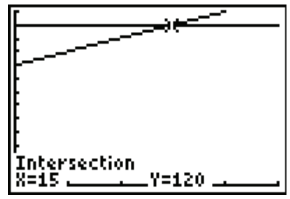
Answers

Principles of Mathematics 10 Exercise and Homework Book

Chapter 1

1.1 Connect English With Mathematics and Graphing Lines, pages 1–2

1. a) $2n + 5$
b) $\frac{1}{4}n - 3$
c) $m(n + 7)$
d) $n - \frac{1}{2}$
2. a) $4l$
b) $3d$
c) $0.30n$
d) $0.06p$
3. a) increased
b) added
c) more than
d) plus
e) subtracted from
f) less than
4. a) $\frac{1}{6}n + 15 = 42$
b) $3n - 4 = 6n + 5$
c) $4p = 320$
d) $b + h = 15$
5. a) (–4, –3)
b) (–1, 5)
c) (–2, 0)
d) (20, 18)
6. a) (4, 2)
b) (2, 0)
c) (2, –1)
d) $(3, -\frac{1}{3})$
7. a) (–0.73, –0.87)
b) (–0.29, 5.86)
c) (8.64, 28.86)
d) (–11, 5.7)
8. a) $C = 10x$
b) $C = 120$
c) (12, 120)
d) Answers may vary. For example: Sarah charges the same price for 12 h of work as Ads-R-Us Delivery Service charges for the season.
9. a) $E = 90 + 2.00n$
b) $E = 120$



c) 15 cellular phones
10. Kristen invested $600 in the account paying 4%/year interest and $400 in the bond paying 6.5% interest.
11. The three lines intersect at the same point, (1, 3).
12. Answers may vary. For example:
a) No. The two lines in the linear system are the same line and intersect everywhere. Alternatively, there are infinitely many solutions instead of just one.
b) No. The two lines in the linear system are parallel and distinct and do not intersect.
c) If the two lines in a linear system have the same slopes and *y*-intercepts, then they are the same lines and there are an infinite number of solutions. If the two lines in a linear system have the same slope and different *y*-intercepts, then the lines are parallel and there is no solution. If the two lines in a linear system have different slopes, then there is one solution.

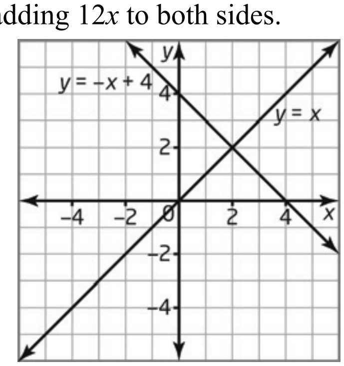
1.2 The Method of Substitution, pages 3–4

1. a) (1, 7)
b) (3, 2)
c) (4, –1)
d) (–4, 2)
2. a) equation 1: $x = -3y + 4$
b) equation 2: $y = -2x + 6$
3. a) No. (1, 1) satisfies the first equation but not the second equation.
b) Yes. (4, –3) satisfies both equations.
4. a) $x = 2, y = 1$
b) $a = 2, b = -6$
c) $x = 7, y = 1$
d) $m = -\frac{1}{2}, n = 3$
5. a) (2, –1)
b) (–2, 2)

6. Answers may vary. For example:
a) Let K represent the number of hours that Kyle read that week, and let S represent the number of hours that Santiago read that week.
b) $K = 2S$
c) $K + S = 24$
d) Kyle read 16 h that week and Santiago read 8 h that week.
7. Answers may vary. For example:
a) Let N represent the number of pairs of shorts Nyiri bought, and let R represent the number of pairs of shorts Raven bought. Then, $N + R = 9$.
b) $R = 2N - 6$
c) Nyiri bought five pairs of shorts and Raven bought four pairs of shorts.
d) Nyiri spent $79.95, before taxes; Raven spent $63.96, before taxes.
8. Answers may vary. For example:
a) Let C represent the cost to rent a hall, and let n represent the number of meals. Then, $C = 450 + 16n$; $C = 330 + 20n$.
b) The charges are the same at both halls for 30 guests.
9. The costs are the same for 2 h of labour.
10. The costs are the same for 2 h of labour.
11. a) (1, 4), (2, 5), (–1, 8)
b) Answers may vary. For example: Yes, because the slope of the first line, m_1, and the slope of the third line, m_3, are negative reciprocals.
12. a) (2, 1)
b) (4.2, –1.4)
c) (1.8, 0.4)
d) (–2, 2)
13. The point of intersection of the three lines is (0, 2); $k = 5$.

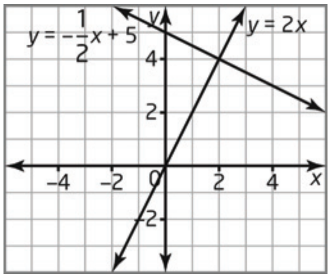
1.3 Investigate Equivalent Linear Relations and Equivalent Linear Systems, pages 5–6

1. B and C
2. C
3. Answers may vary. For example:
a) $2y = 10x - 6$; $3y = 15x - 9$
b) $8x + 6y = 24$
c) $3y = 2x + 15$; $6y = 4x + 30$
d) $2x + 2y = 14$; $3x + 3y = 21$
4. B and C
5. $k = 9$
6. a) $k = 8$
b) $k = -8$
c) $k = 6$
7. Answers may vary. For example:
a) Let m represent the number of males in Endi's math class, and let f represent the number of females in Endi's math class.
b) $m + f = 14$; an equivalent linear equation is $3m + 3f = 42$
8. Answers may vary. For example:
a) Let d represent the number of dimes in Marijan's piggy bank; let q represent the number of quarters in Marijan's piggy bank.
b) $d + q = 82$; an equivalent linear equation is $2d + 2q = 164$
9. Answers may vary. For example:
a) Let l represent the length of the rectangle, and let w represent the width of the rectangle.
b) $2l + 2w = 18$; an equivalent linear equation is $4l + 4w = 36$
10. The systems are equivalent because equation ③ is equation ① divided by two, and equation ④ is equation ② multiplied by two.
11. The systems are equivalent because equation ③ is equation ① multiplied by four, and equation ④ is equation ② multiplied by three.
12. a) Answers will vary. For example: The brackets are expanded using the distributive law, x- and y-terms are left on the left side of the equal sign, and the constants are moved to the right of the equal sign.
b) Equation ③ was obtained by multiplying both sides of equation ① by 15 and then adding $2x$ to both sides. Equation ④ was obtained by multiplying both sides of equation ② by –8 and then adding $12x$ to both sides.
13. a)



b) (2, 2)
c) $m_1 = 1$ and $m_2 = -1$. Since $m_1 \times m_2 = -1$, the lines are perpendicular to each other.
d) Answers may vary. For example: $2y = 2x$ and $2y = -2x + 8$; $-x + y = 0$ and $x + y = 4$.

14. a)



b) (2, 4)

c) $m_1 = 2$ and $m_2 = -\frac{1}{2}$. Since $m_1 \times m_2 = -1$, the lines are perpendicular to each other.

d) Answers may vary. For example: $2y = 4x$ and $2y = -x + 10$; $-2x + y = 0$ and $2x + 4y = 20$.

1.4 The Method of Elimination, pages 7–8

1. a) $x = 3, y = 1$
b) $x = -3, y = -1$
c) $x = -2, y = 3$
d) $x = 4, y = -2$
2. a) $x = 1, y = 2$
b) $x = -1, y = 3$
c) $x = 2, y = 1$
d) $x = -0.75, y = 2$
3. a) $x = 3, y = 1$
b) $x = -3, y = 2$
c) $x = -2, y = 5$
d) $x = -2, y = 2$
4. a) (2, 1)
b) (1, 0)
c) (1, 1)
d) (5, –1)
5. a) $\left(3, \frac{1}{5}\right)$
b) $\left(\frac{17}{5}, -\frac{1}{5}\right)$
c) (1, 1)
d) (23, –13)
e) $\left(\frac{23}{26}, -\frac{7}{26}\right)$
f) $\left(\frac{11}{16}, \frac{1}{8}\right)$
6. a) Zidane sold 18 pairs of hockey skates.
b) Zidane sold 14 pairs of figure skates.
7. a) Sadia sold 40 large boxes of popcorn.
b) Sadia sold 20 small boxes of popcorn.
8. a) $x = 1, y = 1$
b) $x = 1, y = 1$
c) Answers will vary.
9. a) $x = 70, y = 50$
b) $a = 80, b = 60$
10. a) $x = 1, y = -1$
b) $a = -2, b = -8$
c) $m = -2, n = 3$
11. Answers may vary. For example: Multiply the first equation by three and the second equation by five, and then subtract the equations. Solve for *y*, substitute this value of *y* into the first equation, and then solve for *x*.
12. Answers may vary. For example: You get 0 = 16, which is impossible. On a graph, the lines are parallel and distinct so there is no solution.
13. a) $a = 20, b = 6$
b) $m = 0.5, n = 4$
14. $\left(\frac{cn - gm}{dn - em}, \frac{ce - dg}{em - dn}\right)$

1.5 Solve Problems Using Linear Systems, pages 9–10

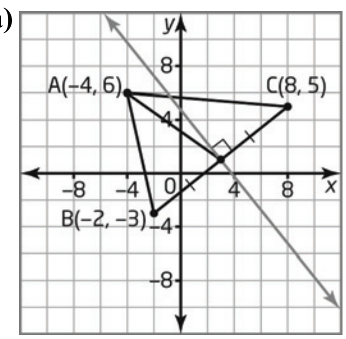
1. There will be 24 carnations and 6 roses in the flower arrangement.
2. Daniel has 82 Canadian stamps and 44 British stamps in his stamp collection.
3. Faiza and Terry sold 80 large bottles of water and 100 small bottles of water at the garage sale.
4. Marley invested $1500 at 7%/year and $550 at 6%/year.
5. In the tropical garden area at the Butterfly Museum there were 30 monarch butterflies and 48 painted lady butterflies.
6. 2.5 L of 1% milk needs to be mixed with 7.5 L of 5% milk to give 10 L of 4% milk.
7. Nathaniel should mix 22 L of 20% sulphuric acid with 8 L of 50% sulphuric acid to make the 30 L of 28% sulphuric acid.
8. 200 g of the chocolate mixture that contains 30% nuts, by mass, needs to be mixed with 300 g of the chocolate mixture that contains 20% nuts, by mass, to make 500 g of a chocolate mixture that will have 24% nuts, by mass.
9. a) The cost is the same at either Bridge Club for ten months of membership.
b) If Taha plans to join the Bridge Club for eight months, he should join the Bid Bridge Club.
c) If Taha plans to join the Bridge Club for one year, he should join the Best Bridge Club.

10. a) Mary ordered 15 large T-shirts.
b) Mary ordered 25 medium T-shirts.
11. 750 g of the 35% aluminum alloy and 250 g of the 55% aluminum alloy should be used to make 1000 g of an aluminum alloy that is 40% aluminum.
12. The students sold 43 apple pies and 40 lemon pies.
13. a) The cost is $15 per meal.
b) The cost is $80 per day for accommodation.
14. Ian's average canoeing speed is 3 km/h; the speed of the current is 1 km/h.
15. The average speed of the plane is 525 km/h and the wind speed is 75 km/h.
16. Donn should mix 400 g of 15-karat gold and 100 g of 10-karat gold.
17. a) Bob jogged at 8 km/h.
b) Bob walked at 5 km/h.
18. There were 170 AA batteries and 50 AA batteries in the recycling bin yesterday.
19. a) There are 15 red roses in the garden.
b) There are five pink roses in the garden.
20. a) There are six shelties in the park today.
b) There are three golden retrievers in the park today.

Chapter 1 Review, pages 11–12
1. a) $5n + 3$
b) $\frac{1}{3}x - 5$
c) $m + 4n$
d) $x - \frac{3}{4}$
2. a) Let n represent the number.
$3n + 4 = \frac{1}{2}n - 1$
b) Let H represent Hannah's age and J represent Jordan's age. $H + 5 = 2J - 7$
c) Let n represent the number of nickels and q the number of quarters.
$0.05n + 0.25q = 1.65$
3. a) The point of intersection of the lines is (–0.71, –1.82).
b) The point of intersection of the lines is (–0.77, –0.85).
4. a) $x = 4, y = 1$
b) $x = 6, y = -1$
c) $x = 1, y = -1$
d) $x = 5, y = -1$
e) $x = 3, y = 4$
f) $x = 1, y = 1$
5. There are 18 goldfish and 10 neon tetras in the aquarium.
6. Answers may vary. For example:
a) Let C represent the cost to rent a digital camera for one day, and let h represent the number of hours the camera is rented. Then, $C = 75$; $C = 35 + 5h$.
b) 8 h
7. C
8. a) $k = 20$
b) $k = 8$
9. Answers may vary. For example:
a) Let g represent the number of graphing calculators in Charlotte's class, and let s represent the number of scientific calculators in Charlotte's class.
b) $g + s = 14$; an equivalent linear equation is $2g + 2s = 28$
10. a) (4, 1)
b) (3, –2)
c) (8, 1)
d) (–0.5, 4)
11. a) $x = 4, y = 0$
b) $a = 0.5, b = 0.5$
c) $m = 2, n = 1$
d) $x = -22, y = 14$
12. 200 g of fertilizer that has 40% nitrogen should be mixed with 600 g of fertilizer that has 20% nitrogen to make 800 kg of fertilizer that has 25% nitrogen.
13. a) The charge is the same for a distance of 4 km.
b) The BLUE Cab Service should be the choice for distances greater than 4 km.
c) GREEN's Taxi Service should be the choice for distances less than 4 km.
14. a) There are 40 fiction books in Jean's library.
b) There are 12 non-fiction books in Jean's library.
15. $2000 at 8%/year and $440 at 7.5%/year
16. average speed of the speedboat in still water is 25 km/h; speed of the current is 5 km/h

Chapter 2

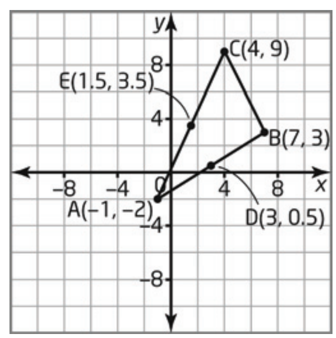
2.1 Midpoint of a Line Segment, pages 13–14

1. a) (–1, –3)
b) (–2, 1)
c) (–1, 1)
d) (1, 1)
2. a) (5, 5)
b) $\left(\frac{3}{2}, \frac{1}{2}\right)$
c) (0.9, 0.7)
d) $\left(-\frac{1}{7}, \frac{1}{9}\right)$
3. a) $m = \frac{3}{5}$
b) $m = -\frac{4}{5}$
4. (1, –2)
5. Answers may vary.
The Geometer's Sketchpad® example: Plot the endpoints, and construct the line segment between them. Construct the midpoint of this line segment. Then, select the midpoint and choose **Coordinates** from the **Measure** menu.
Cabri® Jr. example: Choose **Point** from the **F2** menu to plot the endpoints. Choose **Coord. & Eq.** from the **F5** menu, and check the placement of the endpoints. Adjust the endpoints if necessary. Choose **Segment** from the **F2** menu, and construct the line segment between the endpoints. Choose **Midpoint** from the **F3** menu, and construct the midpoint. Then, choose **Coord. & Eq.** again to display the coordinates of the midpoint.
6. (48.4, 50.1)
7. a) $y = -x + 2$
b) $y = \frac{1}{2}x - 1$
8. Answers may vary.
a) *The Geometer's Sketchpad*® example: Plot the vertices of ΔDEF, and construct the midpoint, M, of side EF. Construct a line through DM. Select the line, and choose **Equation** from the **Measure** menu.
Cabri® Jr. example: Choose **Point** from the **F2** menu, and plot the vertices of ΔDEF. Choose **Coord. & Eq.** from the **F5** menu, and check the placement of the vertices. Adjust the vertices if necessary. Choose **Segment** from the **F2** menu, and construct the line segment between vertices E and F. Select this line segment and choose **Midpoint** from the **F3** menu. Choose **Line** from the **F2** menu, and construct the line through the midpoint and vertex D. Then, choose **Coord. & Eq.** again to display the equation of the line.
b) Use similar procedures as outlined in part a).
9. Q(–4, –9); Answers may vary. For example: Let the coordinates of the other endpoint be Q(x, y). Solving the equation $\frac{x+8}{2} = 2$ gives $x = –4$. Similarly, solving the equation $\frac{y+3}{2} = -3$ gives $y = –9$.
10. a) $y = \frac{7}{4}x - \frac{9}{2}$
b) $y = -\frac{7}{5}x - \frac{26}{5}$
11. a)


b) $y = -\frac{5}{7}x + \frac{22}{7}$
c) $y = -\frac{5}{4}x + \frac{19}{4}$
12. a) (8a, 10b); These coordinates are the mean of the x-coordinates of the endpoints and the mean of the y-coordinates of the endpoints.
b) (1.5a, –1.5b); These coordinates are the mean of the x-coordinates of the endpoints and the mean of the y-coordinates of the endpoints.
13. a) Answers may vary. For example: Any point on the right bisector of a line segment is equidistant from the endpoints. Therefore, points on the right bisector of the line segment joining the two towns are possible locations for the water tower.
b) $y = -\frac{3}{4}x + 10$
14. (4, 8)
15. a) (3, 8) and (5, 13)

b) Answers may vary. For example: For the first dividing point, add $\frac{1}{3}$ of the run to the x-coordinate of the first endpoint and add $\frac{1}{3}$ of the rise to the y-coordinate of the first endpoint. For the second dividing point, add $\frac{2}{3}$ of the run to the x-coordinate of the first endpoint and add $\frac{2}{3}$ of the rise to the y-coordinate of the first endpoint.
16. a) A(2, 1), B(4, 9), C(6, 5)
b) Substituting the coordinates of each pair of vertices should give the coordinates of one of the midpoints.

2.2 Length of a Line Segment, pages 15–16
1. Estimates may vary. Calculated lengths:
a) 5
b) 7
c) $\sqrt{41}$
d) $\sqrt{40}$
2. a) 12.1
b) 11.3
c) 16.4
d) 1.2
3. a) $\sqrt{149}$
b) $2\sqrt{149}$
4. 6.1 km
5. a) DE = $\sqrt{160}$, EF = $\sqrt{136}$, DF = $\sqrt{104}$
b) scalene
c) 34.5 units
6. a) Applying the length formula shows that AB = BC = 5 and AC = 8. Therefore, ΔABC is isosceles.
b) Answers may vary. For example: the triangle with vertices L(–8, 0), M(0, 6), and N(8, 0).
7. a) $\sqrt{52}$
b) Answers may vary.
The Geometer's Sketchpad® example: Plot the points X, Y, and Z. Construct line segment YZ and its midpoint, M. Then, construct and measure line segment XM.
Cabri® Jr. example: Choose **Triangle** from the **F2** menu, and construct ΔXYZ. Choose **Coord. & Eq.** from the **F5** menu, and display the coordinates of the vertices. Adjust the vertices if necessary. Choose **Midpoint** from the **F3** menu, and select side YZ. Choose **Segment** from the **F2** menu, and construct the line segment from the midpoint to vertex X. Choose **Measure**/**D. & Length** from the **F5** menu, and select the median.
8. a) 36 square units
b) Answers may vary.
The Geometer's Sketchpad® example: Construct the triangle with vertices P, Q, and R. Then, select and measure the interior of ΔPQR.
Cabri® Jr. example: Choose **Triangle** from the **F2** menu, and construct ΔPQR. Choose **Coord. & Eq.** from the **F5** menu, and display the coordinates of the points. Adjust the position of a vertex if its coordinates are not correct. Choose **Measure**/**Area** from the **F5** menu, and select ΔPQR.
9. a) M(2, –2)
b) Since both distances are $\sqrt{74}$, point M is the midpoint.
c) Answers may vary. For example: Construct line segment ED. Construct the midpoint, M, of ED. Measure and compare the lengths of EM and MD.
10. \$55.42
11. a), b)



c) DE = $\sqrt{\frac{45}{4}} = \frac{1}{2}\sqrt{45}$ and BC = $\sqrt{45}$
d) $m_{DE} = m_{BC} = -2$. Therefore, DE is parallel to BC.
e) Answers may vary. For example: Use the length formula to show that each side of ΔABC is exactly half the length of the corresponding side of ΔABC.
12. a) 4.2
b) 6.7
c) 9
13. a) M(–1, –2)
b) AB = $\sqrt{128} = 8\sqrt{2}$ and

AM = $\sqrt{32}$ = $4\sqrt{2}$. Therefore, the length of AM is half of the length of AB.

c) AB = $\sqrt{128}$ = $8\sqrt{2}$ and MB = $\sqrt{32}$ = $4\sqrt{2}$. Therefore, the length of MB is half of the length of AB.

14. Answers may vary.

The Geometer's Sketchpad® example: Construct line segment AB. Construct the midpoint, M, of AB. Measure and compare the lengths of AM and MB.

Cabri® Jr. example: Choose **Segment** from the **F2** menu, and construct AB. Choose **Coord. & Eq.** from the **F5** menu, and display the coordinates of the endpoints. Adjust the endpoints if necessary. Choose **Midpoint** from the **F3** menu, and construct the midpoint of AB. Choose **Measure**/**D. & Length** from the **F5** menu. Then, select AM and MB.

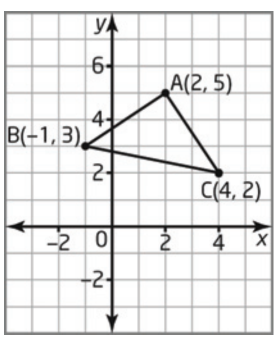
15. a) 3

b) Yes. Explanations may vary. For example: The equation $4 = \sqrt{(2-6)^2 + (y-3)^2}$ simplifies to $(y-3)^2 = 0$, so $y = 3$.

2.3 Apply Slope, Midpoint, and Length Formulas, pages 17–18

1. $y = 2x - 2$

2. a)



b) $m_{BA} \times m_{AC} = -1$, therefore ∠A is a right angle.

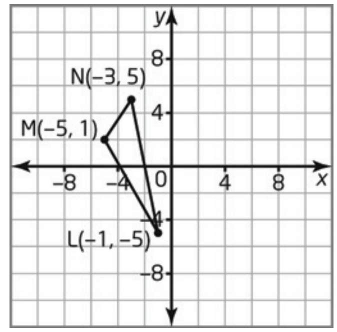
3. 6.5

4. a) P(3, 1), Q(–1, –1)

b) $m_{PQ} = m_{YZ} = \frac{1}{2}$. Since $m_{PQ} = m_{YZ}$, PQ is parallel to XY.

c) PQ = $\sqrt{20}$ = $2\sqrt{5}$ and XY = $\sqrt{80}$ = $4\sqrt{5}$. Therefore, the length of PQ is half the length of XY.

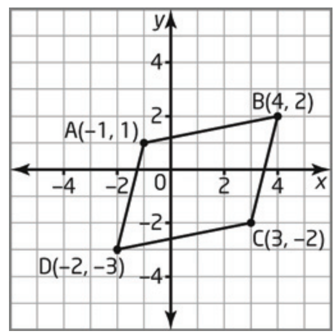
5. a)



b) $y = -\frac{8}{3}x - \frac{23}{3}$

c) $\sqrt{73}$

6. a)



b) $m_{AB} = m_{CD} = \frac{1}{5}$ and $m_{AC} = m_{BD} = 4$. Therefore, opposite sides are parallel and ABCD is a parallelogram.

c) 18.4 units

7. Answers may vary.

The Geometer's Sketchpad® example: Construct ABCD and measure the slope of each side. These slopes show that the opposite sides are parallel. Then, measure the lengths of the sides of ABCD and determine the sum.

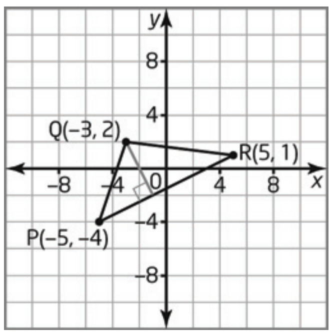
Cabri® Jr. example: Choose **Quad.** from the **F2** menu, and construct ABCD. Choose **Coord. & Eq.** from the **F5** menu, and display the coordinates of the vertices. Adjust the vertices if necessary. Choose **Measure**/**Slope** from the **F5** menu. Then, select AB, BC, CD, and DA. Choose **Measure**/**D. & Length** from the **F5** menu. Then, select AB, BC, CD, and DA. Determine the sum.

8. a) 11.3

b) (–1, 3)

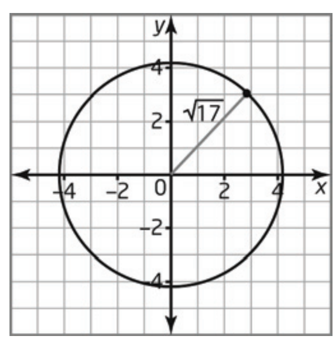
c) 5.7

9. DE = DC = $\sqrt{40}$ and EC = $\sqrt{32}$. Since DE = DC, ΔDEC is isosceles.
10. 4.5
11. 6.7
12. a) D(5, 1)
b) AC = BD = 10
c) The midpoint of both diagonals is M(0, 1). Therefore, the diagonals bisect each other.
13. a)



b) $y = -2x - 4$
c) $\sqrt{20}$
14. Answers may vary.
The Geometer's Sketchpad® example:
a) Construct the triangle with vertices P, Q, and R. Construct the perpendicular from Q to PR. Construct point M, the point of intersection of the perpendicular and PR. Select the perpendicular, and choose **Equation** from the **Measure** menu.
b) Construct line segment QM. Measure the length of QM.
Cabri® Jr. example:
a) Choose **Triangle** from the **F2** menu, and construct ΔPQR. Choose **Coord. & Eq.** from the **F5** menu, and display the coordinates of the vertices. Adjust the vertices if necessary. Choose **Perp.** from the **F3** menu, and construct the perpendicular from Q to PR. Choose **Coord. & Eq.** from the **F5** menu, and select the perpendicular.
b) Choose **Measure**/**D. & Length** from the **F5** menu, and select the endpoints of the altitude.
15. a) 8.2 km
b) Answers may vary. For example: The shortest route might be blocked by fences or thick woods, or it might involve trespassing on private land.
16. Answers may vary.
a) Plot the points P(2, 4), A(–7, 6), and B(–5, –2). Construct line segment AB and the perpendicular from AB to P. Construct point C where the perpendicular meets AB. Measure the length of PC.
17. a) $\sqrt{34}$
b) E(7, –4)
18. 2.8

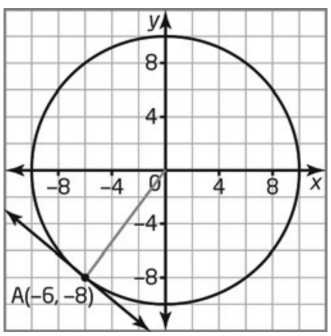
2.4 Equation for a Circle, pages 19–20
1. a) $x^2 + y^2 = 16$
b) $x^2 + y^2 = 49$
c) $x^2 + y^2 = 13$
d) $x^2 + y^2 = 64$
2. a) 3; points include (0, 3), (3, 0), (–3, 0), and (0, –3)
b) 9; points include (0, 9), (9, 0), (–9, 0), and (0, –9)
c) $\sqrt{40}$; points include (6, 2), (–6, 2), (6, –2), and (–6, –2)
d) 1.1; points include (0, 1.1), (1.1, 0), (–1.1, 0), and (0, –1.1)
3. a) $x^2 + y^2 = 25$
b) $x^2 + y^2 = 29$
c) $x^2 + y^2 = 52$
d) $x^2 + y^2 = 50$
4. a) outside the circle
b) on the circle
c) inside the circle
5. $x^2 + y^2 = 25$
6. a) $a = -4$ or $a = 4$
b) Graph the circle $x^2 + y^2 = 25$. The points (4, –3) and (–4, –3) are both on this circle.
7. a) 31.4 m
b) 78.5 m^2
8. a)



b) The coordinates (–4, –1) and (1, 4) both satisfy the equation of the circle.
c) $y = -x$
d) The coordinates (0, 0) satisfy the equation $y = -x$.

9. Answers may vary.
The Geometer's Sketchpad® example: Construct the circle $x^2 + y^2 = 17$ and a line segment between points D and E. Construct the right bisector of the line segment. Select the right bisector, and choose **Equation** from the **Measure** menu. Observe that the right bisector passes through the centre of the circle.
Cabri® Jr. example: Choose **Circle** from the **F2** menu, and construct the circle $x^2 + y^2 = 17$. Choose **Segment** from the **F2** menu, and construct a line segment between points D and E. Choose **Perp. Bis.** from the **F3** menu, and select the line segment. Choose **Coord. & Eq.** from the **F5** menu, and select the right bisector. Observe that the right bisector passes through the centre of the circle.

10. a), **c)**, **d)**



b) The coordinates of point A satisfy the equation $x^2 + y^2 = 100$.

c) $m_{AO} = \frac{4}{3}$

d) $m = -\frac{3}{4}$

e) $y = -\frac{3}{4}x - \frac{25}{2}$

11. a) $x^2 + y^2 = 625$

b) 156.2 s

c) Answers may vary. For example: The maple leaf remains at a constant distance from the place where the pebble was dropped.

12. a) The region is inside the circle defined by the equation $x^2 + y^2 = 36$.

b) The region is outside the circle defined by the equation $x^2 + y^2 = 49$.

Chapter 2 Review, pages 21–22

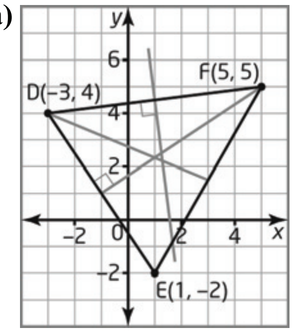
1. a) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

b) $\left(\frac{1}{2}, \frac{1}{2}\right)$

2. a) (–1, –3)

b) $\left(\frac{3}{2}, -\frac{5}{2}\right)$

3. a)



b) $y = -\frac{5}{12}x + \frac{11}{4}$

c) $y = -8x + \frac{25}{2}$

d) $y = \frac{2}{3}x + \frac{5}{3}$

4. a) $\sqrt{65}$

b) $\sqrt{61}$

5. a) 9.9

b) 12.7

6. a) 6.3

b) XY = XZ = $\sqrt{50}$ and YZ = $\sqrt{40}$. Therefore, ΔXYZ is isosceles.

c) 20.5 units

d) Answers may vary. For example: Construct ΔXYZ. Measure the side lengths. Determine the sum of the side lengths.

7. a) $m_{PR} = \frac{1}{2}$ and $m_{RQ} = -2$. Therefore, ∠PRQ = 90° and ΔPQR is a right triangle. Alternatively, use QR = $\sqrt{20}$, PR = $\sqrt{80}$, and PQ = 10 with the Pythagorean theorem to show $QR^2 + PR^2 = PQ^2$.

b) (–1, 1)

c) The distance from the midpoint to each vertex is 5.

8. a) 70.7 m

b) Yes. The coordinates satisfy the equation $y = x + 15$.

c) No. The coordinates do not satisfy the equation $y = x + 15$.

9. a) $x^2 + y^2 = 25$

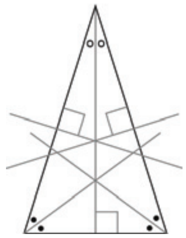
b) $x^2 + y^2 = 20$
c) $x^2 + y^2 = 12.25$
10. a) $x^2 + y^2 = 27.04$
b) $x^2 + y^2 = 18$
c) $x^2 + y^2 = 100$
d) $x^2 + y^2 = 13$
11. a) Since both (–4, –1) and (4, 1) satisfy the equation $x^2 + y^2 = 17$, the line segment connecting them is a chord of the circle.
b) $y = -4x$
c) Since (0, 0) satisfies the equation $y = -4x$, the line passes through the centre of the circle.
12. a) Point D lies on the circle defined by $x^2 + y^2 = 29$.
b) $y = -\frac{5}{2}x$
c) $y = \frac{2}{5}x + \frac{29}{5}$

Chapter 3

3.1 Investigate Properties of Triangles, pages 23–24

1. 8 square units
2. 40 square units
3. a) Diagrams may vary. For example: Let ΔABC be an isosceles right triangle with BA = BC and ∠ABC = 90°. The midpoint of the hypotenuse AC is the point D.
b) Answer will vary.
c) Answers may vary. For example: Construct isosceles right ΔABC with BA = BC and ∠ABC = 90°. Construct the midpoint of AC as point D. Construct the right bisector of AC. Measure ∠ABD and ∠CBD.
4. The median from vertex D, the bisector of ∠D, the right bisector of side EF, and the altitude from vertex D coincide; the median from vertex E, the bisector of ∠E, the right bisector of side DF, and the altitude from vertex E coincide; the median from vertex F, the bisector of ∠F, the right bisector of side DE, and the altitude from vertex F coincide.
5. a) Diagrams will vary.
b) Answers will vary.
c) centroid
6. Answers may vary. For example: Construct any triangle and the median from each of the vertices. Observe the point of intersection of the three medians while dragging the vertices of the triangle. For each median, measure the distance from the point of intersection to each endpoint. Compare these distances while dragging the vertices of the triangle. The medians always meet at a single point and are divided in a 2:1 ratio.
7. a) Diagrams will vary.
b) Answers will vary.
c) incentre
8. Answers may vary. For example: Construct any triangle and the bisector of each of its angles. Observe the point of intersection of the three angle bisectors while dragging the vertices of the triangle. Measure the perpendicular distance from the point of intersection to each side. Compare these distances while dragging the vertices of the triangle. The angle bisectors always meet at a single point, which is equidistant from the sides of the triangle.
9. a) Diagrams will vary.
b) Answers will vary.
c) circumcentre
10. Answers may vary. For example: Construct any triangle and the right bisector of each of its sides. Observe the point of intersection of the three right bisectors while dragging the vertices of the triangle. Measure the distance from the point of intersection to each vertex. Compare these distances while dragging the vertices of the triangle. The right bisectors always meet at a single point, which is equidistant from the vertices of the triangle.
11. a) Diagrams will vary.
b) Answers will vary.
c) orthocentre
12. Answers may vary. For example: Construct any triangle and the altitude from each of its vertices. Observe the point of intersection of the three altitudes while dragging the vertices of the triangle. The altitudes always meet at a single point.
13. Answers may vary. For example: Isosceles right triangles have two equal sides and two equal angles. Each of the equal angles measures 45°. The median drawn from the vertex between the equal sides is the same as the angle bisector of the same vertex. The altitude drawn from the vertex between the equal sides is the same as the perpendicular bisector of the opposite side.

14.



Mary is incorrect. In an isosceles triangle, the incentre does not coincide with the circumcentre.

15. No. Explanations may vary. The triangle could be isosceles since the median from the vertex between the equal sides is also an altitude.

16. No. Explanations may vary. This is true for all right triangles.

17. Yes. Explanations may vary. This is true for all right triangles.

18. a)–c) Answers will vary.

3.2 Verify Properties of Triangles, pages 25–26

1. a) $y = \frac{2}{3}x + \frac{1}{3}$

b) $y = -\frac{4}{5}x + \frac{6}{5}$

c) $y = -\frac{4}{5}x + \frac{3}{5}$

d) $y = -\frac{5}{3}x + \frac{4}{3}$

2. a) $m_{PQ} = m_{YZ} = -\frac{1}{2}$

b) PQ = $2\sqrt{5}$, YZ = $4\sqrt{5}$

3. a) ST = SU = $\sqrt{52}$ and TU = $\sqrt{104}$.

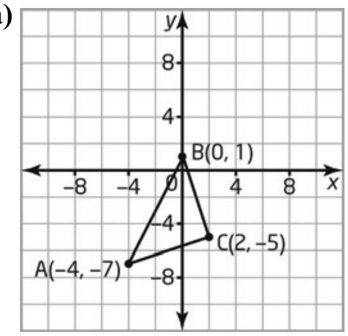
b) The midpoint of TU is M(–3, –3). Then, the median from vertex S is SM. $m_{SM} = \frac{1}{5}$ and $m_{TU} = -5$. Since $m_{SM} \times m_{TU} = -1$, SM is perpendicular to TU, which makes SM an altitude of the triangle.

4. Answers may vary. For example:

a) Construct ΔSTU. Measure and compare the lengths of ST, TU, and SU.

b) Construct the midpoint, M, of side TU. Construct line segment SM. Measure ∠SMU.

5. a)



b) AC = BC = $\sqrt{40}$ and AB = $\sqrt{80}$

c) $m_{BC} = -3$, $m_{AC} = \frac{1}{3}$, $m_{AB} = 2$

d) Since AC = BC and $m_{AC} \times m_{BC} = -1$, ΔABC is an isosceles right triangle.

6. Answers may vary. For example:

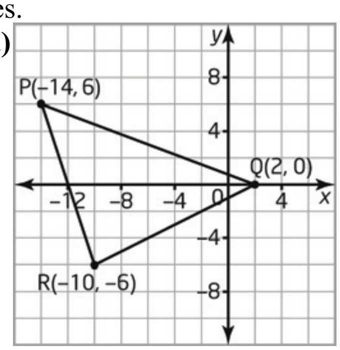
a) Construct ΔABC.

b) Measure the lengths of the three sides.

c) Measure the slopes of the three sides.

d) Compare the lengths and slopes of the three sides.

7. a)



b) Y(–6, 3), Z(–12, 0) **c)** $m_{RQ} = m_{YZ} = \frac{1}{2}$

d) RQ = $6\sqrt{5}$ and YZ = $3\sqrt{5}$.

8. a) scalene right triangle

b) GH = $\sqrt{104}$, GF = $\sqrt{338}$, HF = $\sqrt{234}$, and $m_{GH} \times m_{HF} = -1$.

c) 43.9 units

d) 78 square units

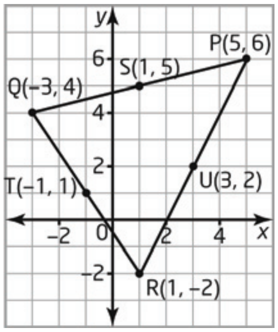
9. Answers may vary. For example:

a) Construct ΔFGH.

b) Measure and compare the lengths and slopes of the three sides.

c) Calculate the sum of the lengths of the sides.

d) Measure the area of ΔFGH.

10. a) 

b) PQ = $2\sqrt{17}$, QR = $2\sqrt{13}$, RP = $4\sqrt{5}$;
TU = $\sqrt{17}$, SU = $\sqrt{13}$, ST = $2\sqrt{5}$.

c) $\frac{TU}{PQ} = \frac{SU}{QR} = \frac{ST}{RP} = \frac{1}{2}$

11. Answers may vary. For example:
a) Construct ΔPQR and the midpoints of its sides, S, T, and U. Display the coordinates of the points.
b) Measure the lengths of the corresponding sides.
c) Calculate and compare the ratio of the side lengths.

12. a) AC = BC = 5, $m_{AC} \times m_{CB} = -1$
b) Since $AC^2 + BC^2 = AB^2$, ΔABC is a right triangle. Since AC = BC, ΔABC is also isosceles.
c) 12.5 square units

13. a) right bisector of side OC: $y = -x + 3$; right bisector of side CD: $y = x - 3$; right bisector of side OD: $x = 3$.
b) (3, 0)
c) isosceles right triangle; OC = CD = $\sqrt{18}$, $m_{OC} \times m_{CD} = -1$
d) The circumcentre is located at the midpoint of the hypotenuse for this triangle.

14. a) median from vertex P: $y = 1$; median from vertex Q: $y = -3x - 4$; median from vertex R: $y = -\frac{6}{7}x - \frac{3}{7}$.
b) $C\left(-\frac{5}{3}, 1\right)$
c) CP = $\frac{10}{3}$ and CT = $\frac{5}{3}$, CQ = $\frac{4}{3}\sqrt{10}$ and CU = $\frac{2}{3}\sqrt{10}$, CR = $\frac{2}{3}\sqrt{85}$ and CS = $\frac{1}{3}\sqrt{85}$.

3.3 Investigate Properties of Quadrilaterals, pages 27–28

1. a) PT = TR, QT = TS
b) WA = AY, XA = AZ

2. a) NQ is parallel to OP; NO is parallel to QP.
b) NQ = OP, NO = QP.

3. Answers may vary. For example: The diagonals of a parallelogram bisect each other.

4. a) DE, KF, and HG are parallel.
b) 4
c) 5

5. a) JI is parallel to KL, JK is parallel to IL.
b) JI = KL, JK = IL

6. a)–c) Answers will vary.

7. Answers may vary. For example:
a) Draw a kite. Draw the diagonals. Measure the angle between the two diagonals.
b) Measure and compare the lengths of the sides of the two triangles formed by the shorter diagonal. Measure and compare the lengths of the sides of the two triangles formed by the longer diagonal.

8. Yes, Greg is correct. Answers will vary.

9. Square: the diagonals are equal in length, the diagonals bisect each other, the diagonals are perpendicular to each other; rectangle: the diagonals are equal in length, the diagonals bisect each other, the diagonals are not perpendicular to each other; parallelogram: the diagonals are not equal in length; the diagonals bisect each other, the diagonals are not perpendicular to each other; rhombus: the diagonals are not equal in length, the diagonals bisect each other, the diagonals are perpendicular to each other; kite: the diagonals are not equal in length, the longer diagonal bisects the shorter diagonal, the diagonals are perpendicular to each other.

10. a) Parallelogram. Explanations may vary. For example: Since PQ and RS bisect each other, but not at right angles, quadrilateral PRQS contains two pairs of congruent triangles (side-angle-side). Therefore, the opposite sides of the quadrilateral are equal in length.
b) Parallelogram. Explanations may vary.

11. Answers may vary. For example:
a) The balance point is at the point of intersection of the diagonals.
b) Find the balance point of a cardboard parallelogram.

12. Answers may vary. For example:
a) The midpoint of AB is M(–4, 0); the midpoint of BC is N(0, –3); the midpoint of CD is O(4, 0); and the midpoint of DA is P(0, 3). MN = NO = OP = PM = 5.
b) The midpoint of PN is the same as the midpoint of MO, (0, 0).
c) The slope of diagonal MO is $m = 0$; the slope of diagonal NP is m = undefined. Therefore, the diagonals are perpendicular to each other.
13. a), **b)** Answers will vary.
14. a), **b)** Answers will vary.

3.4 Verify Properties of Quadrilaterals, pages 29–30

1. Since $m_{AB} = m_{CD} = 1$ and $m_{DA} = m_{BC} = -1$, all adjacent sides are perpendicular.

2. Since $m_{DE} = m_{FG} = -\frac{1}{3}$ and $m_{EF} = m_{GD} = -4$, opposite sides are parallel.

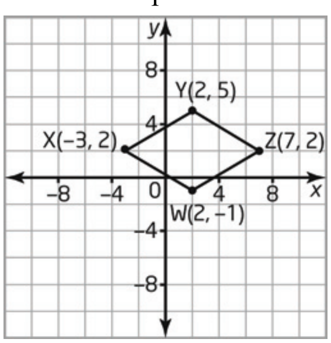
3. $m_{PQ} = m_{SR} = \frac{1}{3}$

4. JK = LK = $\sqrt{17}$, JM = LM = $\sqrt{73}$

5. a) J(–4, 0), K(0, –3), L(4, 0), M(0, 3)

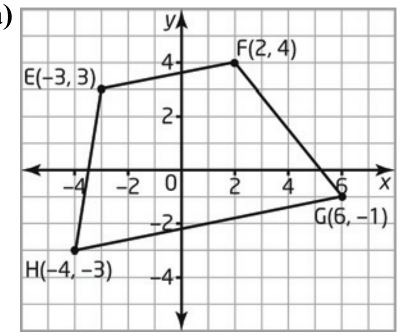
b) Since $m_{JK} = m_{LM} = -\frac{3}{4}$ and $m_{JM} = m_{KL} = \frac{3}{4}$, opposite sides are parallel.

6. a)



b) XY = YZ = ZW = WX = $\sqrt{34}$
c) The midpoint of diagonal XZ and the midpoint of the diagonal YW are both located at (2, 2).
d) Since of the slope of diagonal XZ is zero and the slope of diagonal YW is undefined, the diagonals of XYZW are perpendicular to each other.
7. Answers may vary. For example:
a) Construct quadrilateral XYZW.
b) Measure and compare the lengths of XY, YZ, ZW, and WX.
c) Construct diagonals XZ and YW and their point of intersection, T. Measure and compare the lengths of XT, ZT, YT, and WT.
d) Measure and compare the slopes of XZ and YW.

8. a)



b) $m_{EF} = m_{XY} = m_{GH} = \frac{1}{5}$

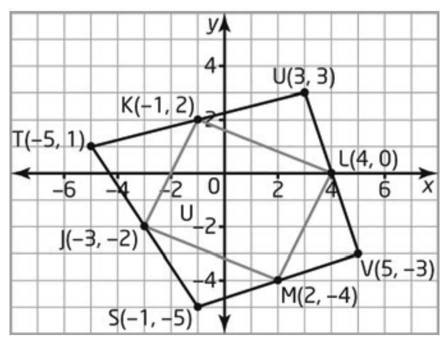
9. Answers may vary. For example: Construct trapezoid EFGH and midpoints, X and Y, of FG and EH, respectively. Construct line segment XY. Measure and compare the slopes of EF, XY, and GH.

10. a) $E\left(-3, \frac{1}{2}\right)$, F(3, 2)

b) $m_{AB} = m_{DC} = m_{EF} = \frac{1}{4}$

c) AB = $\sqrt{17}$, CD = $2\sqrt{17}$, EF = $\frac{3}{2}\sqrt{17}$

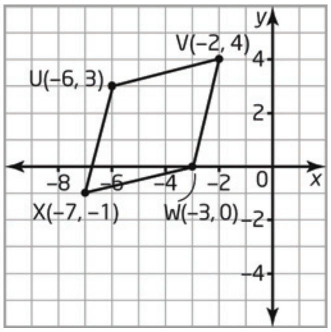
11. a), **b)**



c) $m_{JK} = m_{LM} = 2$, $m_{KL} = m_{MJ} = -\frac{2}{5}$

d) JK = ML = $\sqrt{20}$, JM = KL = $\sqrt{29}$
e) Parallelogram. Answers may vary. For example: The opposite sides of the quadrilateral are parallel and equal in length.
12. Answers may vary. For example:
a) Construct quadrilateral STUV.
b) Construct the midpoint of each side and display the coordinates. Construct line segments joining adjacent midpoints.

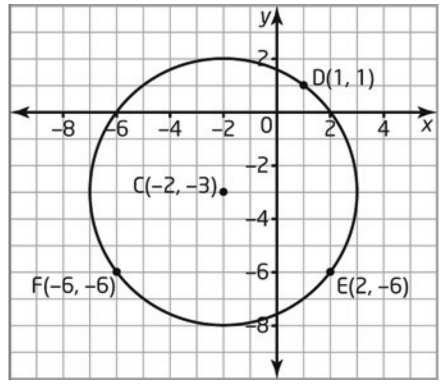
c) Measure and compare the slopes of the sides of JKLM.
d) Measure and compare the lengths of the sides of JKLM.
13. a)



b) Since $m_{AB} = m_{CD} = -1$ and $m_{BC} = m_{DA} = 1$, the slopes of adjacent sides are negative reciprocals.
14. Answers may vary. For example:
a) Construct quadrilateral UVWX.
b) Construct the midpoint of each side and line segments joining the adjacent midpoints. Measure the angle at each vertex of the new quadrilateral.
15. D$\left(\frac{p+r}{2}, \frac{q+s}{2}\right)$, E$\left(\frac{r+t}{2}, \frac{s+u}{2}\right)$,
F$\left(\frac{t+v}{2}, \frac{u+w}{2}\right)$, G$\left(\frac{v+p}{2}, \frac{w+q}{2}\right)$
b) $m_{DE} = m_{FG} = \frac{q-u}{p-t}$ and $m_{EF} = m_{DG} = \frac{w-s}{v-r}$
16. a)–c) Answers will vary.
17. a), **b)**Answers will vary.

3.5 Properties of Circles, pages 31–32

1. a) M$\left(\frac{1}{2}, \frac{7}{2}\right)$
b) $m_{PQ} = -\frac{1}{7}$
c) Since $m_{PQ} \times m_{OM} = -1$, OM is perpendicular to PQ.
2. a) CD = CE = CF = 5
b)



3. a) PS = QS = RS = $\sqrt{26}$
b) $\sqrt{26}$
4. (–2, 5)
5. Answers may vary. For example: Construct line segments DE and EF. Construct the right bisectors of DE and EF. Construct the point of intersection of the right bisectors and display its coordinates.
6. a) The centre of the circle, O(0, 0), satisfies $y = -3x$, an equation for the right bisector of the chord AB.
b) $\sqrt{20}$
7. a) Answers may vary. For example: Substituting into the distance formula shows that the distance from the origin to any point that satisfies $x^2 + y^2 = 41$ is $\sqrt{41}$.
b) The coordinates of points C and D satisfy $x^2 + y^2 = 41$.
c) $m_{CD} = -1$ and $m_{OM} = -1$, so $m_{CD} \times m_{OM} = 1$, where M is the midpoint of the chord CD.
8. (–4, 2); $\sqrt{10}$
9. Answers may vary. For example: Draw any two chords on the circular piece of wood. Then, draw the right bisector of each chord. Mark the point of intersection of the right bisectors as the location for the marker.
10. Answers may vary. For example: Fold Ottawa onto Kingston, and fold Toronto onto Kingston. Look for a ski hill near the intersection of the two folds.
11. 6.4 m^2
12. a) (9, 10)
b) Answers will vary. For example: Distances from the libraries to the school are minimized.
c) Answers will vary. For example: No suitable site may be available at the centre of the circle.
13. Answers may vary. For example: Join point D to point O. Since CO = DO = EO,

∠ODC = ∠OCD and ∠ODE = ∠OED. The sum of the angles in ΔCDE is

180° = ∠ODC + ∠OCD + ∠ODE + ∠OED
180° = 2∠ODC + 2∠ODE
90° = ∠ODC + ∠ODE

Since ∠CDE = ∠ODC + ∠ODE,
∠CDE = 90°.

14. Answers may vary. For example: Construct a circle with diameter CE. Construct any point D on the circle and measure ∠CDE.

15. a) The coordinates of points A and B satisfy $x^2 + y^2 = 25$; AB = 10.

b) Answers may vary. For example:
C(4, –3).

c) AC = BC = $\sqrt{50}$ and AB = 10. Use the Pythagorean theorem to show
$AC^2 + BC^2 = AB^2$.

16. a) The coordinates of points P and Q satisfy $x^2 + y^2 = 65$.

b) The centre of the circle, O(0, 0), satisfies $y = -x$, an equation for the right bisector of the chord PQ.

17. a)–c) Diagrams may vary.

d) Since O(0, 0) is the midpoint of CD, EO is the median of vertex E.

18. Answers may vary. For example: Construct a circle with diameter CD centred at O(0, 0). Construct any point E on the circle. Construct line segments CE, ED, CD, and EO, and measure CO and DO.

19. a)–c) Diagrams may vary.

d) Answers will vary. For example:
OA and OB are equal radii, AM = BM, and OM is common to ΔOMA and ΔOMB. Therefore, the triangles are congruent (side-side-side).

20. Answers will vary. For example: Construct a circle centred at O(0, 0). Construct any chord AB on the circle and its midpoint, M. Measure and compare the side lengths in ΔOMA and ΔOMB.

Chapter 3 Review, pages 33–34

1. Answers may vary. For example:

a) The right bisector of a side of a triangle is a line passing through the midpoint of a side of the triangle that is perpendicular to the side.

b) The three right bisectors of the sides of a triangle meet at a single point, called the circumcentre. The circumcentre is the centre of the circle passing through all three vertices. If the circumcentre is located on one side of the triangle, then the opposite angle is a right angle.

c) Construct a triangle and the right bisector of each side. Measure the distance from the point of intersection of the right bisectors to each vertex. Observe the point of intersection of the right bisectors and its distance from each vertex while dragging the vertices of the original triangle.

2. Answers will vary. For example:

a) Consider ΔABC that is an isosceles triangle with AB = AC and midpoints, M and N, of sides AB and AC, respectively. Since AB = AC, ∠MBC = ∠NCB. MB = NC, and side BC is common to ΔMBC and ΔNCB. Therefore, ΔMBC ≅ ΔNCB (side-angle-side), and MC = NB.

b) Consider ΔABC that is an isosceles triangle with AB = AC and midpoint P of side BC. Since AB = AC, ∠ABC = ∠ACB, and BP = CP, ΔABP ≅ ΔACP (side-angle-side) and ∠BAP = ∠CAP. The sum of the angles in ΔABC is

180° = ∠ABC + ∠BAP + ∠CAP + ∠ACB
180° = 2∠ABP + 2∠BAP
90° = ∠ABP + ∠BAP

Since ∠BPA = 180° – (∠ABP + ∠BAP), ∠BPA = 90°. The median from vertex A is also an altitude.

3. Answers may vary. For example:

a) isosceles triangle, equilateral triangle, isosceles right triangle

b) equilateral triangle

4. a) PQ = PR = $\sqrt{34}$ and QR = 6.

b) $M\left(-\frac{1}{2}, \frac{1}{2}\right)$, $N\left(-\frac{1}{2}, \frac{7}{2}\right)$

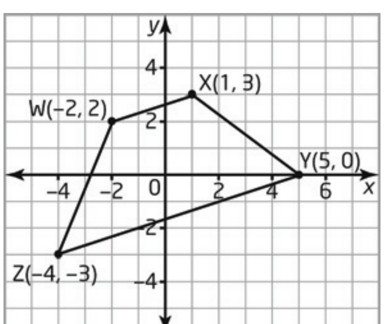
c) RM = QN = $\sqrt{\frac{53}{2}}$

5. a) isosceles triangle; DE = EF = $\sqrt{53}$ and DF = $\sqrt{50}$.

b) The midpoint of DF is $M\left(-\frac{3}{2}, -\frac{1}{2}\right)$. Since DE = EF, DM = FM, and EM = EM, ΔDEM ≅ ΔFEM (side-side-side) and ∠DEM = ∠FEM.

6. a) Since $m_{BA} = \frac{4}{3}$ and $m_{AC} = -\frac{3}{4}$,

$m_{PQ} \times m_{QR} = -1$ and ΔABC is a right triangle.
b) Answers may vary. For example: You could also check that the sides satisfy the Pythagorean theorem, where the hypotenuse of the triangle is BC.
c) midpoint of BC is $M\left(\frac{1}{2}, \frac{3}{2}\right)$;
$AM = BM = CM = \sqrt{\frac{25}{2}}$
d) 12.5 square units
7. Answers may vary. For example:
a) The diagonals of a rectangle are equal in length and bisect each other.
b) The diagonals of a rhombus are perpendicular and bisect each other.
8. a) square, parallelogram, kite
b) square: the diagonals of a square are equal and bisect each other at right angles; parallelogram: the diagonals of a parallelogram are not equal in length and bisect each other; kite: the diagonals of a kite are perpendicular and the longer diagonal bisects the shorter diagonal.
9. Answers may vary. For example:
b) The longer diagonal is AC.
c) Since AB = AD, CB = CD, and AC = AC, $\Delta ABC \cong \Delta ADC$ (side-side-side) and the diagonal AC of the kite bisects the area of the kite ABCD.
10. $m_{JK} = m_{LM} = 5$ and $m_{KL} = m_{JM} = \frac{1}{6}$.
11. a)–c) Answers will vary.
12. Answers may vary. For example: Construct trapezoid QRST and midpoints, M and N, of QR and ST, respectively. Construct line segment MN. Measure and compare the slopes of RS, QT, and MN.
13. a)



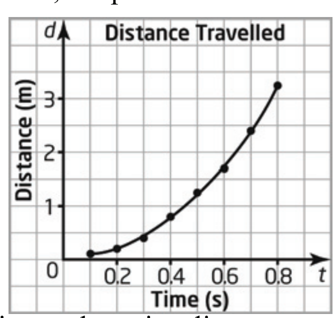
b) trapezoid; $m_{WX} = m_{YZ} = \frac{[illegible]}{3}$

c) The midpoint of side WZ is $M\left(-3, -\frac{1}{2}\right)$, and the midpoint of side XY is $N\left(3, \frac{3}{2}\right)$. Then, $m_{MN} = \frac{1}{3}$.
14. a) The coordinates of points A and B satisfy $x^2 + y^2 = 625$ and AB = 50.
b) Answers may vary. For example: C(7, 24).
c) $AC = BC = \sqrt{1250}$ and AB = 50.
$AC^2 + BC^2 = AC^2$, so ΔABC is a right triangle.
15. a) The coordinates of points J and K satisfy $x^2 + y^2 = 45$.
b) O(0, 0) satisfies $y = x$, an equation of the right bisector of chord JK.
16. a) Answers may vary. For example: Fold the first friend's home onto the second friend's home, and fold the third friend's home onto the first friend's home. Look for a restaurant near the intersection of the two folds.
b) Answers may vary.

Chapter 4

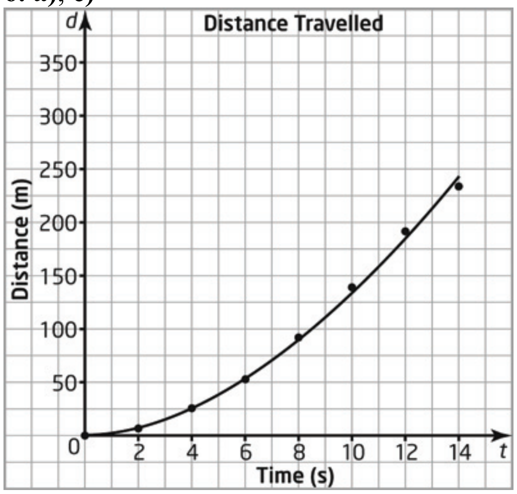
4.1 Investigate Non-Linear Relations, pages 35–36

1. a) No, because the points follow a curve.
b) Yes, because the points lie close to the line.
c) Yes, because the points lie close to the curve.
d) Yes, because the points lie close to the line.
2. Parts a) and d) could be modelled using a curve because the points do not lie along a line.
3. Linear; the points lie on a line.
4. Non-linear; the points lie on a curve.
5. a), c)

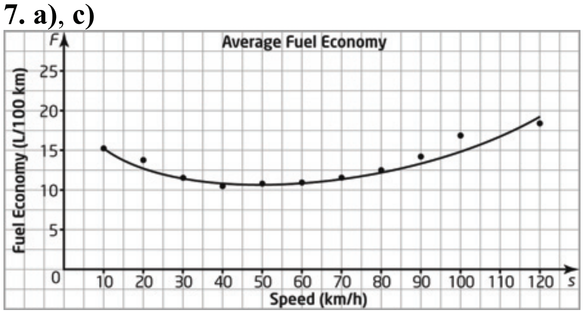


b) Non-linear; the points lie on a curve.

6. a), **c)**



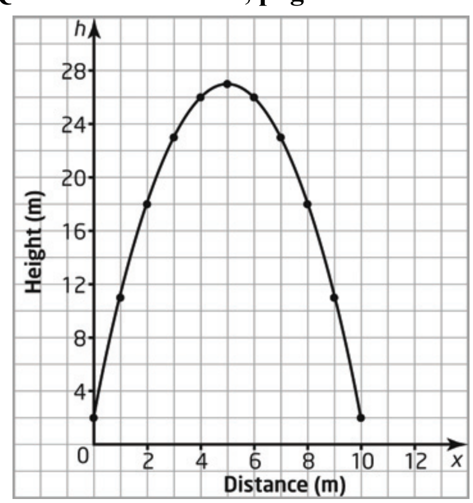
b) Non-linear; the points lie on a curve.
d) Answers may vary. For example: 310 m
7. a), **c)**



b) Non-linear; the points lie on a curve.
d) Answers may vary. For example:
23 L/100 km

4.2 Quadratic Relations, pages 37–38

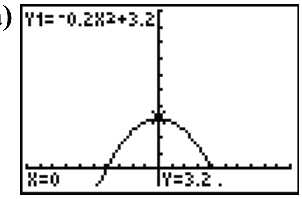
1. a)



b) The flight path of the baseball is parabolic. The axis of symmetry is $x = 5$; vertex is (5, 27).
c) The maximum height reached is 27 m.

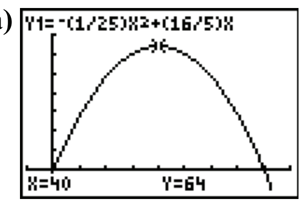
d) A table of values for $h = -x^2 + 10x + 2$ is the same as the table of values given.

2. a)



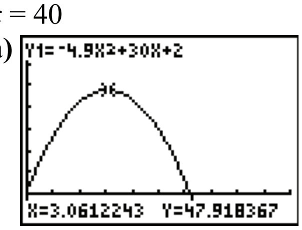
b) The shape of the arch is parabolic.
c) The arch is 3.2 m tall and 8 m wide.
3. a) The relation is linear, since the first differences are constant.
b) The relation is quadratic, since the second differences are constant.
c) The relation is quadratic, since the second differences are constant.
d) The relation is neither linear nor quadratic, since neither the first nor the second differences are constant.

4. a)



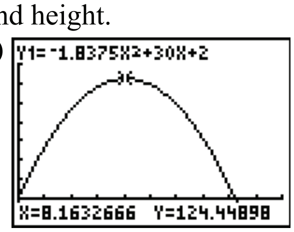
b) 39 m
c) 80 m
d) The maximum height is 64 m at a horizontal distance of 40 m.
e) $x = 40$

5. a)

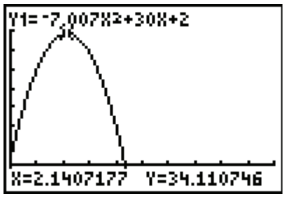


b) There is a quadratic relation between time and height.

c)



There is a quadratic relation between time and height.

d) 

There is a quadratic relation between time and height.

e) Answers may vary. For example: All three show a quadratic relation between time and height, but the parabolas have different axes of symmetry and vertices. Since the ball falls to the ground faster on Neptune than on Earth and Mars, Neptune has a stronger force of gravity than Earth and Mars.

6. a) 4.5 m

b) 1.5 s

7. a) 18 m

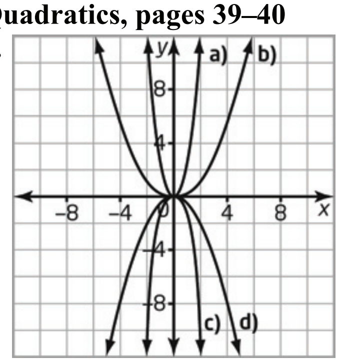
b) 3 s

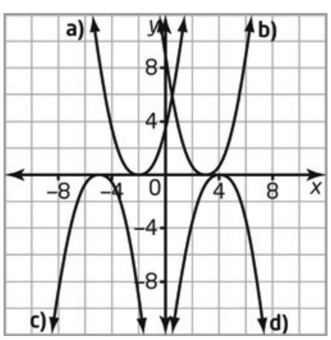
c) 6 s

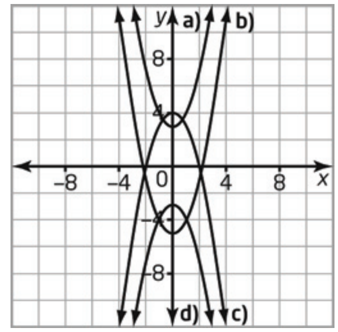
d) 2 s and 4 s

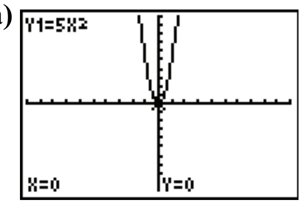
8. $y = n^2 + n$

4.3 Investigate Transformations of Quadratics, pages 39–40

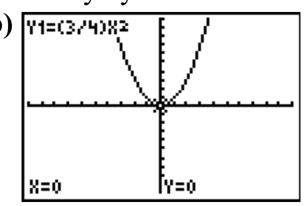
1. 

2. 

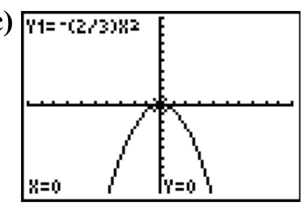
3. 

4. a) 

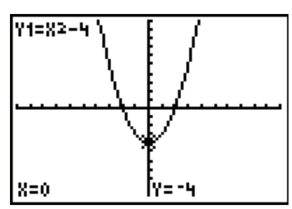
This graph is the graph of $y = x^2$ stretched vertically by a factor of 5.

b) 

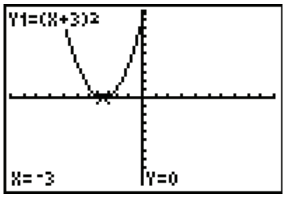
This graph is the graph of $y = x^2$ compressed vertically by a factor of $\frac{3}{4}$.

c) 

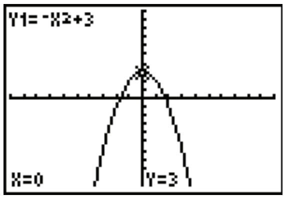
This graph is the graph of $y = x^2$ reflected in the x-axis and compressed vertically by a factor of $\frac{2}{3}$.

5. a) 

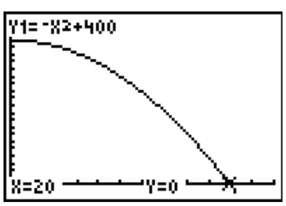
This graph is the graph of $y = x^2$ translated 4 units downward.

b) 

This graph is the graph of $y = x^2$ translated 3 units to the left.

c) 

This is the graph of $y = x^2$ reflected in the x-axis and then translated 3 units upward.

6. a) $y = x^2 + 5$
b) $y = x^2 - 3$
c) $y = x^2 + 6$
d) $y = x^2 - 8$
7. a) $y = (x + 4)^2$
b) $y = (x - 7)^2$
c) $y = (x + 6)^2$
d) $y = (x - 2)^2$
8. a) $y = 3x^2$
b) $y = \frac{1}{4}x^2$
c) $y = 5x^2$
d) $y = \frac{1}{6}x^2$
e) $y = -6x^2$
f) $y = -\frac{3}{5}x^2$
9. a) 

b) The y-intercept is 400. This represents the area of the backyard if there is no swimming pool. The x-intercept is 20. This represents the side length of the swimming pool, in metres, if the pool completely fills the backyard.
c) x must be greater than or equal to zero but less than or equal to 20
10. a) fourth diagram: 20 squares; fifth diagram: 30 squares.

b)

h	*A*	First Differences	Second Differences
1	2		
2	6	4	
3	12	6	2
4	20	8	2
5	30	10	2

Since the second differences are constant, the relation is a quadratic.
c) $y = x^2 + x$
d) Answers will vary.

4.4 Graph $y = a(x - h)^2 + k$, pages 41–42

1. a)

Property	$y = (x + 3)^2$
Vertex	(–3, 0)
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

b)

Property	$y = (x - 4)^2$
Vertex	(4, 0)
Axis of symmetry	$x = 4$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

c)

Property	$y = (x + 2)^2 + 5$
Vertex	(–2, 5)
Axis of symmetry	$x = -2$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 5$

d)

Property	$y = (x + 5)^2 - 3$
Vertex	(–5, –3)
Axis of symmetry	$x = -5$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -3$

e)

Property	$y = (x - 6)^2 + 7$
Vertex	(6, 7)
Axis of symmetry	$x = 6$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 7$

f)

Property	$y = (x - 1)^2 - 8$
Vertex	(1, –8)
Axis of symmetry	$x = 1$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -8$

g)

Property	$y = -(x + 8)^2 - 4$
Vertex	(–8, –4)
Axis of symmetry	$x = -8$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -4$

h)

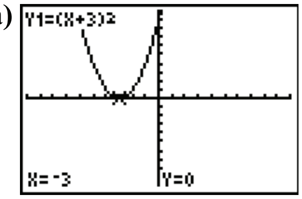
Property	$y = 3(x + 7)^2 - 2$
Vertex	(–7, –2)
Axis of symmetry	$x = -7$
Stretch or compression factor relative to $y = x^2$	3
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -2$

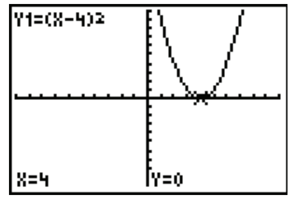
i)

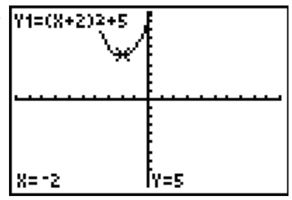
Property	$y = -2(x + 3)^2 - 6$
Vertex	(–3, –6)
Axis of symmetry	$x = -3$
Stretch or compression factor relative to $y = x^2$	2
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -6$

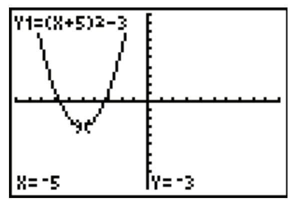
j)

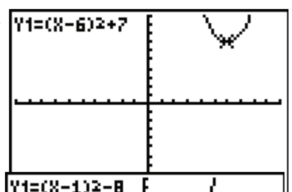
Property	$y = -\frac{1}{2}(x + 5)^2 - 3$
Vertex	(–5, –3)
Axis of Symmetry	$x = -5$
Stretch or compression factor relative to $y = x^2$	$\frac{1}{2}$
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq -3$

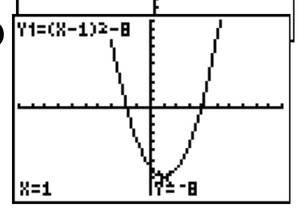
2. a)


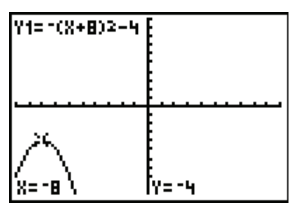
b)


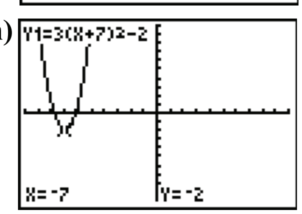
c)


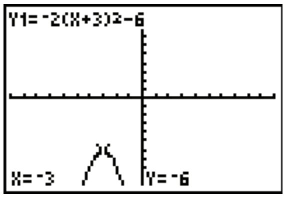
d)


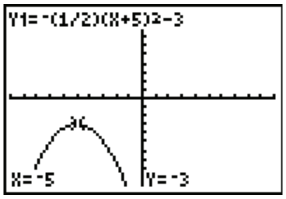
e)


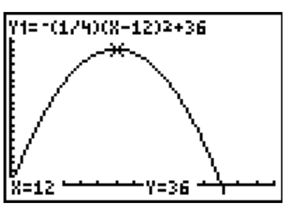
f)


g)


h)


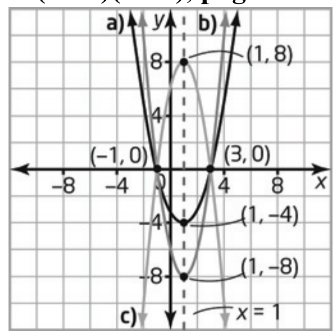
i) 

j) 

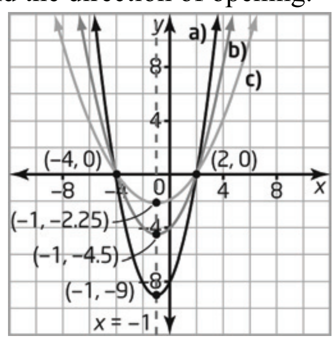
3. $y = (x - 3)^2 + 5$
4. $y = -(x - 6)^2 - 2$
5. $y = -3(x + 4)^2 + 5$
6. $y = 0.4(x + 1)^2 - 7$
7. a) $y = (x - 2)^2$
b) $y = -(x + 1)^2$
c) $y = x^2 - 3$
8. a) $y = -x^2 + 4$
b) $y = 2(x - 1)^2 - 3$
c) $y = -3(x + 2)^2 + 2$
9. a) $y = -\frac{1}{3}(x - 2)^2 + 6$
b) $y = \frac{2}{5}(x + 3)^2 - 4$
c) $y = -\frac{3}{4}(x + 1)^2 + 3$
d) $y = -2(x - 2)^2 + 5$
e) $y = -(x + 6)^2 - 2$
f) $y = -\frac{1}{2}(x - 6)^2 + 4$
10. a) 

b) 36 m
c) 12 m
d) 35 m
e) 14 m
11. a) $y = -3(x + 2)^2 - 4$
b) $y = 3(x - 4)^2 + 4$
c) $y = -3(x + 2)^2 - 7$
d) $y = 3(x - 2)^2 + 4$
12. a) $(x - 6)^2 + y^2 = 16$
b) $x^2 + (y + 2)^2 = 25$
c) $(x + 7)^2 + (y - 3)^2 = 9$
d) $(x + 5)^2 + (y + 4)^2 = 36$

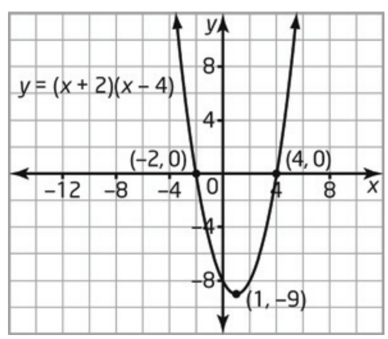
4.5 Quadratic Relations of the Form $y = a(x - r)(x - s)$, pages 43–44

1. 

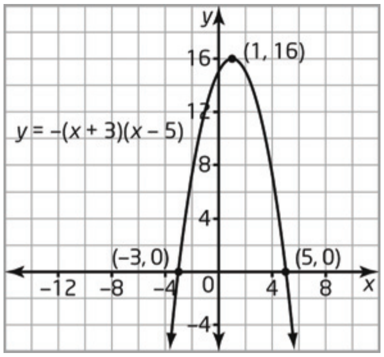
The graphs all have the same x-intercepts and axis of symmetry. They differ in the vertical stretch of the parabola, the y-value of the vertex, and the direction of opening.

2. 

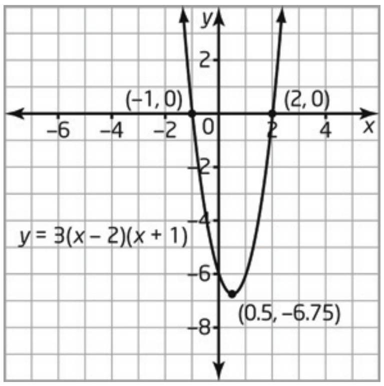
The graphs all have the same x-intercepts, axis of symmetry, and direction of opening. They differ in the vertical stretch of the parabola and the y-value of the vertex.

3. a) 

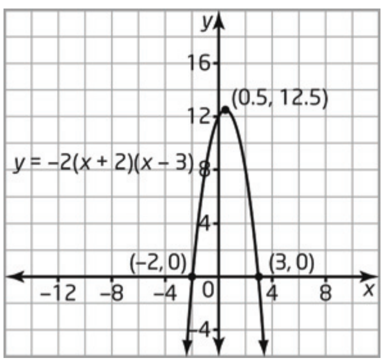
b)



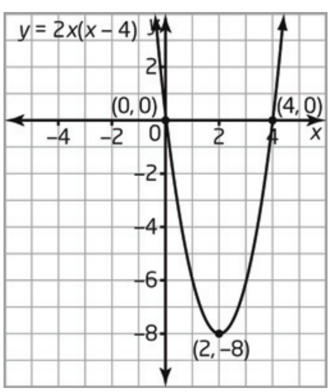
c)



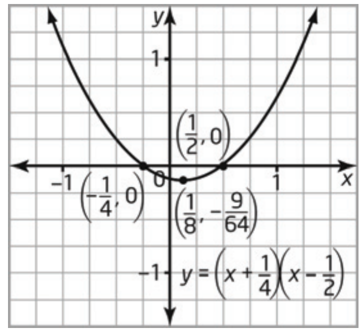
d)



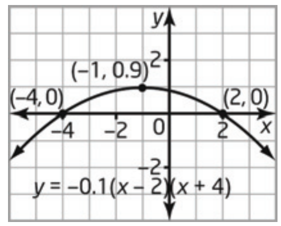
4. a)



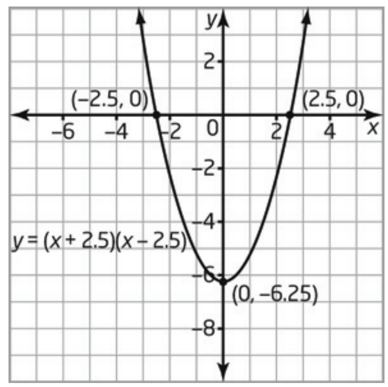
b)



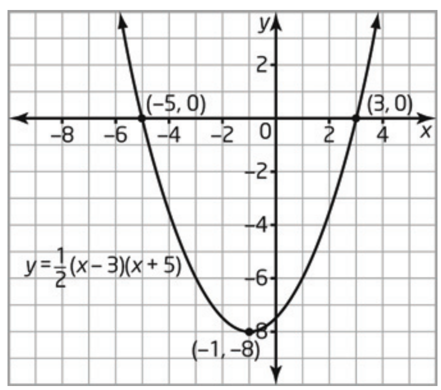
c)



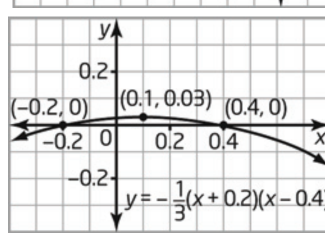
5. a)



b)



c)



6. a) $y = \frac{2}{9}(x-1)(x-7)$

b) $y = -\frac{5}{4}(x+5)(x+1)$

7. a) upward

b) (–3, 0)

c) –3

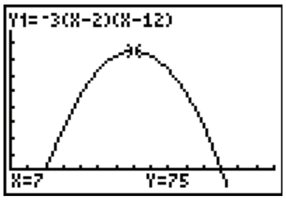
d) $y = (x + 3)(x + 3)$

8. a) downward

b) (4, 0)

c) 4

d) $y = -(x - 4)(x - 4)$

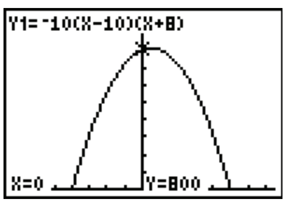
9. a) 

b) 2 m

c) 12 m

d) 75 m when it is 7 m from the wall

10. a) $R = -10(x - 10)(x + 8)$

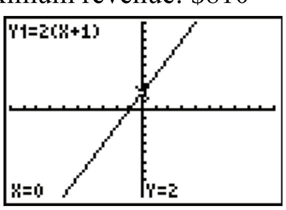
b) 

c) The R-intercept represents the current revenue with the current price of a T-shirt at $16 each. The x-intercepts represent the number of price increases or decreases that would give a revenue of $0.

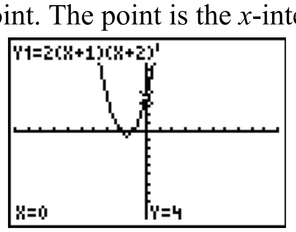
d) A negative x-value represents a decrease in T-shirt price.

e) $18 per T-shirt

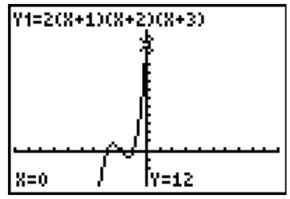
f) maximum revenue: $810

11. a) 

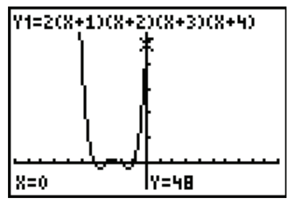
The equation of the relation has one factor. The graph of the relation crosses the x-axis at one point. The point is the x-intercept of the relation.

b) 

The equation of the relation has two factors. The graph of the relation crosses the x-axis at two points. The points are the x-intercepts of the relation.

c) 

The equation of the relation has three factors. The graph of the relation crosses the x-axis at three points. The points are the x-intercepts of the relation.

d) 

The equation of the relation has four factors. The graph of the relation crosses the x-axis at four points. The points are the x-intercepts of the relation.

4.6 Negative and Zero Exponents, pages 45–46

1. a) $\frac{1}{5^3}$

b) $\frac{1}{3^4}$

c) $\frac{1}{10^5}$

d) $\frac{1}{7^2}$

e) $\frac{1}{(-5)^1}$

f) $\frac{1}{(-4)^2}$

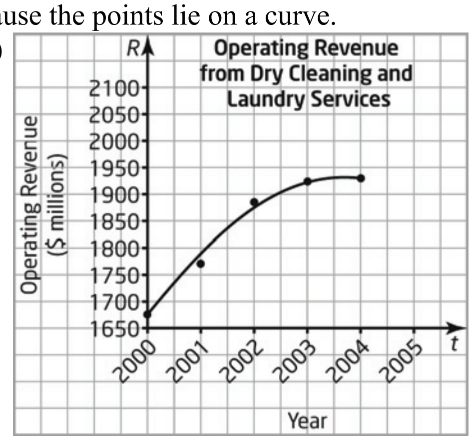
2. a) $\frac{1}{9}$

b) $\frac{1}{125}$

c) 1

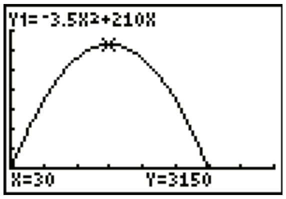
d) $\frac{1}{8}$

e) $\frac{1}{64}$
f) $\frac{1}{16}$
g) $-\frac{1}{216}$
h) –1
i) $-\frac{1}{25}$
j) –1
3. a) 16
b) –8
c) $\frac{3}{2}$
d) $\frac{16}{9}$
e) $-\frac{343}{125}$
f) undefined
g) $\frac{1}{32}$
h) $\frac{1}{36}$
i) $\frac{25}{9}$
j) $-\frac{1}{9}$
4. a) $\frac{2}{25}$
b) $\frac{1}{100}$
c) $1\frac{1}{9}$
d) 1
e) $\frac{17}{72}$
f) $3\frac{3}{4}$
5. a) $\frac{1}{8}$
b) $\frac{1}{16}$
c) $\frac{1}{32}$
d) 2^{-3}, 2^{-4}, 2^{-5}
6. a) 0.2 kg
b) 0.1 kg
7. a) 2^{-5}
b) 0.125 mg
8. a) 2 or –2
b) $\frac{3}{2}$ or $-\frac{3}{2}$
c) 4
d) $\frac{4}{3}$
e) –4
f) –3
9. a) 4000
b) 8000
c) 16 000
d) 32 000
e) 10:00 a.m.
f) There were 250 bacteria 3 h before 10:00 a.m., which was 7:00 a.m.
10. a) $\frac{1}{16}$
b) $\frac{1}{256}$
c) 2^{-4}, 2^{-8}
d) $781.25
11. a) $N = 500 \times 2^t$
b) 12 h
12. a) 13 h after the initial count
b) 13 h before the initial count

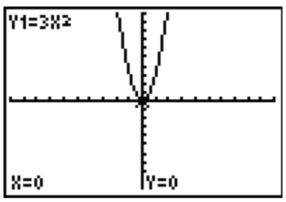
Chapter 4 Review, pages 47–48
1. Graph a) can be modelled using a curve because the points lie on a curve.
2. a)


b) There is a non-linear relation between the variables.
c) Answers may vary. For example: $1 940 000 000

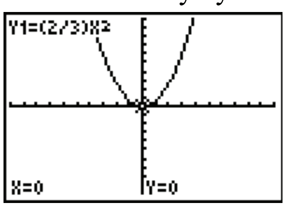
3. a) The relation is quadratic, since the second differences are constant.
b) The relation is linear, since the first differences are constant.
c) The relation is neither linear nor quadratic, since neither the first nor the second differences are constant.

4. a)


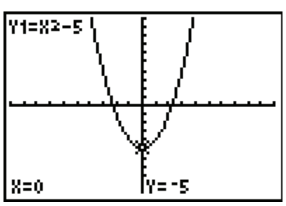
b) 60 min
c) 3150 m; after 30 min

5. a)


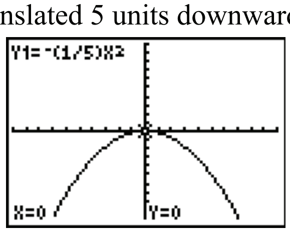
The graph of $y = 3x^2$ is the graph of $y = x^2$ stretched vertically by a factor of 3.

b)


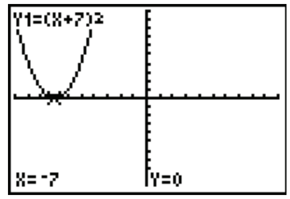
The graph of $y = \frac{2}{3}x^2$ is the graph of $y = x^2$ compressed vertically by a factor of $\frac{2}{3}$.

c)


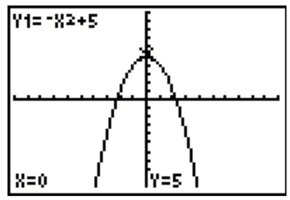
The graph of $y = x^2 - 5$ is the graph of $y = x^2$ translated 5 units downward.

d)


The graph of $y = -\frac{1}{5}x^2$ is the graph of $y = x^2$ reflected in the x-axis and compressed vertically by a factor of $\frac{1}{5}$.

e)


The graph of $y = (x + 7)^2$ is the graph of of $y = x^2$ translated 7 units to the left.

f)


The graph of $y = -x^2 + 5$ is the graph of $y = x^2$ reflected in the x-axis and translated 5 units upward.

6. a)

Property	$\boldsymbol{y = 2(x + 2)^2}$
Vertex	(–2, 0)
Axis of symmetry	$x = -2$
Stretch or compression factor relative to $y = x^2$	2
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq 0$

b)

Property	$\boldsymbol{y = -(x - 5)^2}$
Vertex	(5, 0)
Axis of symmetry	$x = 5$
Stretch or compression factor relative to $y = x^2$	none
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \leq 0$

c)

Property	$\boldsymbol{y = 3x^2 - 4}$
Vertex	(0, –4)
Axis of symmetry	$x = 0$
Stretch or compression factor relative to $y = x^2$	3
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \geq -4$

d)

Property	$y = -5x^2 + 3$
Vertex	(0, 3)
Axis of symmetry	$x = 0$
Stretch or compression factor relative to $y = x^2$	5
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \le 3$

e)

Property	$y = 4(x + 5)^2 + 2$
Vertex	(–5, 2)
Axis of symmetry	$x = -5$
Stretch or compression factor relative to $y = x^2$	4
Direction of opening	upward
Values x may take	set of real numbers
Values y may take	$y \ge 2$

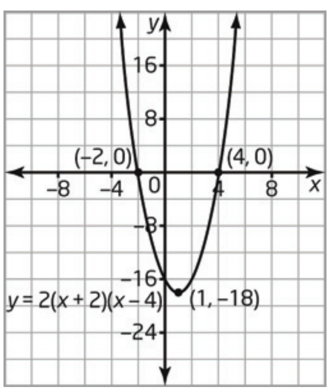
f)

Property	$y = -\frac{1}{2}(x - 4)^2 - 5$
Vertex	(4, –5)
Axis of symmetry	$x = 4$
Stretch or compression factor relative to $y = x^2$	$\frac{1}{2}$
Direction of opening	downward
Values x may take	set of real numbers
Values y may take	$y \le -5$

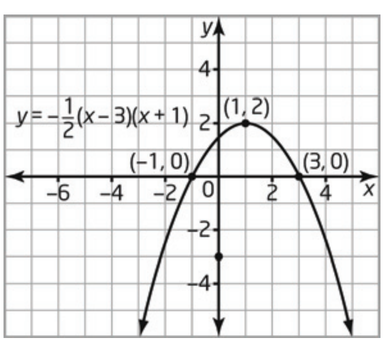
7. a) $y = 2(x - 3)^2 - 1$

b) $y = \frac{1}{2}(x + 5)^2 - 5$

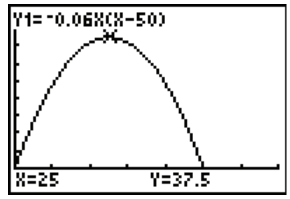
8. a)



b)



9. a)



b) 50 m

c) distance of 25 m; 37.5 m in height

10. a) $\frac{1}{16}$

b) $\frac{1}{243}$

c) 1

d) $-\frac{1}{3}$

e) $\frac{64}{27}$

f) 1

11. a) $\frac{1}{32}, \frac{1}{1024}$

b) $625

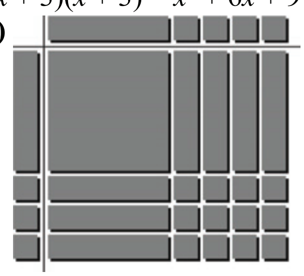
Chapter 5

5.1 Multiply Polynomials, pages 49–50

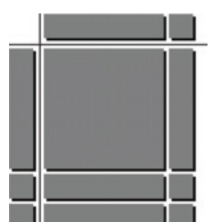
1. a) $(x + 3)(2x + 1) = 2x^2 + 7x + 3$

b) $(x + 3)(x + 3) = x^2 + 6x + 9$

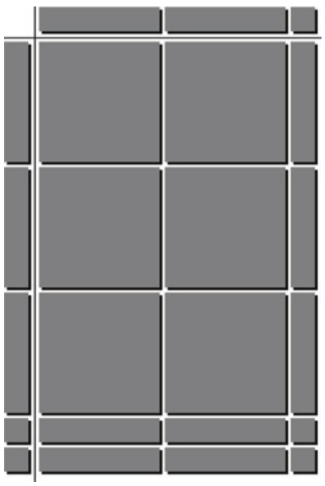
2. a)



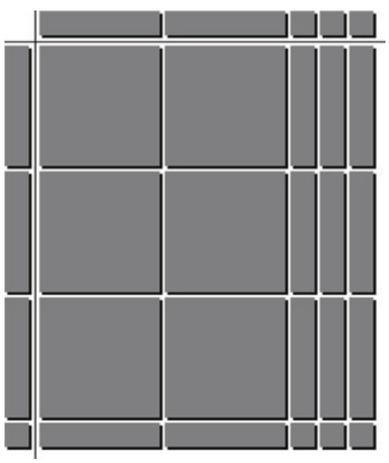
b)



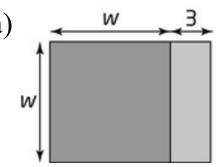
c)



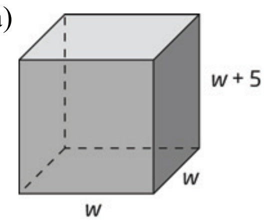
d)



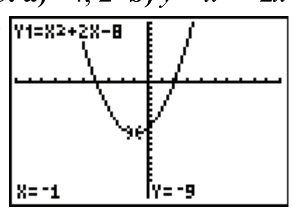
3. a) $x^2 + 7x + 10$
b) $x^2 + 4x + 3$
c) $d^2 + 5d + 6$
d) $q^2 + 15q + 50$
e) $y^2 + 7y + 6$
f) $z^2 + 16z + 64$
4. a) $x^2 - 7x + 12$
b) $x^2 - 11x + 30$
c) $m^2 - 9m + 14$
d) $a^2 - 14a + 48$
e) $h^2 - 5h + 4$
f) $k^2 - 12k + 36$
5. a) $x^2 + x - 20$
b) $x^2 - 4x - 21$
c) $n^2 + 2n - 24$
d) $r^2 - 5r - 24$
e) $h^2 + 4h - 45$
f) $e^2 + e - 30$
6. a) $5x^2 + 50x + 120$
b) $3y^2 + 12y - 63$
c) $-2t^2 - 2t + 60$
d) $-h^2 + 10h - 16$
7. a) $2x^2 + 14x + 23$
b) $2y^2 + 2y - 38$
c) $3c - 18$
d) $-21k - 17$
8. a) $98x^2 + 25x - 69$
b) $6y^2 - 79y + 63$
c) $24g$
d) $-30r^2 - 39r - 76$
9. a) $h = -2d^2 + 22d - 56$
b) 4 m
10. a) original area: x^2
b) new area: $(x + 2)(x + 3)$
c) $x^2 + 5x + 6$
d) increase in area: $5x + 6$
e) 36 m^2
11. a)



b) area: $w(w + 3)$
c) $w^2 + 3w$
12. a)



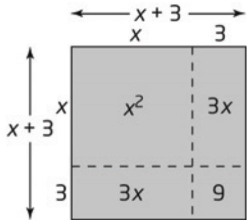
b) volume: $w(w)(w + 5)$
c) $w^3 + 5w^2$
13. a) –4, 2 **b)** $y = x^2 + 2x - 8$
c)



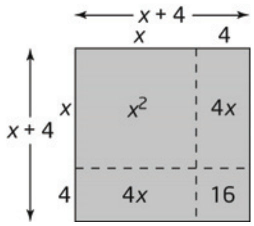
14. Methods may vary. For example:
a) Area: $3(x + 5) + 4(x) = 7x + 15$
Alternative method:
$(x + 5)(x + 3) - x(x + 1) = 7x + 15$
b) Area: $3(4) + x(x + 5) = x^2 + 5x + 12$
Alternative method:
$x(x + 9) - 4(x - 3) = x^2 + 5x + 12$
15. a) $p = 6 - \frac{n}{50}$
b) $R = 12n - \frac{n^2}{25}$

5.2 Special Products, pages 51–52

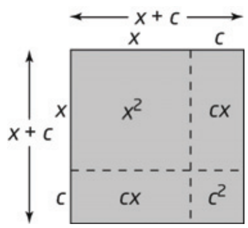
1. a)



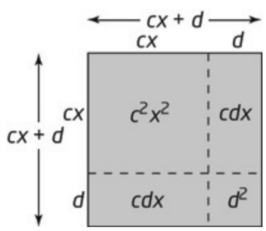
b)



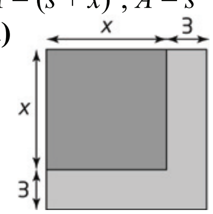
c)



d)



2. a) $x^2 + 4x + 4$
b) $y^2 + 6y + 9$
c) $a^2 + 16a + 64$
d) $n^2 + 2n + 1$
e) $w^2 + 24w + 144$
f) $m^2 + 10m + 25$
g) $p^2 + 26p + 169$
h) $z^2 + 30z + 225$
3. a) $x^2 – 2x + 1$
b) $r^2 – 12r + 36$
c) $f^2 – 14f + 49$
d) $b^2 – 8b + 16$
e) $e^2 – 10e + 25$
f) $k^2 – 28k + 196$
g) $s^2 – 22s + 121$
h) $h^2 – 18h + 81$
4. a) $x^2 + 10xy + 25y^2$
b) $4x^2 + 4xy + y^2$
c) $9a^2 + 24ab + 16b^2$
d) $16c^2 – 40cd + 25d^2$
e) $25d^2 + 70dm + 49m^2$
f) $36r^2 – 96rj + 64j^2$
g) $4q^2 + 12qr + 9r^2$
h) $9s^2 – 30st + 25t^2$
i) $16g^2 – 72gh + 81h^2$
j) $49u^2 – 84uw + 36w^2$
5. a) $x^2 – 16$
b) $y^2 – 49$
c) $v^2 – 64$
d) $u^2 – 36$
e) $t^2 – 9$
f) $e^2 – 25$
g) $i^2 – 256$
h) $u^2 – 625$
6. a) $a^2 – b^2$
b) $25c^2 – d^2$
c) $s^2 – 16t^2$
d) $q^2 – 36n^2$
e) $16p^2 – 49w^2$
f) $64h^2 – 9f^2$
7. a) $x^2 – 4$; 5
b) $x^2 + 12x + 36$; 81
The results are the same because the expressions are equivalent.
8. a) $x^2 – 9$
b) $x^2 – 8x + 16$
The results are the same because the expressions are equivalent.
9. a) $x^2 – 36$
b) $x^2 – 24x + 144$
The results are the same because the expressions are equivalent.
10. $A = (s + x)^2$; $A = s^2 + 2sx + x^2$
11. a)

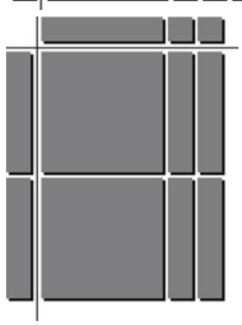
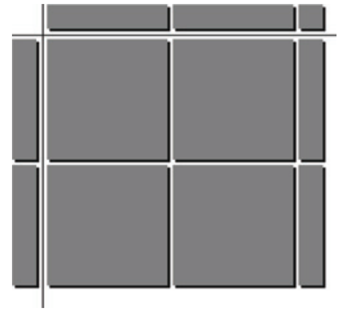


b) $x^2 + 6x + 9$
c) $6x + 9$
12. a) area: $(8a + 3b)(8a – 3b) = 64a^2 – 9b^2$
b) change in area:
$(8a + 3b)(8a – 3b) – (8a)^2 = – 9b^2$
c) 988 cm^2; 36 cm^2 less
13. a) (3, 0)
b) $y = x^2 – 6x + 9$
c) **L.S.** $= y$ **R.S.** $= x^2 – 6x + 9$
$= 0$ $= 3^2 – 6(3) + 9$
$= 0$
L.S. = R.S.
Therefore, the point (3, 0) satisfies the equation $y = x^2 – 6x + 9$.

14. a) (–4, 0)
b) $y = x^2 + 8x + 16$
c) **L.S.** $= y$ **R.S.** $= x^2 + 8x + 16$
$= 0$ $= (-4)^2 + 8(-4) + 16$
$= 0$
L.S. = R.S.
Therefore, the point (–4, 0) satisfies the equation $y = x^2 + 8x + 16$.
15. Methods may vary. For example:
Area: $(x + 4)(x - 4) + 4(8) = x^2 + 16$
Alternative method:
$(x - 4)^2 + 8(x) = x^2 + 16$
16. a) $x^4 + 8x^3 + 24x^2 + 32x + 16$
b) $x^3 - 12x^2 + 48x - 64$
c) $6x^3 + 31x^2 + 30x + 8$
d) $9x^4 + 24x^3 + 28x^2 + 16x + 4$

5.3 Common Factors, pages 53–54
1. a) x
b) $2c$
c) x^3
d) m^5
e) h
f) $-2y^3$
2. a)

b)

c)

d)

3. a) $7(2m + 3n)$
b) $5(c + 2d)$
c) $a(13b - 7c)$
d) $x^2(3x - 5)$
e) not possible
f) $6r^3(2r^2 + 3)$
g) $2h^2(h^3 + 3h^2 - 2)$
h) not possible
4. a) $2xy(8x^2 + 9y^2)$
b) $5a^2b^3(3a^2 - 2b^2)$
c) not possible
d) $5r^3st(6rs + 5t)$
e) $de(5d^2e + 3d - 7e^2)$
f) not possible
g) $3h^2k^2(h^2 + 2hk^2 - 3k)$
h) $2m^2n^3(6m^3 - 5n + 7mn^2)$
5. a) $(x + 5)(2x + 3)$
b) $(x - 3)(x - 2)$
c) $(a + 3b)(5a + 4b)$
d) $(2s + 5t)(3s - 7t)$
e) not possible
f) $(2h + 5)(6h - 7)$
6. a) $(x + y)(g + 3)$
b) $(x + 4)(x + 2)$
c) $(d + 5)(cd + 3)$
d) $(3m - 4)^2$
e) $(2p - 3)(5p + 4)$
f) $(4r - 3)(3r - 2)$
7. Answers may vary. For example:
a) $16x + 24y$
b) $3y^2 + 5y$
c) $6a^4 - 9a^2$
d) $10m^4n^5 + 5m^3n^4$
8. a) $A = \frac{1}{2}h(b_1 + b_2)$
b) $A = 9$ cm^2. The areas are the same using the original and the factored form of the formula because the formulas are equivalent.
9. a) $SA = \pi r(r + s)$

b) $SA = 24\pi$ cm^2, or approximately 75.4 cm^2.
The surface areas are the same using the original and the factored form of the formula because the formulas are equivalent.

10. a) $V = \pi h(R^2 - r^2)$

b) $V = 140\pi$ cm^3, or approximately 439.8 cm^3.
The surface areas are the same using the original and the factored form of the formula because the formulas are equivalent.

11. length 1, width $4x^2 + 8x$;
length 2, width $2x^2 + 4x$;
length 4, width $x^2 + 2x$;
length x, width, $4x + 8$;
length $2x$, width $2x + 4$;
length $4x$, width $x + 2$

12. a) $(y - 3)(4x - 3)$
b) $(y - 4)(6x + 5)$

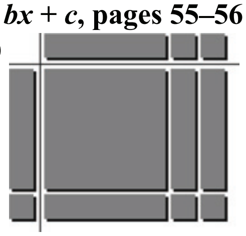
13. $15(3x^2 - y^2)$

14. $y = x(3x - 5)$; x-intercepts are 0 and $\frac{5}{3}$

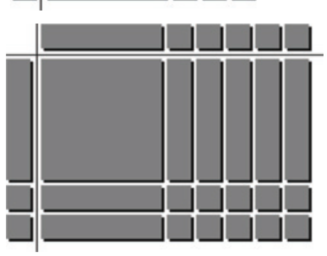
15. $y = x(6x + 7)$; x-intercepts are 0 and $-\frac{7}{6}$

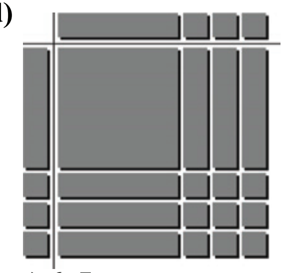
16. a) $\frac{1}{4}\left(x^2 + 3y^2\right)$
b) $\frac{1}{5}ab\left(2a - b\right)$

5.4 Factor Quadratic Expressions of the Form $x^2 + bx + c$, pages 55–56

1. a)


b)


c)


d)


2. a) 6, 7
b) 4, 2
c) –3, 2
d) –6, 3

3. a) $(x + 1)(x + 5)$
b) $(m + 3)(m + 5)$
c) not possible
d) $(k + 2)(k + 6)$
e) not possible
f) $(g + 5)(g + 6)$
g) $(w + 2)(w + 4)$
h) $(a + 1)(a + 6)$
i) not possible
j) $(n + 1)(n + 7)$

4. a) $(n - 2)(n - 8)$
b) not possible
c) $(r - 2)(r - 5)$
d) $(z - 7)(z - 8)$
e) $(w - 3)(w - 6)$
f) not possible
g) $(r - 1)(r - 2)$
h) $(p - 1)(p - 12)$
i) $(k - 1)(k - 10)$
j) $(c - 3)(c - 4)$

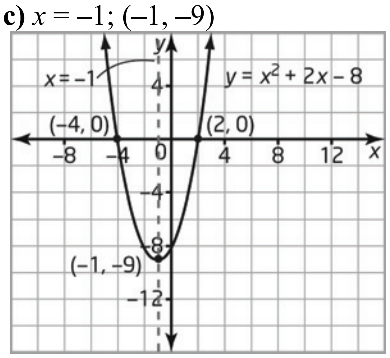
5. a) $(a - 8)(a + 3)$
b) $(q + 6)(q - 1)$
c) $(h - 9)(h + 2)$
d) not possible
e) $(k - 4)(k + 3)$
f) $(b + 3)(b - 1)$
g) not possible
h) $(t + 13)(t - 2)$

6. a) $A = (x + 12)(x + 5)$; 24 cm by 17 cm
b) $A = (x + 14)(x + 2)$; 26 cm by 14 cm

7. a) $3(x + 2)(x + 3)$
b) $4(m - 6)(m - 2)$
c) $5(k + 8)(k + 2)$
d) $4(p - 5)(p - 4)$
e) $x(x + 21)(x - 2)$
f) $c(x - 9)(x + 3)$

8. Answers may vary. For example:
a) 9, 25
b) 36, 20

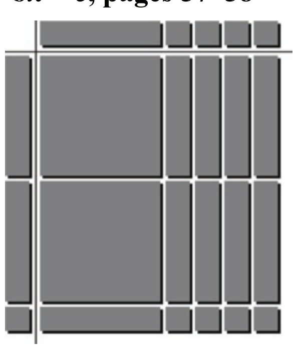
c) 12, 20
d) 10, 28
9. Answers may vary. For example:
a) 11, 10
b) 5, –7
c) 11, 1
d) 1, 8
10. a) $(m + 12n)(m + 2n)$
b) $(p - 3q)(p - 5q)$
c) $(r + 8s)(r - 6s)$
d) $(w - 5z)(w + 2z)$
11. a) $y = (x + 4)(x - 2)$
b) –4, 2
c) $x = -1$; (–1, –9)



12. a) $h = -5(t + 2)(t - 6)$
b) 6 s
13. a) $x^4 + 2x^2 + 1 = (x^2 + 1)(x^2 + 1)$
$x^4 + 4x^2 + 4 = (x^2 + 2)(x^2 + 2)$
$x^4 + 6x^2 + 9 = (x^2 + 3)(x^2 + 3)$
$x^4 + 8x^2 + 16 = (x^2 + 4)(x^2 + 4)$
b) Answers will vary.
c) Answers will vary.
d) $x^4 + 10x^2 + 25 = (x^2 + 5)(x^2 + 5)$
$x^4 + 12x^2 + 36 = (x^2 + 6)(x^2 + 6)$
14. a) $(x^2 + 10)(x^2 + 2)$
b) $(x^2 + y^2)(x^2 + 7y^2)$
c) $(x^3y^3 - 6z^2)(x^3y^3 + 2z^2)$

5.5 Factor Quadratic Expressions of the Form $ax^2 + bx + c$, pages 57–58

1. a)



b)

c)

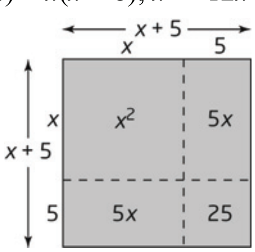
d)

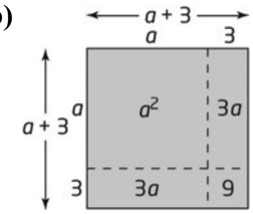
2. a) $(3x + 1)(2x + 3)$
b) $(2y + 5)(y + 3)$
c) $(5m + 4)(m + 2)$
d) $(3d + 2)(d + 5)$
e) not possible
f) $(4s + 5)(3s + 4)$
3. a) $(2m - 5)(3m - 1)$
b) not possible
c) $(4r - 1)(3r - 2)$
d) $(b - 2)(5b - 3)$
e) $(2k - 7)(k - 3)$
f) $(6h - 5)(2h - 3)$
4. a) $(4k - 3)(2k + 1)$
b) $(3g + 2)(4g - 5)$
c) $(8c - 5)(2c + 3)$
d) not possible
e) $(5a + 7)(2a - 1)$
f) $(3v - 5)(2v + 3)$
5. a) $(2x + y)(x + 4y)$
b) $(2a - 3b)(3a - b)$
c) $(2r + 3s)(4r + 5s)$
d) $(3g - 4h)(2g + 5h)$
e) $(3p - 2q)(3p + 7q)$
f) $(3c - 5d)(4c + 3d)$
6. a) $2(2d - 5)(4d + 3)$
b) $3(j + 3)(2j - 5)$
c) $4(4b - 1)(2b + 3)$
d) $2(5z + 1)(3z - 4)$

e) $4(3v + 1)(v - 1)$
f) $2(5t - 7)(t + 1)$
7. a) $(2x + 1)(x + 3)$; 42
b) $(2x - 3)(2x - 5)$; 3
c) $(2x + 1)(x - 3)$; 0
d) $(3x + 5)(x - 2)$; 14
e) $(2x + 3)(3x - 5)$; 36
f) $(5x - 1)(3x + 2)$; 154
g) $(2x + 3)(5x - 3)$; 108
h) $(4x - 1)(2x + 3)$; 99
i) $(2x + 3)(x - 1)$; 18
j) $(3x + 1)(2x + 5)$; 110
The results are the same because the expressions are equivalent.
8. Answers may vary. For example:
a) 9, 6
b) 14, 16
c) 16, 23
d) 1, 6
9. Answers may vary. For example:
a) 6, 4
b) 2, 6
c) 12, 15
d) 7, 8
10. a) $(2x + 7)(3x - 5)$
length $2x + 7$, width $3x - 5$
b) $P = 84$ cm; $A = 437$ cm^2
11. a) $h = -2(t + 4)(t - 6)$
b) 6 s
12. a) $(3m^2 + 8)(2m^2 + 5)$
b) $(4q^2 - 5)(2q^2 - 7)$
c) $(5a^2 + 4)(2a^2 - 3)$
13. a) $(3x + 3c + 1)(x + c + 2)$
b) $(2x - 2d + 5)(x - d + 2)$
c) $(3x + 3m + 1)(2x + 2m + 1)$

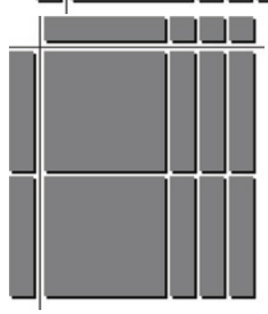
5.6 Factor a Perfect Square Trinomial and a Difference of Squares, pages 59–60
1. a) $(x + 6)(x - 6)$
b) $(y + 5)(y - 5)$
c) $(2h + 7)(2h - 7)$
d) $(11k + 8)(11k - 8)$
e) $(9a + 2)(9a - 2)$
f) $(10r + 3)(10r - 3)$
g) $(12y + 13)(12y - 13)$
h) $(14u + 1)(14u - 1)$
2. a) $(a + 6b)(a - 6b)$
b) $(r + 7s)(r - 7s)$
c) $(5 + 6c)(5 - 6c)$
d) $(8 + 9d)(8 - 9d)$
e) $(3q + 2r)(3q - 2r)$
f) $(g + 4h)(g - 4h)$
g) $2(3v + 7b)(3v - 7b)$
h) $3(2k + 3c)(2k - 3c)$
3. a) $x^2 + 10x + 25 = (x)^2 + 2(x)(5) + (5)^2$;
$(x + 5)^2$
b) $f^2 + 14f + 49 = (f)^2 + 2(f)(7) + (7)^2$;
$(f + 7)^2$
c) $r^2 - 8r + 16 = (r)^2 - 2(r)(4) + (4)^2$;
$(r - 4)^2$
d) $u^2 - 18u + 81 = (u)^2 - 2(u)(9) + (9)^2$;
$(u - 9)^2$
e) $e^2 + 20e + 100 = (e)^2 + 2(e)(10) + (10)^2$;
$(e + 10)^2$
f) $36 - 12y + y^2 = (6)^2 - 2(6)(y) + (y)^2$;
$(6 - y)^2$
4. a) $16m^2 + 24m + 9$
$= (4m)^2 + 2(4m)(3) + (3)^2$; $(4m + 3)^2$
b) $4k^2 + 28k + 49 = (2k)^2 + 2(k)(7) + (7)^2$;
$(2k + 7)^2$
c) $25w^2 + 10w + 1 = (5w)^2 + 2(5w)(1) + (1)^2$;
$(5w + 1)^2$
d) $81y^2 + 180y + 100$
$= (9y)^2 + 2(9y)(10) + 10^2$; $(9y + 10)^2$
e) $36p^2 + 60p + 25 = (6p)^2 + 2(6p)(5) + (5)^2$;
$(6p + 5)^2$
5. a) $(3x + 4y)^2$
b) $(2m - 5n)^2$
c) $(7a + 3b)^2$
d) $(5p + 8q)(5p - 8q)$
e) not possible
f) $(10r + 9s)(10r - 9s)$
g) not possible
h) $(8j - 7k)^2$
6. a) $(3x + 4)^2 - (x - 5)^2$
b) $(4x - 1)(2x + 9)$
7. a) 12, –12
b) 24, –24
c) 70, –70
d) 36, –36
8. a) 64
b) 25
c) 2, –2
d) 225
9. Answers may vary. For example:
a) 9, 25
b) 4, 36
c) 16, 81
d) 25, 49
10. The figure could be a square or a parallelogram with base equal to height.
11. a) $y = (x - 4)^2$; the vertex is (4, 0).

b) $y = (x + 3)^2$; the vertex is (–3, 0).
12. a) (20 + 13)(20 – 13) = 231
b) (57 + 23)(57 – 23) = 2720
c) (104 + 103)(104 – 103) = 207
d) (88 + 77)(88 – 77) = 1815
e) (67 + 39)(67 – 39); 2968
f) (49 + 47)(49 – 47); 192
13. a) $(x - 2)(x - 8)$
b) $(x + 5)(x + 6)$
c) $(6x^2 + 7y^2)(6x^2 - 7y^2)$
d) $(m^3 - 3)^2$
e) $(a^4 + 7)^2$
f) $\left(\frac{x^3}{4} + \frac{y^3}{5}\right)\left(\frac{x^3}{4} - \frac{y^3}{5}\right)$
14. a) By expanding,
$(x - 1)(x^2 + x + 1) = x^3 - 1$.
b) By expanding,
$(x + 1)(x^2 - x + 1) = x^3 + 1$.
c) By expanding,
$(x - 2)(x^2 + 2x + 4) = x^3 - 8$.
d) By expanding,
$(x + 2)(x^2 - 2x + 4) = x^3 + 8$.
15. a) $(y - 3)(y^2 + 3y + 9)$
b) $(y + 3)(y^2 - 3y + 9)$

Chapter 5 Review, pages 61–62
1. a) $x^2 + 11x + 28$
b) $x^2 - 8x + 15$
c) $x^2 - 3xy - 10y^2$
d) $8a^2 + 14ab - 15b^2$
2. a) $-k^2 - 2k + 15$
b) $3m^2 - 3mn - 18n^2$
c) $t^3 - 7t^2v + 12tv^2$
d) $14y$
e) $2x^3 - 4x^2 - 6x$
f) $6x^3 - 18x^2 + 28x$
3. $4(x) + x(x + 8)$; $x^2 + 12x$
4. a)


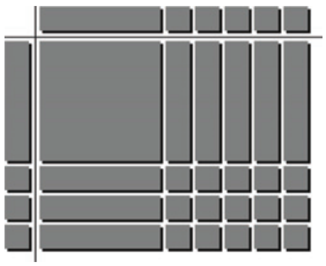
b)


5. a) $x^2 + 8x + 16$
b) $q^2 + 14q + 49$
c) $r^2 - 16r + 64$
d) $t^2 - 10t + 25$
e) $n^2 + 16n + 64$
f) $p^2 - 12p + 36$
6. a) $k^2 - 121$
b) $r^2 - 64$
c) $s^2 - 169$
d) $u^2 - 25$
e) $x^2 - 81$
f) $t^2 - 16$
7. a) $9x^2 + 6xy + y^2$
b) $16m^2 - 8mn + n^2$
c) $36p^2 + 60pq + 25q^2$
d) $64p^2 - 112pq + 49q^2$
e) $4g^2 - 9h^2$
f) $-16r^2 + 25s^2$
8. a)

b)

9. a) $5(x + y)$
b) $b(13a - 15c)$
c) $t(t + 5)$
d) $3m^2(1 - 2m)$
10. a) $(3x + 2)(5x + 4)$
b) $(m - 5)(3m - 2)$
c) $(5p + 1)(2p - 1)$
d) $(3x - 1)(2x - 5)$
11. a) $2(3m^2 - 4m + 2)$
b) $(x + y)(c + d)$
c) not possible
d) $ghz(z - g + 1)$

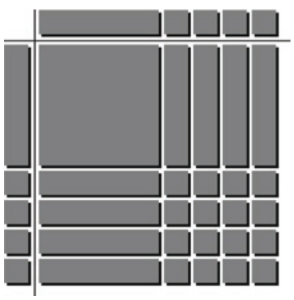
12. length $14x^2 + 7x$, width 1
length $2x^2 + x$, width 7
length $14x + 7$, width x
length $2x + 1$, width $7x$

13. a)

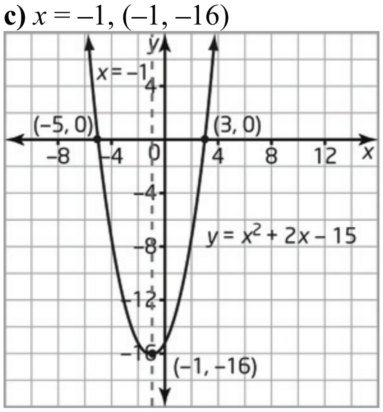


b)

c)



14. a) $(d + 5)(d + 6)$
b) $(q + 2)(q + 8)$
c) $(m – 7)(m – 8)$
d) $(z – 6)(z – 2)$
e) $(r + 7)(r – 1)$
f) $(w + 8)(w – 3)$
g) $(p – 9)(p + 2)$
h) $(e – 5)(e + 2)$
15. a) $y = (x + 5)(x – 3)$
b) –5, 3
c) $x = –1$, (–1, –16)



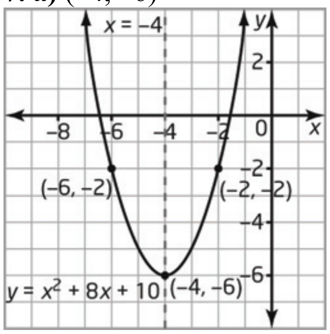
16. a) $(2x + 1)(4x + 7)$
b) $(5y + 2)(3y + 4)$
c) $(3c – 5)(2c – 1)$
d) $(4h – 3)(2h – 1)$
e) $(2w + 7)(2w – 3)$
f) $(3p + 1)(2p – 7)$
17. a) $(2x + 5y)(3x – 5y)$
b) $(3m – 4n)(4m + 3n)$
c) $(4p + 3q)(p + q)$
d) $(3k – v)(k – 4v)$
e) not possible
f) $(2h – k)(3h – 5k)$
18. a) length $(4x + 3)$, width $(3x + 2)$
b) $P = 150$ cm, $A = 1376$ cm^2
19. a) $(x + 4)(x – 4)$
b) $(y + 8)(y – 8)$
c) $(3a + 4b)(3a – 4b)$
d) $(5m + 7n)(5m – 7n)$
20. a) $x^2 + 12x + 36 = (x)^2 + 2(x)(6) + (6)^2$;
$(x + 6)^2$
b) $q^2 – 10q + 25 = (q)^2 – 2(q)(5) + (5)^2$;
$(q – 5)^2$
c) $16m^2 + 24m + 9= (4m)^2 + 2(4m)(3) + (3)^2$;
$(4m + 3)^2$
d) $9a^2 – 30a + 25= (3a)^2 – 2(3a)(5) + (5)^2$;
$(3a – 5)^2$
e) $4a^2 + 28ab + 49b^2$
$= (2a)^2 + 2(2a)(7b) + (7b)^2$; $(2a + 7b)^2$
f) $64p^2 – 80pq + 25q^2$
$= (8p)^2 – 2(8p)(5q) + (5q)^2$; $(8p – 5q)^2$

Chapter 6

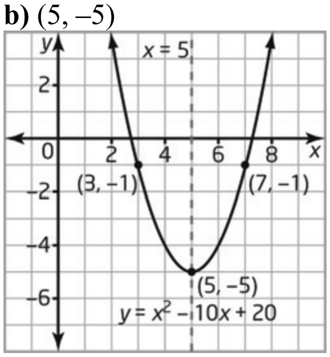
6.1 Maxima and Minima, pages 63–64

1. a) $y = (x + 4)^2 – 13$
b) $y = (x + 5)^2 – 18$
2. a) 16
b) 9
c) 49
d) 64
e) 1
f) 4225
3. a) $y = (x + 4)^2 – 12$
b) $y = (x + 2)^2 + 1$
c) $y = (x + 3)^2 – 12$
d) $y = (x – 1)^2 – 9$
e) $y = (x – 5)^2 – 32$
f) $y = (x – 2)^2 – 14$
4. a) $y = –(x – 4)^2 – 14$
b) $y = –(x – 5)^2 + 45$
c) $y = –(x + 2)^2 + 14$
d) $y = –(x + 1)^2 – 12$
5. a) $y = 2(x + 4)^2 – 29$
b) $y = 3(x – 2)^2 – 17$

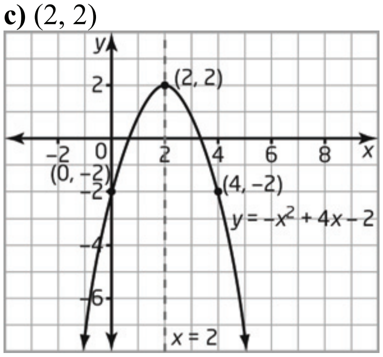
c) $y = -2(x - 2)^2 + 15$
d) $y = -4(x + 1)^2 + 3$
6. a) B
b) C
c) D
d) A
7. a) (–4, –6)



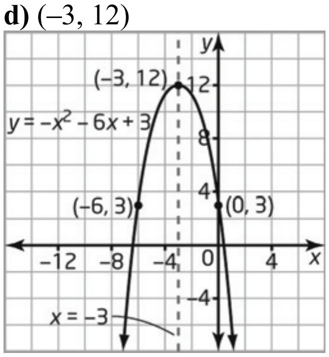
b) (5, –5)



c) (2, 2)



d) (–3, 12)



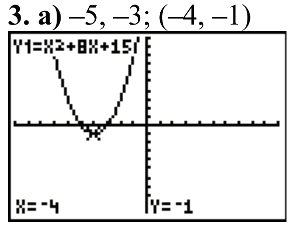
8. a) minimum point at (–2, –7)
b) minimum point at (6, –30)
c) maximum point at (3, 6)
d) maximum point at (–4, 21)
9. a) minimum point at (–15, –453)
b) minimum point at (8, –180)
c) maximum point at (7, 83)
d) maximum point at (–4, 73)
10. a) minimum point at (–0.3, –4.7)
b) maximum point at (0.1, –6.8)
c) minimum point at (–0.2, –3.5)
d) maximum point at (–2.7, 26.3)
e) minimum point at (–4, –8.9)
f) maximum point at (–1.5, 7.1)
g) minimum point at (–0.9, –0.8)
h) maximum point at (0.4, 0.4)
11. The maximum height of 11 m occurs at a horizontal distance of 3 m.
12. The minimum cost of $78 occurs when the appliance runs for 6 months.
13. a) $c = 9$, $h = 3$
b) $b = 14$, $h = 7$
c) $b = 4$, $c = 7$
d) $b = -10$, $c = -7$
14. The relation $y = ax^2 + bx + c$ can be expressed as $y = a\left(x - \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$ by completing the square. Therefore, the y-coordinate of the vertex is $\frac{4ac - b^2}{4a}$.

6.2 Solve Quadratic Equations, **pages 65–66**
1. a) –3, –4
b) –2, 5
c) 6, –1
d) 7, 8
e) 0, –10
f) 0, 15
g) $-\frac{5}{3}, \frac{7}{2}$
h) $\frac{9}{4}, -\frac{1}{6}$
2. a) –2, –3
b) –3, –4
c) 2, 6
d) 4, 5
e) –8, 3
f) –2, 9
g) 0, –5
h) 0, 21

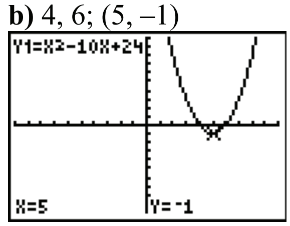
3. a) $-\frac{3}{2}, \frac{3}{2}$
b) $-\frac{4}{5}, \frac{4}{5}$
c) $-\frac{7}{3}, \frac{7}{3}$
d) $-\frac{9}{8}, \frac{9}{8}$
4. a) $-\frac{3}{2}, -4$
b) $-5, -\frac{4}{3}$
c) $\frac{2}{5}, \frac{3}{2}$
d) $\frac{1}{3}, \frac{5}{2}$
e) $\frac{3}{4}, -5$
f) $-\frac{2}{3}, \frac{5}{4}$
5. a) $-\frac{4}{3}$
b) $\frac{5}{2}$
6. a) –2, –1
b) –8, –7
c) –9, –3
d) 3, 5
e) –10, 2
f) 7, –3
7. a) $-\frac{5}{2}, \frac{4}{3}$
b) $\frac{1}{5}, \frac{1}{3}$
c) $-\frac{2}{3}, \frac{3}{2}$
d) $\frac{2}{5}, -\frac{3}{4}$
e) $-\frac{1}{3}, -3$
f) $-\frac{7}{4}, \frac{3}{2}$
8. a) –5, 2
b) –4, 3
c) –4, 5
d) 2, 6
9. 4 cm
10. a) $(x - 3)(x - 7) = 0$
b) $(x - 4)(x + 2) = 0$
11. a) $x^2 - x - 30 = 0$
b) $x^2 + 17x + 72 = 0$
12. a) $6x^2 - 5x + 1 = 0$
b) $15x^2 + x - 6 = 0$
13. 5 cm and 12 cm
14. 4 or 67
15. a) $y = -7x, y = -x$
b) $y = 4x, y = 2x$
c) $y = -4x, y = x$
d) $y = -3x, y = 13x$

6.3 Graph Quadratics Using the *x*-Intercepts, pages 67–68

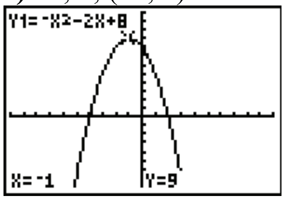
1. a) –6, –2
b) 4, 8
c) –9, 2
d) –2, 11
e) 0, –6
f) 0, 11
2. a) $-\frac{5}{3}, -1$
b) $\frac{1}{4}, \frac{2}{3}$
c) $\frac{1}{4}, -1$
d) $\frac{8}{5}, -1$
e) $\frac{3}{2}, \frac{4}{3}$
f) $\frac{2}{5}, \frac{1}{3}$
3. a) –5, –3; (–4, –1)



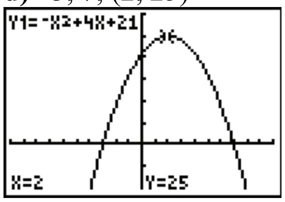
b) 4, 6; (5, –1)



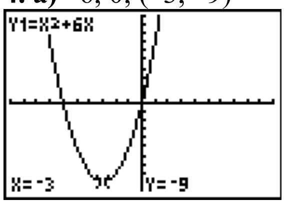
c) –4, 2; (–1, 9)



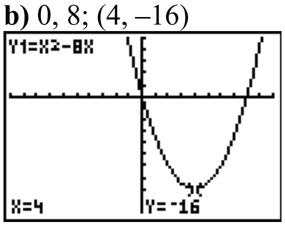
d) –3, 7; (2, 25)



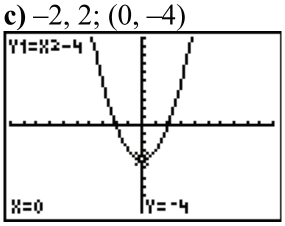
4. a) –6, 0; (–3, –9)



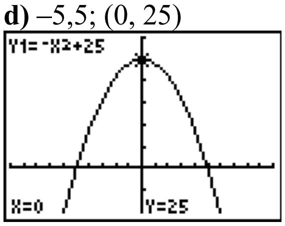
b) 0, 8; (4, –16)



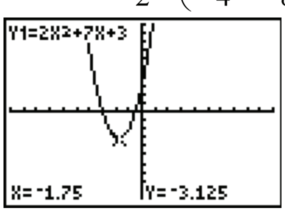
c) –2, 2; (0, –4)



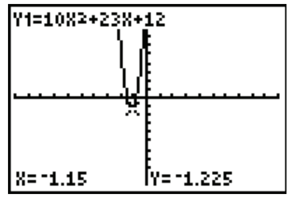
d) –5,5; (0, 25)



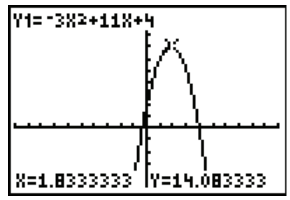
5. a) –3, $-\frac{1}{2}$; $\left(-\frac{7}{4}, -\frac{25}{8}\right)$



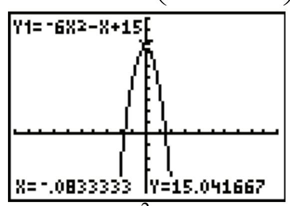
b) $-\frac{3}{2}$, $-\frac{4}{5}$; $\left(-\frac{23}{20}, -\frac{49}{40}\right)$



c) $-\frac{1}{3}$, 4; $\left(\frac{11}{6}, \frac{169}{12}\right)$



d) $-\frac{5}{3}$, $\frac{3}{2}$; $\left(-\frac{1}{12}, \frac{361}{24}\right)$



6. a) $y = 2x^2 - 12x + 16$

b) $y = -3x^2 - 6x$

7. a) $y = \frac{1}{2}x^2 - 4x + 6$

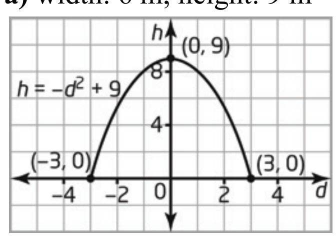
b) $y = -3x^2 - 12x - 9$

c) $y = \frac{1}{2}x^2 + 3x + \frac{5}{2}$

d) $y = -\frac{1}{3}x^2 + \frac{10}{3}x - 7$

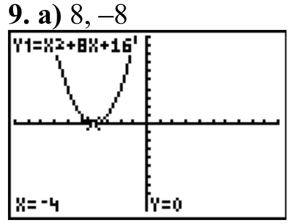
8. a) width: 6 m; height: 9 m

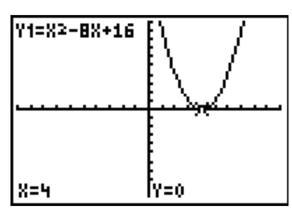
b)



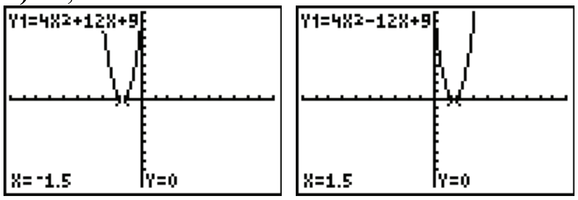
c) $-3 \le d \le 3$ so that $h \ge 0$

9. a) 8, –8

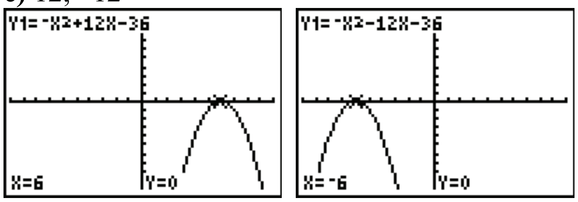




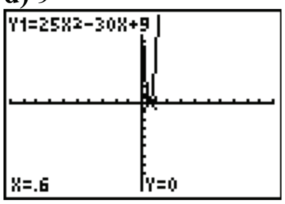
b) 12, –12



c) 12, –12



d) 9



10. –34

6.4 The Quadratic Formula, pages 69–70

1. a) 1, $\frac{2}{5}$

b) $\frac{4}{3}$

c) $\frac{-9 \pm \sqrt{105}}{6}$

d) $\frac{-4 \pm \sqrt{8}}{4}$

e) $\frac{-3 \pm \sqrt{89}}{10}$

f) $\frac{4}{5}$

2. a) $\frac{-9 \pm \sqrt{57}}{4}$; –4.14, –0.36

b) $\frac{-13 \pm \sqrt{113}}{14}$; –1.69, –0.17

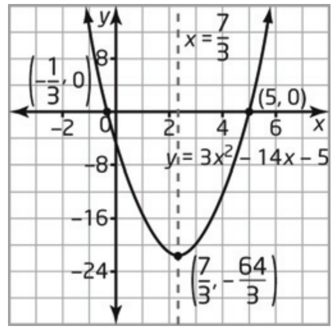
c) $\frac{8 \pm \sqrt{76}}{6}$; –0.12, 2.79

d) $\frac{48 \pm \sqrt{2544}}{24}$; –0.10, 4.10

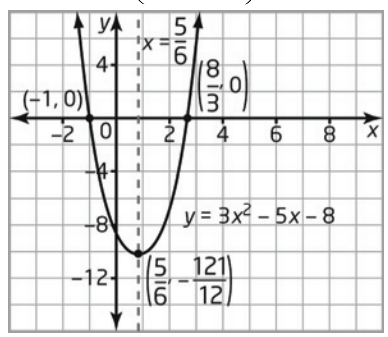
e) $\frac{-22 \pm \sqrt{428}}{-14}$; 3.05, 0.09

f) $\frac{-30 \pm \sqrt{780}}{-12}$; 4.83, 0.17

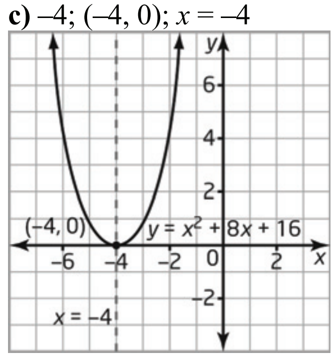
3. a) 5, $-\frac{1}{3}$; $\left(\frac{7}{3}, -\frac{64}{3}\right)$; $x = \frac{7}{3}$



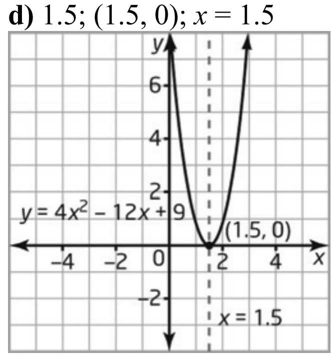
b) $\frac{8}{3}$, –1; $\left(\frac{5}{6}, \frac{-121}{12}\right)$; $x = \frac{5}{6}$



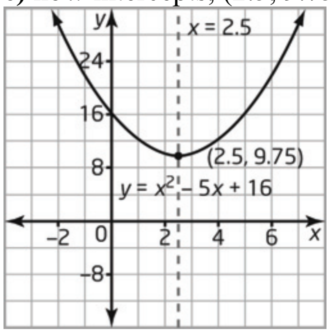
c) –4; (–4, 0); $x = -4$



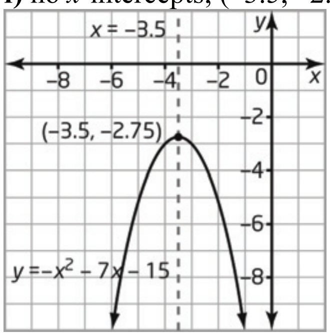
d) 1.5; (1.5, 0); $x = 1.5$



e) no x-intercepts; (2.5, 9.75); $x = 2.5$

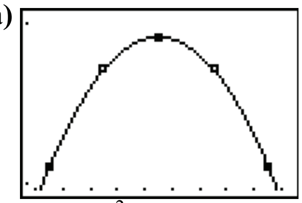


f) no x-intercepts; (–3.5, –2.75); $x = –3.5$



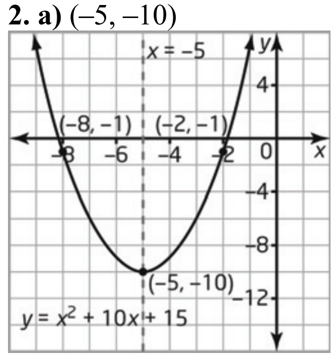
4. a) (2, 3); opens upward; no x-intercepts
b) (–4, –2); opens upward; two x-intercepts
c) (5, –4); opens downward; no x-intercepts
d) (–3, 5); opens downward; two x-intercepts.
e) (–6, 0); opens upward; one x-intercept
f) (8, 0); opens downward; one x-intercept
5. a) 10.1 m
b) 0.6 m or 9.4 m
c) maximum height: 5.3 m; horizontal distance: 5 m
6. a) 2.0 s
b) 0.4 s and 1.5 s
c) maximum height: 5.9 m; time: 0.9 s
7. Answers will vary.
8. a) 135.4 m
b) 25.2 m
c) 15.5 m
9. a) –3.32, –0.48
b) –1.66, 0.91
c) –3.41, –0.59
d) –9.85, –3.15
e) –0.12, 4.24
f) –0.93, 1.73
10. a) $x^2 – 3x + 1 = 0$
b) $2x^2 + 5x – 14 = 0$

6.5 Solve Problems Using Quadratic Equations, **pages 71–72**
1. a) $h = –4.9t^2 + 8t + 1.2$
b) 1.77 s
2. a) 10.43 s
b) 2.41 s and 8 s
c) maximum height: 134.0 m; time: 5.2 s
3. 8 cm and 13 cm
4. 38 and 39 or –39 and –38
5. 15 and 16 or –16 and –15
6. 7 cm, 24 cm, and 25 cm
7. a)

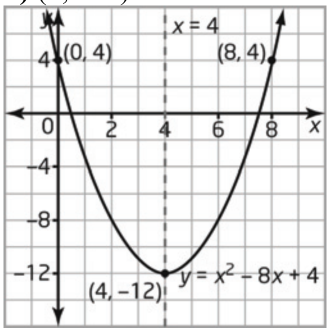


b) $y = –0.05x^2 + 0.85x + 0.2975$
8. 41.2 mm
9. 4.68 cm
10. 11, 12, and 13 or –13, –12, and –11
11. 9.65 cm by 4.35 cm
12. 15 m by 30 m
13. 3 units, 4 units, and 5 units
14. a) $h = –4.9t^2 + 38.25t + 0.71$
b) 7.8 s
c) maximum height: 75.4 m; time: 3.9 s
15. 6 m by 7 m
16. a) Revenue, R, in dollars, is $R = (15 + 2x)(20 – x)$.
b) maximum revenue: $378.13; price: $13.75

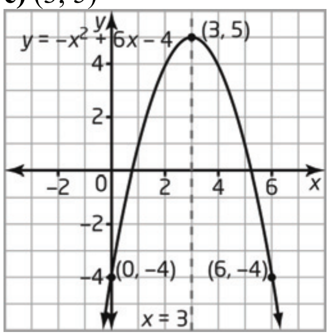
Chapter 6 Review, **pages 73–74**
1. a) $y = (x + 3)^2 – 12$
b) $y = (x + 2)^2 + 1$
c) $y = (x + 5)^2 – 7$
d) $y = (x + 6)^2 – 10$
2. a) (–5, –10)



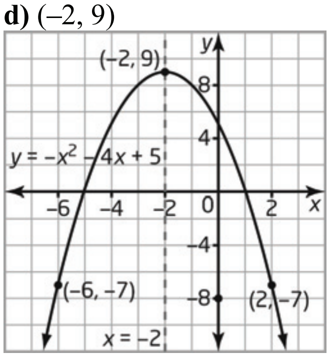
b) (4, –12)



c) (3, 5)



d) (–2, 9)



3. a) minimum point at (–0.2, 1.4)

b) maximum point at (–0.4, 1.4)

c) minimum point at (–0.8, –4.5)

d) maximum point at (–0.7, 1.0)

4. a) –8, –3

b) –9, 4

c) 1, 6

d) 8

e) –6, 6

f) $\frac{7}{2}$, –3

5. a) 3, 4

b) –6, –4

c) $-\frac{3}{4}$, $\frac{1}{2}$

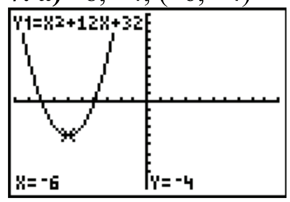
d) $\frac{5}{2}$, $\frac{4}{3}$

e) $\frac{7}{4}$, $-\frac{3}{2}$

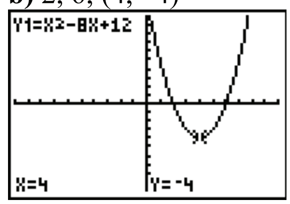
f) $-\frac{3}{2}$, 2

6. 5 cm, 12 cm, and 13 cm

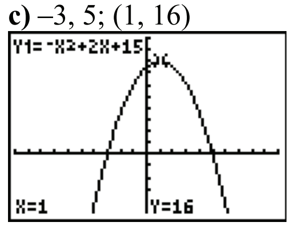
7. a) –8, –4; (–6, –4)



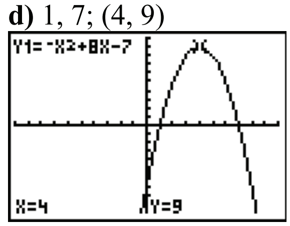
b) 2, 6; (4, –4)



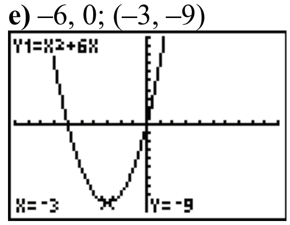
c) –3, 5; (1, 16)



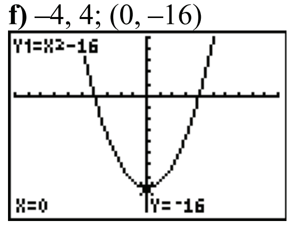
d) 1, 7; (4, 9)



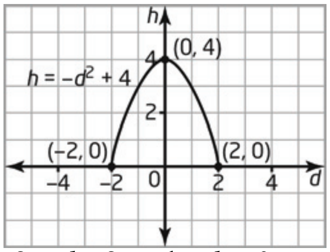
e) –6, 0; (–3, –9)



f) –4, 4; (0, –16)

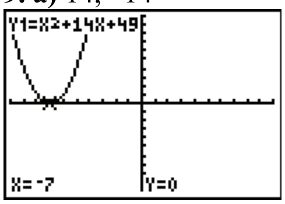


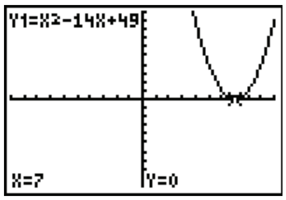
8. a) width 4 m; height 4 m

b) 

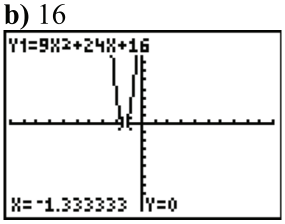
c) $-2 \le d \le 2$ so that $h \ge 0$

9. a) 14, –14

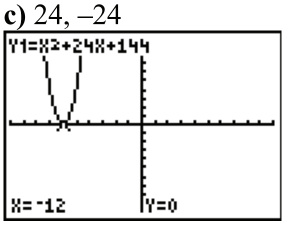


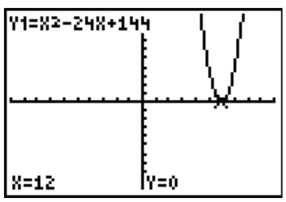


b) 16

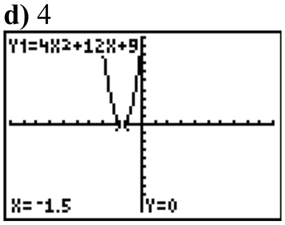


c) 24, –24





d) 4



10. a) $2, -\frac{3}{4}$

b) $\frac{3}{2}$

c) $\frac{-9 \pm \sqrt{57}}{4}$

d) $\frac{17 \pm \sqrt{181}}{18}$

e) $\frac{-7 \pm \sqrt{109}}{10}$

f) $-\frac{5}{4}$

11. a) 1.8 s

b) maximum height: 4.8 m; time: 0.9 s

12. 18 and 19 or –19 and –18

13. 5.3 cm by 9.3 cm

Chapter 7

7.1 Investigate Properties of Similar Triangles, pages 75–76

1. a) Answers will vary.

b) Answers will vary.

2. Answers may vary. For example:

a) Congruent figures, because the tiles will be the same, or similar figures if the sides are in proportion.

b) Similar figures, because the logo on the poster would be larger than the logo on the letterhead, but the same shape.

c) Neither, the photograph would probably be rectangular and the painting might be a square. The figures could also be similar figures, or congruent figures.

d) Similar figures, because the three-dimensional model will be smaller than the real living room, but the same shape.

3. a) ΔDEF ~ ΔGHI

b) ΔPQR ~ ΔSTU

4. a) ΔABC ~ ΔAXY

b) ΔJKL ~ ΔNKM

5. a) $\frac{AB}{AX} = \frac{BC}{XY} = \frac{AC}{AY}$

b) $\frac{JK}{NK} = \frac{KL}{KM} = \frac{JL}{NM}$

6. a) ΔADE ~ ΔABC; ∠A is common to both triangles, ∠ADE = ∠B because they are both right angles. Also, ∠AED = ∠C because they are corresponding angles of parallel lines.

b) ΔPQR ~ ΔTSR; ∠P = ∠T and ∠Q = ∠S because they are alternate angles. Also, ∠PRQ = ∠TRS because they are opposite angles.

c) ΔXYZ ~ ΔXVW; ∠X is common to both triangles; ∠XYZ = ∠V and ∠XZY = ∠W because they are corresponding angles of parallel lines.

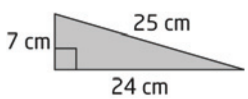
7. a) ΔPQR ~ ΔXYZ; ratio of corresponding sides are all equal to $\frac{1}{2}$.

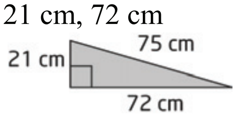
b) ΔADB ~ ΔBDC; ratio of corresponding sides are all equal to 3.

c) ΔQSP ~ ΔPSR; ratio of corresponding sides are all equal to 2.

8. a) ∠P = ∠X, ∠Q = ∠Y, ∠R = ∠Z;
PQ:XY = QR:YZ = RP:ZX
b) ∠ADB = ∠BDC, ∠DBA = ∠DCB,
∠BAD = ∠CBD;
AD:BD = DB:DC = BA:CB
c) ∠PQS = ∠RPS, ∠QSP = ∠PSR,
∠SPQ = ∠SRP; QS:PS = SP:SR = QP:PR
9. a) Answers will vary.
b) Answers will vary.
10. Answers may vary. For example: Yes. The three angles in an isosceles right triangle will equal the three angles in another isosceles right triangle.
11. a) width: 10 cm; length: 16 cm
b) width: 15 cm; length: 24 cm
c) Answers may vary. For example: The area of each piece of artwork equals the area of the original piece of artwork multiplied by the square of the scale factor.
12. a) l = 1 m, w = 0.5 m, h = 2 m
b) l = 0.5 m, w = 0.25 m, h = 1 m

7.2 Use Similar Triangles to Solve Problems, pages 77–78

1. a)


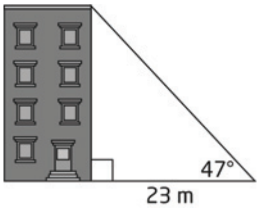
b) 3
c) 21 cm, 72 cm
d)


2. a) area of first triangle: 84 cm^2; area of similar triangle: 756 cm^2
b) The area of the larger triangle is 9 times the area of the smaller triangle.
c) Answers may vary. For example: 9 is the square of the scale factor, 3.
3. a) Δ ABC ~ Δ DEC; ∠CAB = ∠CDE and ∠CBA = ∠CED because they are alternate angles of parallel lines. Also, ∠ACB = ∠DCE because they are opposite angles.
b) x = 40.5 cm, y = 14 cm
4. a) e = 2.4 mm; f = 3.2 mm
b) r = 12 cm; t = 45 cm
5. a) j = 9.6 m; k = 8.8 m
b) s = 48 km; t = 32 km
6. a) 10 cm
b) 16 m
7. a) 108 cm^2
b) 225 m^2
c) 576 cm^2
d) 18 m^2
8. 9.3 m
9. 4.5 m
10. 3:5
11. 4:3
12. Answers will vary.
13. 8 km^2

7.3 The Tangent Ratio, pages 79–80

1. a) 0.6250
b) 0.7451
c) 0.6563
d) 0.8641
e) 0.7157
f) 1.2105
2. a) 1.6000
b) 1.3421
c) 1.5238
d) 1.1573
e) 1.3973
f) 0.8261
3. a) 1.2799
b) 8.1443
c) 0.1405
d) 1.7321
e) 0.5384
f) 0.8754
g) 4.8716
h) 0.4142
4. a) 27°
b) 50°
c) 72°
d) 37°
e) 31°
f) 23°
g) 75°
h) 62°
5. a) ∠A = 35°, ∠B = 55°
b) ∠D = 59°, ∠E = 31°
6. a) 3.4 m
b)24.3 cm
7. a) 10.2 mm
b) 17.7 cm
8. a) 10.9 m
b) 3.1 km

9. a)



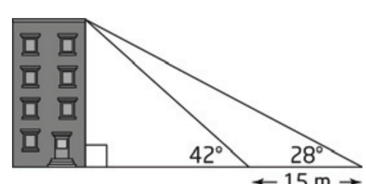
b) 24.7 m
10. a) 25°
b) Answers will vary. For example: There are no obstacles along the path.
11. 38°
12. 52°

7.4 The Sine and Cosine Ratios, pages 81–82

1. a) $\sin \theta = \frac{3}{5}$, $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$
b) $\sin \theta = \frac{7}{25}$, $\cos \theta = \frac{24}{25}$, $\tan \theta = \frac{7}{24}$
c) $\sin \theta = \frac{25}{27}$, $\cos \theta = \frac{10}{27}$, $\tan \theta = \frac{5}{2}$
d) $\sin \theta = \frac{35}{38}$, $\cos \theta = \frac{15}{38}$, $\tan \theta = \frac{7}{3}$
e) $\sin \theta = \frac{230}{259}$, $\cos \theta = \frac{120}{259}$, $\tan \theta = \frac{23}{12}$
f) $\sin \theta = \frac{25}{39}$, $\cos \theta = \frac{10}{13}$, $\tan \theta = \frac{5}{6}$
2. a) sin A = 0.8471, cos A = 0.5294, tan A = 1.6000
b) sin A = 0.8701, cos A = 0.4935, tan A = 1.7632
3. a) 0.6157
b) 0.8829
c) 0.4226
d) 0.9986
e) 1.0000
f) 0.6428
g) 0.8090
h) 0.9455
4. a) 0.8746
b) 0.9903
c) 0.4226
d) 0.8290
e) 0.1392
f) 0.6428
g) 0.7314
h) 0.3746
5. a) 51°
b) 13°
c) 27°
d) 67°
e) 26°
f) 18°
g) 49°
h) 47°
6. a) 59°
b) 77°
c) 39°
d) 55°
e) 73°
f) 52°
g) 82°
h) 32°
7. a) 7.7 cm
b) 23.4 m
c) 18.7 mm
d) 5.6 km
8. a) 15.9 km
b) 34.1 cm
c) 11.7 m
d) 3.1 mm
9. a) ∠A = 54°, a = 14.6 cm, b = 10.6 cm
b) ∠D = 65°, e = 12.6 m, f = 29.8 m
c) ∠Y = 28°, ∠Z = 62°, x = 9.3 km
10. ∠A = 51°, ∠C = 39°, a = 6.2 m
11. 11.1 m

7.5 Solve Problems Involving Right Triangles, pages 83–84

1. a) 2.5 m
b) 2.6 m
2. a) 9.1 m
b) 29.4 m
3. 25.7 m
4. 59.7 m
5. 14.9 m
6. 22.8 m
7. a) 31°, 50°
b) 5.8 m, 7.8 m
8. 95.2 m
9. a) 6.4 km
b) 51°
10. 44°
11. a)



b) 19 m

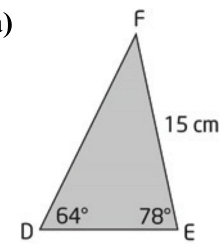
Chapter 7 Review, pages 85–86

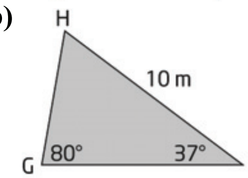
1. a) Answers will vary.
b) Answers will vary.
c) Answers will vary.
d) Answers will vary.
e) Answers will vary.
f) Answers will vary.
2. Answers may vary. For example: Yes, because they have the same shape.
3. ΔABE ~ ΔDCE because corresponding pairs or angles are equal $\angle$A = $\angle$D (alternate angles), $\angle$B = $\angle$C (alternate angles), and $\angle$AEB = $\angle$DEC (opposite angles).
4. $\Delta PQT \sim \Delta RST$; $\frac{PQ}{RS}=\frac{QT}{ST}=\frac{PT}{RT}=\frac{2}{1}$
5. x = 24 cm; z = 10 cm
6. l = 8 m; g = 27 m
7. 7.4 m
8. 24 m^2
9. a) 0.4615
b) 0.7442
10. a) 35°
b) 53°
c) 33°
d) 62°
11. a) 17.3 m
b) 31.1 m
12. a) 30°
b) 55°
13. 2.5 m
14. a) $\sin\theta = \frac{9}{41}$, $\cos\theta = \frac{40}{41}$, $\tan\theta = \frac{9}{40}$
b) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$
15. a) 55°
b) 27°
c) 61°
d) 65°
16. a) 7.6 m
b) 16.1 m
17. a) c = 14.2 m, $\angle$A = 38°, $\angle$B = 52°
b) $\angle$D = 26°, e = 13.5 cm, d = 6.6 cm
18. 9.4 m
19. 23.6 m

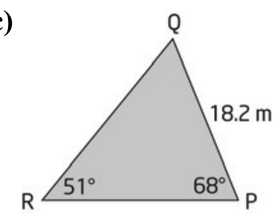
Chapter 8

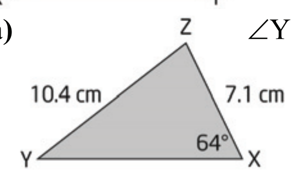
Note: Slightly different answers may be obtained if measures are calculated in a different order.

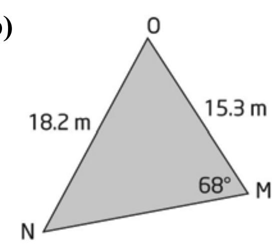
8.1 The Sine Law, pages 87–88

1. a) b = 7 cm
b) f = 13 m
c) r = 11 mm
d) y = 21 km
2. a) $\angle$S = 63°
b) $\angle$Y = 44°
c) $\angle$C = 40°
d) $\angle$F = 49°
3. a) 

e = 16.3 cm
b) 

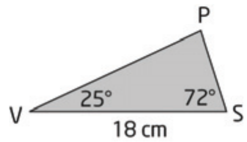
i = 6.1 m
c) 

p = 21.7 m
4. a) 

$\angle$Y = 38°
b) 

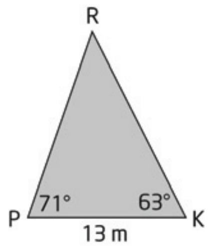
$\angle$N = 51°
5. a) $\angle$C = 68°, b = 22 cm, c = 23 cm
b) $\angle$F = 57°, d = 20 m, e = 25 m
6. a) $\angle$G = 40°, $\angle$I = 74°, i = 18 cm
b) $\angle$K = 52°, $\angle$L = 42°, l = 9 m

7. a)



$\angle P = 83°$, $v = 8$ cm, $s = 17$ cm

b)



$\angle R = 46°$, $k = 16$ m, $s = 17$ m

8. Answers will vary.

9. a) 2.6 m

b) 2.4 m

10. a) 68°

b) 14.9 m

c) 19.4 m

11. a) 6.6 km

b) 7.4 km

c) 6.3 km

8.2 The Cosine Law, pages 89–90

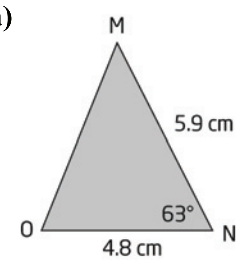
1. a) $a = 24$ cm

b) $f = 14$ m

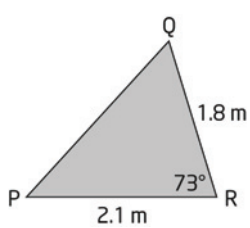
c) $g = 9$ mm

d) $l = 21$ km

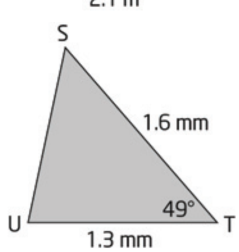
2. a) $n = 5.7$ cm



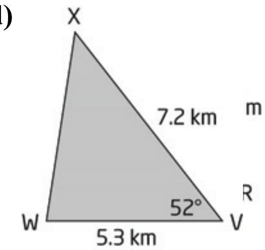
b) $r = 2.3$ m



c) $t = 1.2$ mm



d) $v = 5.7$ km



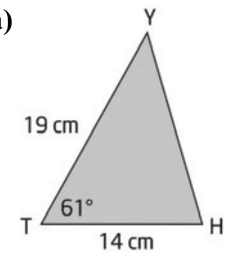
3. a) $q = 17.6$ cm, $\angle X = 70°$, $\angle V = 43°$,

b) $r = 16.6$ m, $\angle D = 41°$, $\angle S = 57°$

c) $n = 20.4$ mm, $\angle P = 86°$, $\angle L = 48°$

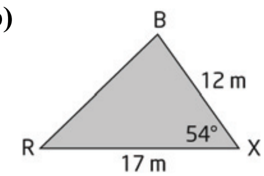
d) $w = 7.2$ km, $\angle M = 78°$, $\angle T = 44°$

4. a)



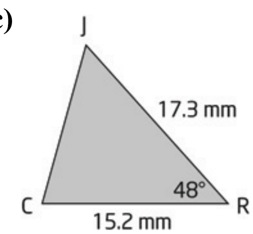
$t = 17.3$ cm, $\angle Y = 45°$, $\angle H = 74°$

b)



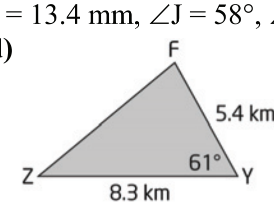
$x = 13.9$ m, $\angle R = 44°$, $\angle B = 82°$

c)



$r = 13.4$ mm, $\angle J = 58°$, $\angle C = 74°$

d)



$y = 7.4$ km, $\angle Z = 40°$, $\angle F = 79°$

5. Answers will vary.

6. 51 m

7. a) 86.7 m

b) 42°

c) 63°

8. 868 m

9. a) 2.8 km

b) 5.7 km

c) Answers will vary.

10. 2.8 km

11. a)–c) Answers will vary.

8.3 Find Angles Using the Cosine Law, pages 91–92

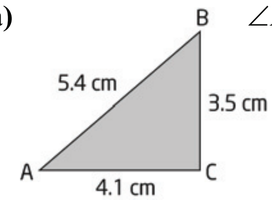
1. a) $\angle Q = 52°$

b) $\angle P = 38°$

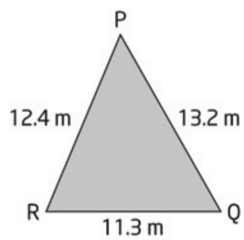
c) $\angle R = 76°$

d) $\angle H = 56°$

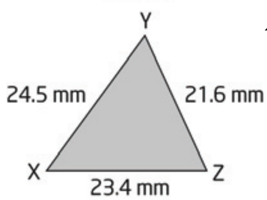
2. a) $\angle A = 40°$



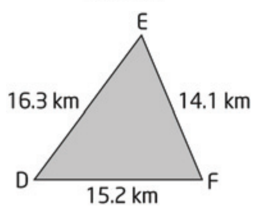
b) $\angle Q = 60°$



c) $\angle X = 54°$



d) $\angle D = 53°$



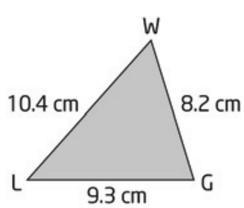
3. a) $\angle K = 77.2°$, $\angle T = 58.8°$, $\angle W = 44.0°$

b) $\angle M = 69.0°$, $\angle C = 61.0°$, $\angle J = 50.0°$

c) $\angle P = 68.4°$, $\angle N = 60.1°$, $\angle V = 51.5°$

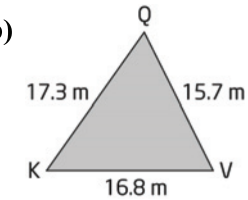
d) $\angle X = 79.5°$, $\angle A = 56.0°$, $\angle F = 44.5°$

4. a)



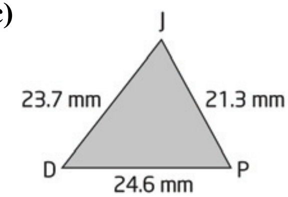
$\angle G = 72.6°$, $\angle W = 58.6°$, $\angle L = 48.8°$

b)



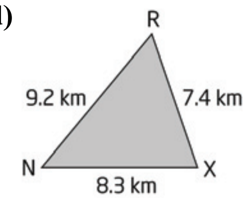
$\angle V = 64.2°$, $\angle Q = 61.0°$, $\angle K = 54.8°$

c)



$\angle J = 66.0°$, $\angle P = 61.7°$, $\angle D = 52.3°$

d)



$\angle X = 71.5°$, $\angle R = 58.8°$, $\angle N = 49.7°$

5. Answers will vary.

6. a) 67°

b) 54°

c) 59°

7. a) 45°

b) 79°

c) 56°

8. 36°, 72°, 72°

9. 87.1°, 48.5°, 44.4°

10. 53°, 57°, 70°

11. a)
$$\begin{aligned}\cos C &= \frac{c^2 - a^2 - b^2}{-2ab} \\ &= \frac{c^2 - a^2 - (2a)^2}{-2a(2a)} \\ &= \frac{c^2 - a^2 - 4a^2}{-4a^2} \\ &= \frac{c^2 - 5a^2}{-4a^2} \\ &= \frac{5}{4} - \frac{c^2}{4a^2}\end{aligned}$$

b) $$\begin{aligned}\cos A &= \frac{a^2 - b^2 - c^2}{-2bc}\\ &= \frac{a^2 - b^2 - (3b)^2}{-2b(3b)}\\ &= \frac{a^2 - b^2 - 9b^2}{-6b^2}\\ &= \frac{a^2 - 10b^2}{-6b^2}\\ &= \frac{5}{3} - \frac{a^2}{6b^2}\end{aligned}$$

8.4 Solve Problems Using Trigonometry, pages 93–94
1. a) primary trigonometric ratios
b) sine law
c) sine law
d) cosine law
2. a) 0.95 cm
b) Answers will vary.
3. a) 12.1 m
b) Answers will vary.
4. a) 2.7 km
b) 7.5 km
c) 59°, 53°
5. a) Answers will vary.
b) 2.4 km
6. a) 20 m
b) 29.7 m
7. a) Answers will vary.
b) 4.5 m
c) 30.3 m
8. 209 m
9. a) 418 m
b) 51°, 75°
10. a) 0.08 km
b) 0.26 km

Chapter 8 Review, pages 95–96
1. a) 1.3 m
b) 1.0 m
2. a) 35°
b) 49°
3. a) ∠R = 64°, q = 18.9 cm, r = 19.2 cm
b) ∠Z = 52°, y = 9.1 m, z = 8.6 m
4. a) 28 m
b) 58°, 50°
5. 30 cm
6. a) m = 25 mm ∠D = 58°, ∠Y = 67°
b) r = 17 cm, ∠D = 69°, ∠K = 49°
7. a) Answers will vary.
b) 11 m
8. a) 44°
b) 49°
9. a) ∠G = 76.9°, ∠C = 55.9°, ∠R = 47.2°
b) ∠M = 84.0°, ∠H = 52.0°, ∠F = 44.0°
10. 27.5 m
11. a) Answers will vary.
b) 33 m
c) 16 m, 17 m
d) 17 m, 21 m
12. water tower 44°, hydro tower 30°

Enrichment Questions

Enrichment Questions 1, page 97
1. 253 × 14
2. Answers will vary.
3. 36 cm
4. 28
5. 8
6. a) 33 h 29 min **b)** 174 km/h

Enrichment Questions 2, page 98
1. 32
2. 001
3. 15
4. 32, 31
5. a) 10 **b)** 10
6. a) Newfoundland **b)** Prince Edward Island
7. Donna
8. Move 7 from the 11 pile to the 7 pile and you have 4, 14, 6. Move 6 from the 14 pile to the 6 pile and you have 4, 8, 12. Move 4 from the 12 pile to the 4 pile and you have 8, 8, 8.
9. Answers will vary.

Enrichment Questions 3, page 99
1. 05:00 Wednesday
2. G, P, W; R
3. Answers will vary.
4. (1, 15), (2, 14), (3, 13), (4, 12), (5, 11), (6, 10), (7, 18), (8, 17), (9, 16)
5. 60 cm^2
6. Terry Bob Sand, Thomas Bill Stone, Tim Bevan Silver, Trip Brooks Sol
7. 9
8. About 3.8 billion, assuming tiles are 8.5 cm by 8.5 cm.
9. 10

Enrichment Questions 4, page 100
1. 321 cm^2
2. a) It is about 0.5 times the surface area of Mercury, assuming both bodies are spherical.
b) about 590 times
3. 08:45 Saturday
4. a) Answers will vary. **b)** twice
c) Answers will vary.
5. 10
6. 1596 cm

Enrichment Questions 5, page 101
1. 9
2. 43
3. 120
4. a) 10
b) 8
5. 22
6. Rohana
7. Answers will vary.
8. Sharif
9. 4 blocks east and 7 blocks south
10. 300

Enrichment Questions 6, page 102
1. 12
2. a) 1 **b)** 2
3. 6
4. $143
5. Answers will vary.
6. Weigh three coins against three other coins. If the scales balance, then the real coin is among the two not weighed and can be determined with another weighing. If the scales do not balance, weigh two of the three coins that tipped the scales most. If one side tips more, the real coin is found. If the scales balance, the real coin was the other of the three.
7. 18, 20, 24
8. a) Québec **b)** Manitoba
9. Answers will vary.
10. 10
11. $h = 22$, $g = 3$

Enrichment Questions 7, page 103
1. a) 94 cm^2 **b)** 50
2. Answers will vary.
3. 4 cm by 8 cm by 18 cm
4. Answers will vary.
5. $3 \times 54 = 162$
6. a) 4 **b)** 1
7. Answers will vary.
8. Toronto and Calgary, Boston and Vancouver, Edmonton and Detroit, Chicago and Montréal.

Enrichment Questions 8, page 104
1. L shape or Z shape
2. Tia: carpenter, turtle; Jane: lawyer, dog; Fran: police officer, parrot; Marta: teacher, cat
3. Between 6 a.m. and 8 a.m., Paris time. Answers will vary.
4. Mr. Smith: farmer, Vancouver;
Mr. Wong: accountant, London;
Ms. Bevan: baker, Toronto;
Ms. Lee: writer, Paris;
Ms. Kostash: biologist, Melbourne
5. a)–c) Answers will vary.

Enrichment Questions 9, page 105
1. a) Answers will vary.
b) The square is being rotated 90° counterclockwise.
c) same as Square 3 **d)** same as Square 1
2. Alexi: green, Kala: blue, Lisa: yellow, Jamal: red
3. Answers will vary.
4. A = 6, B = 2, C = 5, D = 3;
missing row sum = 21,
missing column sum = 22
5. a) 28 **b)** 168
6. a)–c) Answers will vary.
7. Answers will vary.
8. a) one thousand **b)** one billion
9. 6, 1, 10, 8; 5, 9, 2; 4, 7; 3
10. Answers will vary.

Enrichment Questions 10, page 106
1. $\frac{1}{57}$
2. 68
3. Answers will vary.
4. a), **b)** Answers will vary.
5. If true, the average length of the bicycles must be $\frac{71\text{ km}}{45\,000\text{ bicycles}}$, or about 1.58 m.
6. 8
7. a), **b)** Answers will vary.
8. Alberta. Answers will vary.
9. Answers will vary.
10. 25, 85