the graph of $y = x^2 - 3x - 4$. One of the factors has a value of zero at each of the x-intercepts. Answers will vary.

4. a) 

b) Answers may vary. For example: The graph of $y = 2(x + 1)(x - 4)$ is different from the graph of $y = (x + 1)(x - 4)$. The graph is stretched vertically by a factor of 2. The graph is similar to the graph of $y = (x + 1)(x - 4)$. The parabola opens upward and has the same x-intercepts.

5. The x-intercepts of the graph of the quadratic relation are the roots of the corresponding quadratic equation.

Communicate Your Understanding Responses (page 288)

C1. Let $y = 0$, then factor to solve the right side of the equation to find the x-intercepts.

C2. The vertex of a quadratic relation is halfway between the x-intercepts and will have the same x-coordinate as the midpoint of the line segment connecting the x-intercepts of a quadratic relation.

Common Errors

- Some students may misinterpret the factors of a quadratic and not properly solve for x.

R_x Have students review solving quadratic equations from Section 6.2. You can also have the students verify their answers by substituting into the relation to solve for y and seeing if they get $y = 0$.

- Although this will mostly occur on summary tests or assignments, students may factor but not continue to solve.

R_x Remind students to read the question carefully. Review the meaning of the words "solve," "factor," "x-intercepts," and "roots."

Accommodations

Visual—Let students graph the x-intercepts of a quadratic relation on a Cartesian graph on a sheet of grid paper. Then, they can use paper folding to find the midpoint of the two points that are the x-intercepts to find the x-coordinate of the vertex of the parabola.

Perceptual—Encourage students who are having difficulty finding the x-intercepts of quadratic relations and/or zeros of quadratic equations to label the points on the x-axis with integers. This will show clearly that the y-value of each point on the x-axis is zero.

Language—Allow students to complete the questions in the language lab, where the questions are read to the students.

Memory—Provide students with opportunities to work with a partner to review the steps for finding the midpoint of two points on a graph.

Student Success

Using an inside/outside circle to have students describe how to graph quadratics using intercepts.

Refer to the introduction of this Teacher's Resource for more information about how to use an inside/outside circle strategy.

Practise

- **Questions 1** through **7** focus on specific skills. Parts of each question should be assigned and taken up in class before moving on to Connect and Apply.
- Impress upon students the importance of multiple representations: words, numbers, graphs, and algebraic expressions. **Questions 8** through **11**, and **16** are excellent applications. Stress the importance of defining the set of values for which each relation is valid, and when this must be considered. Use **A–8 Application General Scoring Rubric** when assessing students.
- **Questions 12** and **13** can be simplified by multiplying through by the common denominator. This will remove the fractions.
- **Question 14** provides a good opportunity for students to communicate their understanding. Use **A–9 Communication General Scoring Rubric** to assess responses.
- **Question 15** has students consider what makes the quadratic a perfect square.
- **Question 17** encourages students to think about more than just factoring. They must include other information, such as the height of the airplane and the width of the hangar at that height.
- **Question 18** can be done if students consider the form $y = a(x - h)^2 + k$ of a quadratic.
- **Question 20** requires students to think at a highly abstract level. They need to communicate their understanding of the vertex form of a parabola, and combine it with their understanding of x-intercepts.

Literacy Connections

As an extension, show the movie *October Sky*. This movie is based on a true story of teenagers who built rockets and used mathematics and quadratic equations to calculate how far their rockets flew.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	16, 17, 21, 22
Reasoning and Proving	6–9, 11–16, 18–20
Reflecting	5, 8, 12, 13, 17
Selecting Tools and Computational Strategies	11, 16, 17, 19, 21
Connecting	8–10, 12–17, 21, 22
Representing	3–8, 10–13, 15, 18, 20
Communicating	8, 12–14, 18, 20, 22

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.