1.1 Connect English With Mathematics and Graphing Lines

***Principles of Mathematics 10*, pages 8–19**

A

1. Translate each phrase into an algebraic expression.

a) five more than twice a number

b) three less than one quarter a value

c) the product of a number and another number increased by seven

d) a value decreased by the fraction one half

2. Translate each phrase into an algebraic expression.

a) four times a length

b) triple a distance

c) thirty percent of a number

d) six percent of a price

3. For each of the following, write a word or phrase that has the opposite meaning.

a) decreased

b) subtracted

c) less than

d) minus

e) added to

f) more than

4. Translate each sentence into an algebraic equation.

a) One sixth of a number, increased by 15, is 42.

b) Three times a number, decreased by four, is five more than six times the number.

c) When tickets to a soccer game cost $4 each, the revenue at the gate is $320.

d) The total length of the base and height of a triangle 15 cm.

B

5. Find the point of intersection for each pair of lines by graphing. Check your answers.

a) $y = 2x + 5$
$y = x + 1$

b) $y = -2x + 3$
$y = 3x + 8$

c) $y = 4x + 8$
$y = \frac{1}{2}x + 1$

d) $y = \frac{4}{5}x + 2$
$y = \frac{3}{4}x + 3$

6. Find the point of intersection for each pair of lines by graphing. Check your answers.

a) $x + y = 6$
$2x - y = 6$

b) $3x + 4y = 6$
$2x - 4y = 4$

c) $x - y = 3$
$3x + y = 5$

d) $2x + 3y = 5$
$x - 3y = 4$

7. **Use Technology** Use a graphing calculator or *The Geometer's Sketchpad*® to find the point of intersection for each pair of lines. Where necessary, round answers to the nearest hundredth.

a) $y = 8x + 5$
$y = -7x - 6$

b) $y = -3x + 5$
$y = 4x + 7$

c) $y = 2.3x + 9$
$y = 4.5x - 10$

d) $y = -0.3x + 2.4$
$y = -0.2x + 3.5$

8. Sarah delivers flyers in the summer to make some extra money. She charges $10.00 per hour. Ads-R-Us Delivery Service charges $120 for the season.

a) Write an equation for the amount Sarah charges to deliver the flyers for the season.

b) Write an equation for the amount Ads-R-Us Delivery Service charges.

c) What is the intersection point of the two linear equations?

d) In the context of this question, what does the point of intersection represent?

9. **Use Technology** Savio works for a cellular phone company. He is paid $90/day plus $2.00 for each cellular phone that he sells. Aimee also works for the cell phone company, but she makes $120 per day and no extra money for selling cellular phones.

a) Write an equation to represent the amount that Savio earns in one day. Graph the equation.

b) Write an equation to represent the amount that Aimee earns in one day. Graph this equation on the same grid you used in part a).

c) How many cellular phones must Savio sell in order to make as much in a day as Aimee?

C

10. Kristen has a total of $1000 to invest. She puts part of it in an account paying 4% interest/year and the rest in a bond paying 6.5% interest. If she has $50 in simple interest at the end of the year, how much was invested at each rate?

11. Graph the equations $y = 2x + 1$, $y = -3x + 6$, and $y = \frac{1}{2}x + \frac{5}{2}$ on the same grid. Explain what you find.

12. **a)** Can you solve the linear system $y = 3x - 2$ and $6x - 2y - 4 = 0$? Explain your reasoning.

b) Can you solve the linear system $y = 4x - 3$ and $8x - 2y + 5 = 0$? Explain your reasoning.

c) Explain how you can tell, without solving, how many solutions a linear system has.

1.2 The Method of Substitution

***Principles of Mathematics 10*, pages 20–28**

A

1. Solve each linear system using the method of substitution. Check your answers.

a) $y = 2x + 5$
$x + y = 8$

b) $y = 3x - 7$
$x + 2y = 7$

c) $y = -x + 3$
$2x + 3y = 5$

d) $3x + 4y = -4$
$x = 2 - 3y$

2. In each pair, decide which equation you will rewrite to express one variable in terms of the other variable. Do that step. Do not solve the linear system.

a) $x + 3y = 4$
$4x + 2y = 7$

b) $2x + 5y = 8$
$2x + y = 6$

3. a) Is (1, 1) the solution for the following linear system? Explain how you can tell.

$3x + 4y = 7$
$2x + 5y = 8$

b) Is (4, –3) the solution for the following linear system? Explain how you can tell.

$3x - 2y = 18$
$2x + 3y = -1$

B

4. Solve by substitution. Check your answers.

a) $x + 3y = 5$
$4x + 2y = 10$

b) $5a + b = 4$
$3a + 2b = -6$

c) $x - 2y = 5$
$2x + 3y = 17$

d) $2m - 3n = -10$
$4m + n = 1$

5. Find the point of intersection for each pair of lines. Check your answers.

a) $5x = y + 11$
$2x + y = 3$

b) $m + 3n = 4$
$4m + 2n + 4 = 0$

6. Kyle reads for twice as many hours per week as Santiago. Together they read for a total of 24 hours in one week.

a) State how you will assign variables.

b) Write an equation to represent the information in the first sentence.

c) Write an equation to represent the information in the second sentence.

d) Use the method of substitution to find the number of hours each person spent reading that week.

7. Nyiri and Raven go to a sports store. The two girls buy a total of nine pairs of shorts. Raven purchases six less than twice as many pairs of shorts as Nyiri.

a) Assign variables. Write an equation to represent the information in the second sentence.

b) Write an equation to represent the information in the third sentence.

c) Solve the linear system by substitution to find the number of pairs of shorts each girl bought.

d) If the shorts cost $15.99 each, how much did each girl spend before taxes?

8. The president of the Athletic Council decides to rent a hall for the annual Athletic Banquet. Sportsmania charges $450 for the hall and $16 per meal. Sports-To-Go charges $330 for the hall and $20 per meal.

a) Assign variables. Write two equations to represent the information.

b) Solve the linear system to find the number of guests for which both halls charge the same amount.

9. Ron makes two types of comforters. To make the first type he charges $35 for the material and $60/h for labour. For the second type he charges $105 for the material and $25/h for labour. For what number of hours are the costs the same?

10. Christina makes two types of leather belts. For the first type she charges $80 for the material and $50/h for labour. For the second type she charges $100 for the material and $40/h for labour. For what number of hours are the costs the same?

C

11. The following three lines intersect to form a triangle.

$y = x + 3$

$2x + y = 6$

$x + y = 7$

a) Find the coordinates of each vertex.

b) Is this a right triangle? Explain how you know.

12. Simplify each equation, and then solve the linear system by substitution. Round your answers to the nearest tenth, if necessary.

a) $2(x + 1) + 3(y + 2) = 15$
$x + 3(y - 1) = 2$

b) $2(x - 1) + y = 5$
$4x - 3(y + 2) = 15$

c) $3(x - 1) - (y + 1) = 1$
$4(x + 1) + 2(y - 1) = 10$

d) $2(x + 1) + 3(y + 2) = 10$
$-(x + 3) + 2(y - 1) = 1$

13. The following three lines all intersect at one point.

$3x + 5y = 10$

$x + 2y = 4$

$5x + ky = 10$

Find the coordinates of the point of intersection of the three lines and the value of k.

1.3 Investigate Equivalent Linear Relations and Equivalent Linear Systems

***Principles of Mathematics 10*, pages 29–33**

A

1. Which two equations are equivalent?

A $y = x + 3$

B $y = \frac{1}{2}x + 4$

C $2y = x + 8$

2. Which is *not* an equivalent linear relation?

A $6y = 2x + 4$

B $y = \frac{1}{3}x + \frac{2}{3}$

C $9y = 2x + 3$

D $3y = x + 2$

3. Write two equivalent equations for each.

a) $y = 5x - 3$

b) $4x + 3y = 12$

c) $y = \frac{2}{3}x + 5$

d) $x + y = 7$

4. Which two of the following linear equations will have the same graph?

A $2y = \frac{1}{2}x + 4$

B $y = \frac{1}{4}x + 1$

C $4y = x + 4$

B

5. If $y = 3x - 7$ and $3y = kx - 21$ are equivalent linear equations, what is the value of k?

6. **a)** If $y = 2x - 8$ and $4y = kx - 32$ are equivalent linear equations, what is the value of k?

b) If $y = 5x - 4$ and $2y = 10x + k$ are equivalent linear equations, what is the value of k?

c) If $3y = 4x - 2$ and $ky = 8x - 4$ are equivalent linear equations, what is the value of k?

7. The total number of males and females in Endi's math class is 14.

a) State how you will assign variables.

b) Write an equation to represent this situation. Then, write an equivalent linear equation.

8. The total number of dimes and quarters in Marijan's piggy bank is 82.

a) State how you will assign variables.

b) Write an equation to represent this situation. Then, write an equivalent linear equation.

9. The perimeter of a rectangle is 18 m.

a) State how you will assign variables.

b) Write an equation to represent this situation. Then, write an equivalent linear equation.

10. A linear system is given.

$4x - 8y = 10$ ①

$x + y = 5$ ②

Explain why the following is an equivalent linear system.

$2x - 4y = 5$ ③

$2x + 2y = 10$ ④

11. A linear system is given.

$y = \frac{3}{4}x - 2$ ①

$y = -\frac{2}{3}x + 1$ ②

Explain why the following is an equivalent linear system.

$4y = 3x - 8$ ③

$3y = -2x + 3$ ④

C

12. a) A linear system is given.

$4(2x - 3) + 5(3y + 8) = 4$

$3(4x + 2) - 4(2y - 3) = 7$

Show that the following is an equivalent linear system.

$8x + 15y = -24$

$12x - 8y = -11$

b) A linear system is given.

$y = -\frac{8}{15}x - \frac{24}{15}$ ①

$y = \frac{3}{2}x + \frac{11}{8}$ ②

Explain why the following is an equivalent linear system.

$8x + 15y = -24$ ③

$12x - 8y = -11$ ④

13. a) Graph the following linear system on grid paper.

$y = x$

$y = -x + 4$

b) State the point of intersection of the two lines.

c) Show that the two lines are perpendicular to each other.

d) Write two equivalent linear systems to the following linear system.

$y = x$

$y = -x + 4$

14. a) Graph the following linear system on grid paper.

$y = 2x$

$y = -\frac{1}{2}x + 5$

b) State the point of intersection of the two lines.

c) Show that the two lines are perpendicular to each other.

d) Write two linear systems equivalent to the following linear system.

$y = 2x$

$y = -\frac{1}{2}x + 5$

1.4 The Method of Elimination

***Principles of Mathematics 10*, pages 34–41**

A

1. Solve using the method of elimination. Check each solution.

a) $x + y = 4$
$2x - y = 5$

b) $x - y = -2$
$3x + y = -10$

c) $x + 4y = 10$
$x + y = 1$

d) $4x + 3y = 10$
$2x + 3y = 2$

2. Solve using the method of elimination. Check each solution.

a) $x + 3y = 7$
$-x + 2y = 3$

b) $2x + 4y = 10$
$-2x + 3y = 11$

c) $x + 5y = 7$
$x + 3y = 5$

d) $4x + 5y = 7$
$4x + 2y = 1$

3. Solve using the method of elimination. Check each solution.

a) $x + 5y = 8$
$x + y = 4$

b) $x + 3y = 3$
$2x + 4y = 2$

c) $3x + 4y = 14$
$2x + y = 1$

d) $3x + 5y = 4$
$3x + 6y = 6$

4. Find the point of intersection of each pair of lines. Check each solution.

a) $2x + 3y = 7$
$3x - 2y = 4$

b) $2x + 4y = 2$
$5x - 3y = 5$

c) $2x + 3y = 5$
$3x + 2y = 5$

d) $2a + 5b = 5$
$3a + 2b = 13$

B

5. Find the point of intersection of each pair of lines. Where necessary, express answers as fractions in lowest terms. Check each solution.

a) $3x + 5y = 10$
$2x + 5y = 7$

b) $3x + 6y = 9$
$2x - 6y = 8$

c) $3x + 4y = 7$
$5x + 3y = 8$

d) $3x + 5y = 4$
$2x + 3y = 7$

e) $4x + 2y = 3$
$3x - 5y = 4$

f) $2m - 3n = 1$
$4m + 2n = 3$

6. Zidane works in a sporting goods store selling skates. A pair of hockey skates costs $58.00 and a pair of figures skates costs $56.00. One shift, Zidane sold 32 pairs of skates. His receipts totalled $1828, not including taxes.

a) How many pairs of hockey skates did Zidane sell?

b) How many pairs of figure skates did Zidane sell?

7. Sadia works at a theatre selling popcorn. Large boxes of popcorn sell for $5.00 and small boxes of popcorn sell for $3.00. During her shift last night she sold 60 boxes of popcorn. Her receipts totalled $260, not including taxes.

a) How many large boxes of popcorn did Sadia sell?

b) How many small boxes of popcorn did she sell?

8. Consider this system.
$3x - 4y = -1$
$4x + y = 5$

a) Solve by elimination.

b) Solve by substitution.

c) Which method do you prefer? Why?

9. Solve each linear system. Check each solution.

a) $0.3x - 0.2y = 11$
$0.5x + 0.4y = 55$

b) $0.4a - 0.2b = 20$
$0.3a + 0.5b = 54$

10. Expand and simplify each equation. Then, solve the linear system. Check each solution.

a) $3(2x - 1) - (y + 3) = 1$
$4(1 + 2x) - 3(2 - y) = 3$

b) $4(a - 2) - 2(b + 3) = -6$
$3(a - 2) - 2(b - 1) = 6$

c) $2(m + 1) + 4(n - 1) = 6$
$3(m - 1) + 4(n + 1) = 7$

C

11. Explain how you would solve the system $5x + 4y = 3$ and $3x + 5y = 8$ using the method of elimination. Do not actually solve the system.

12. What happens when you solve this system by elimination?
$3x + 4y = 8$
$6x + 8y = 0$
Use a graph in your explanation.

13. Solve by elimination.

a) $\frac{3a}{5} - \frac{b}{2} = 9$
$\frac{3a}{4} + \frac{b}{3} = 17$

b) $\frac{m-3}{5} + \frac{n+2}{4} = 1$
$\frac{m+4}{3} + \frac{n-3}{2} = 2$

14. Solve by elimination.
$dx + my = c$
$ex + ny = g$

1.5 Solve Problems Using Linear Systems

***Principles of Mathematics 10*, pages 42–47**

A

1. Zara works part-time at a flower shop. She is making a flower arrangement for a customer with a total of 30 flowers, using two types of flowers. She is asked to use four times as many carnations as roses. How many of each will be in the arrangement?

2. Daniel has a total of 126 Canadian and British stamps in his stamp collection. He determines that he has 38 more Canadian stamps than British stamps. How many of each type of stamp does he have?

3. Faiza and Terry held a fundraising garage sale for the student council. At the garage sale they sold large bottles of water for $3 and small bottles of water for $2. They sold 180 bottles of water at the garage sale and collected $440. How many of each size of bottle did they sell?

4. Marley invests the earnings of $2050 from her part-time job. She invests part of the money at 7%/year, and the rest at 6%/year. After one year, these investments earn $138 simple interest. How much did she invest at each rate?

B

5. On the weekend, Shirley went to visit the Butterfly Museum. There were 78 butterflies in the tropical garden area. Shirley calculated that there were 18 more painted lady butterflies than monarch butterflies. How many of each type of butterfly were in the tropical garden?

6. Two cartons of milk contain different percents of butterfat. How much 1% milk needs to be mixed with how much 5% milk to give 10 L of 4% milk?

7. Nathaniel needs to make 30 L of 28% sulphuric acid solution. In the Chemistry office, he finds bottles of 20% sulphuric acid and 50% sulphuric acid. What volume of each should he mix in order to make the 28% solution?

8. One type of chocolate mixture contains 30% nuts, by mass. A second type of chocolate mixture contains 20% nuts, by mass. What mass of each type of chocolate mixture needs to be mixed to make a 500-g chocolate mixture that will have 24% nuts, by mass?

9. **Use Technology** To join the Best Bridge Club, Taha must pay an initial fee of $150 and a monthly fee of $20. If he joins the Bid Bridge Club, he must pay an initial fee of $100 and a monthly fee of $25.

a) After how many months is the cost the same at either bridge club?

b) If Taha plans to join a bridge club for eight months, which club should he join?

c) If Taha decides to join a bridge club for one year, which club should he join?

10. For a mathematics conference, Mary decides to order royal blue T-shirts for all of the volunteers. It will cost $10 per shirt for the large size and $8 per shirt for the medium size. Mary orders a total of 40 T-shirts and spends $350.

a) How many large T-shirts did Mary order?

b) How many medium T-shirts did Mary order?

11. One aluminum alloy is 35% aluminum. Another aluminum alloy is 55% aluminum. How much of each should be used to make 1000 g of an aluminum alloy that is 40% aluminum?

12. Some students at Monarch Park Collegiate held a food sale to raise money for the Kenya School Building Project. They charged $8 for apple pies and $7 for lemon pies. They sold a total of 83 pies and earned $624. How many of each type of pie did they sell?

13. Aljohn and Tamia are planning a trip for their class. For one option, each student will pay $875 for three meals a day and seven nights accommodation. For the second option, each student will pay $770 for two meals a day and seven nights accommodation.

a) What is the cost per meal?

b) What is the cost per day for accommodation?

14. Ian canoes 20 km downstream in 5 h. On the return trip it takes him 8 h to travel 16 km. Determine his average canoeing speed and the speed of the current.

15. With a tailwind, a plane flew the 1800 km from Saskatoon to Toronto in 3 h. The return flight, against the wind, took 4 h. Find the average speed of the plane and the wind speed.

C

16. Donn has some 15-karat gold ($\frac{15}{24}$ pure gold) and some 10-karat gold ($\frac{10}{24}$ pure gold). What mass of each type of gold should he mix to make 500 g of 14-karat gold ($\frac{14}{24}$ pure gold)?

17. Bob jogged for 2 h then walked for 3 h, covering a total distance of 31 km. The next day he jogged for 3 h then walked for 2 h, covering a distance of 34 km. Assume that his running and jogging speeds were the same both days.

a) Find the speed at which Bob jogged.

b) Find the speed at which Bob walked.

18. Tera is a member of the environmental club. Yesterday, there were 220 batteries in the recycling bin. Tera claims that the number of AA-size batteries was 20 more than three times the number of AAA-size batteries.

a) How many AA batteries were in the recycling bin yesterday?

b) How many AAA batteries were in the recycling bin yesterday?

19. A flower garden has three times as many red roses as pink roses. Twice the number of red roses is equal to four times the number of pink roses increased by ten.

a) How many red roses are there?

b) How many pink roses are there?

20. In the dog park today there are twice as many shelties as golden retrievers. Three times the number of golden retrievers added to two times the number of shelties is 21.

a) How many shelties are there?

b) How many golden retrievers are there?

Chapter 1 Review

***Principles of Mathematics 10*, pages 48–49**

1. Translate each sentence into an algebraic expression.

a) three more than five times a number

b) five less than one third of a value

c) one number increased by four times another number

d) a value decreased by the fraction three quarters

2. Translate each sentence into an equation. Tell how you are assigning the variables in each.

a) Three times a number, increased by four, is one half the same number, decreased by one.

b) Hannah's age increased by five is twice Jordan's age, decreased by seven.

c) Zoe has nickels and quarters that total $1.65 in her piggy bank.

3. Use Technology

a) Use a graphing calculator or *The Geometer's Sketchpad*® to find the point of intersection of the lines $y = 4x + 1$ and $y = -\frac{1}{4}x - 2$.

If necessary, round your answer to the nearest hundredth.

b) Use a graphing calculator or *The Geometer's Sketchpad*® to find the point of intersection of the lines $y = 5x + 3$ and $y = -\frac{1}{5}x - 1$.

If necessary, round your answer to the nearest hundredth.

4. Solve each linear system using the method of substitution. Check each solution.

a) $x + y = 5$
$y = x - 3$

b) $x - y = 7$
$y = -x + 5$

c) $2x + y = 1$
$x - 3y = 4$

d) $3x - 4y = 19$
$x + 2y = 3$

e) $x + 3y = 15$
$4x - y = 8$

f) $5x - y = 4$
$-x + 3y = 2$

5. There are 28 fish in an aquarium. There are eight more goldfish than neon tetras in the aquarium. How many goldfish are in the aquarium? How many neon tetras are in the aquarium?

6. Madeleine would like to rent a digital camera this weekend. One company charges a flat rate of $75/day. A second company charges $35/day plus $5/h.

a) Write two equations to represent the information.

b) Solve the linear system to find the number of hours for which the cost of renting a digital camera is the same for both companies.

7. Which is *not* an equivalent equation for $6x + 3y = 15$?

A $2x + y = 5$

B $12x + 6y = 30$

C $9x + 6y = 18$

D $x + \frac{1}{2}y = \frac{5}{2}$

8. **a)** If $y = 4x - 5$ and $5y = kx - 25$ are equivalent linear equations, what is the value of k?

b) If $y = 6x + 2$ and $4y = 24x + k$ are equivalent linear equations, what is the value of k?

9. The total number of graphing calculators and scientific calculators in Charlotte's classroom is 14.

a) State how you will assign variables.

b) Write an equation to represent this situation. Then, write an equivalent linear equation.

10. Find the point of intersection of each pair of lines using the method of elimination. Check each solution.

a) $x + y = 5$
$3x - y = 11$

b) $5x + 3y = 9$
$2x - 3y = 12$

c) $x + 3y = 11$
$-x + 4y = -4$

d) $4x + 5y = 18$
$4x + y = 2$

11. Solve each linear system. Check each solution.

a) $5x + 4y = 20$
$4x - 3y = 16$

b) $5a + 3b = 4$
$2a - 4b = -1$

c) $0.3m + 0.2n = 0.8$
$0.5m - 0.3n = 0.7$

d) $2(x + 3) + 4(y - 2) = 10$
$3(x - 2) + 5(y + 2) = 8$

12. One type of fertilizer has 40% nitrogen and a second type of fertilizer has 20% nitrogen. How much of each type of fertilizer should be mixed to make 800 kg of fertilizer that has 25% nitrogen?

13. **Use Technology** The BLUE Cab Service charges \$6 plus \$2.00/km travelled. GREEN's Taxi Service charges \$4 plus \$2.50/km.

a) For what distance is the charge the same using either taxi company?

b) For what number of kilometres would you choose the BLUE Cab Service?

c) For what number of kilometres would you choose GREEN's Taxi Service?

14. There are 52 books in Jean's library. There are 28 more fiction books than non-fiction books.

a) How many fiction books are in Jean's library?

b) How many non-fiction books are in Jean's library?

15. Murray invests his summer earnings of \$2440. He invests part of the money at 8%/year, and the rest at 7.5%/year. After one year, these investments earn \$193 in simple interest. How much did he invest at each rate?

16. A speedboat took 3 h to travel a distance of 60 km up a river, against the current. The return trip took 2 h. Find the average speed of the boat in still water and the speed of the current.