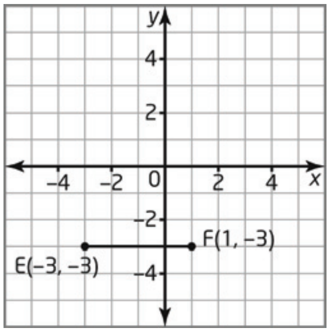
2.1 Midpoint of a Line Segment

***Principles of Mathematics 10*, pages 56–69**

A

1. Determine the coordinates of the midpoint of each line segment.

a)



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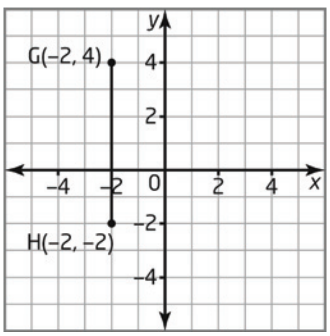

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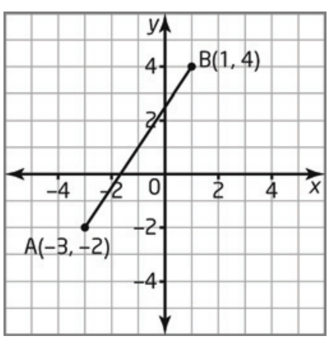
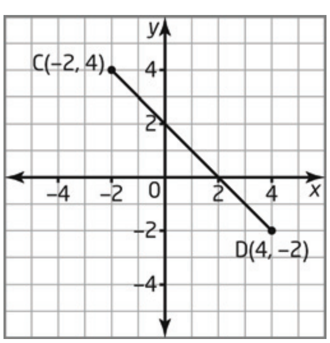


d)



2. Determine the coordinates of the midpoint of the line segment defined by each pair of endpoints.

a) A(2, 3) and B(8, 7)

b) C(–3, 4) and D(6, –3)

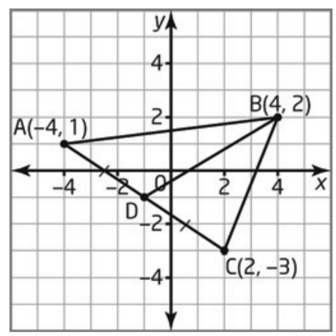
c) E(5.3, –3.4) and F(–3.5, 4.8)

d) $G\left(-\frac{3}{7}, -\frac{5}{9}\right)$ and $H\left(\frac{1}{7}, \frac{7}{9}\right)$

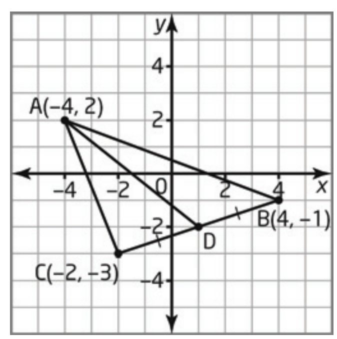
B

3. Find the slope of each median shown.

a)



b)



4. The endpoints of the diameter of a circle are A(–3, –7) and B(5, 3). Find the coordinates of the centre of this circle.

5. **Use Technology** Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to question 4. Describe the method you used.

6. Rachel is organizing a bicycle rally. On the grid of a roadmap, the starting point is at (14.5, 21.4) and the finish line is at (82.3, 78.8). Rachel has decided to set up a checkpoint at the halfway point. Find the coordinates of this checkpoint.

7. The vertices of ΔDEF are D(–4, 6), E(6, 2) and F(4, –8).
   a) Find an equation in slope y-intercept form for the median from vertex D.
   b) Find an equation in slope y-intercept form for the median from vertex E.

8. **Use Technology** Use geometry software to check your answer to question 7. Describe your method.

9. A line segment with one end at P(8, 3) has midpoint M(2, –3). Determine the coordinates of the other endpoint, Q. Explain your solution.

10. **a)** Determine an equation for the right bisector of the line segment with endpoints J(–5, 3) and K(9, –5).
    b) Determine an equation for the right bisector of the line segment with endpoints J(–4, –7) and K(3, –2).

11. **a)** Draw ΔABC with vertices A(–4, 6), B(–2, –3), and C(8, 5).
    b) Draw the median from vertex A. Then find an equation in slope y-intercept form for this median.
    c) Draw the right bisector of BC. Then find an equation in slope y-intercept form for this right bisector.

12. **a)** Write an expression for the coordinates of the midpoint of the line segment with endpoints A($5a$, $7b$) and B($11a$, $13b$). Explain your reasoning.
    b) Write an expression for the coordinates of the midpoint of the line segment with endpoints A($-6a$, $3b$) and B($9a$, $-6b$). Explain your reasoning.

C

13. The county planning team wants to build a water tower that is the same distance from two adjacent towns. On a local map, the towns have coordinates (1, 3) and (7, 11).
    a) Explain how you could use a right bisector to find possible locations for the water tower.
    b) Find an equation for this right bisector.

14. One endpoint of a diameter of a circle centred on the origin is (–4, –8). Find the coordinates of the other endpoint of this diameter.

15. A line segment has endpoints A(1, 3) and B(7, 18).
    a) Find the coordinates of the two points that divide the segment into three equal parts.
    b) Describe the method that you used in part a).

16. In ΔABC, D(3, 5) is the midpoint of side AB, E(5, 7) is the midpoint of BC, and F(4, 3) is the midpoint of AC.
    a) Find the coordinates of A, B, and C.
    b) Use the midpoint formula to check the coordinates you calculated in part a).

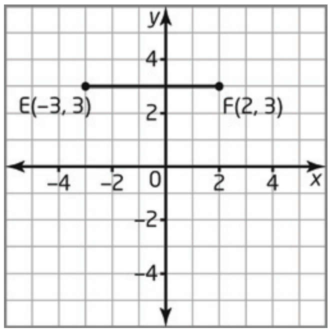
2.2 Length of a Line Segment

***Principles of Mathematics 10*, pages 70–79**

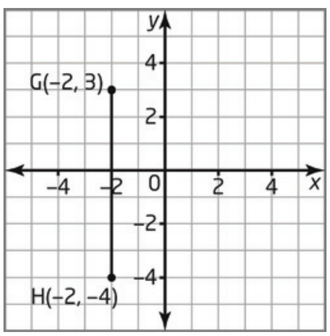
A

1. Estimate the length of each line segment from its graph. Then calculate its exact length.

a)



b)



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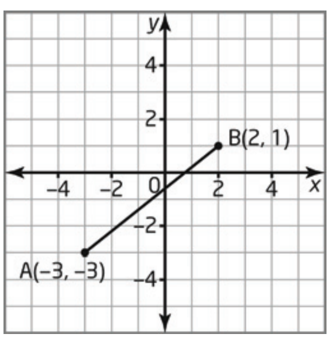
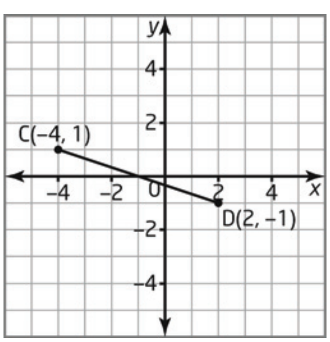


d)



2. Calculate the length of each line segment defined by each pair of endpoints. Round answers to the nearest tenth of a unit.

a) A(–3, 8) and B(2, –3)

b) C(–5, –6) and D(3, 2)

c) E(–6.4, 5.2) and F(4.3, –7.2)

d) $G\left(-\frac{2}{7}, -\frac{1}{4}\right)$ and $H\left(\frac{3}{7}, \frac{3}{4}\right)$

B

3. A circle has a radius with endpoints P(–2, 3) and Q(5, –7).

a) Find the length of the radius of this circle.

b) Find the length of the diameter of this circle.

4. On a city map, the coordinates of two libraries are (1, 4) and (7, 3). How far apart are the libraries if each unit on the map represents 1 km? Round your answer to one decimal place.

5. The vertices of ΔDEF are D(–3, 11), E(1, –1), and F(7, 9).

a) Determine the length of each side of this triangle.

b) Classify the triangle.

c) Determine the perimeter of the triangle. Round your answer to the nearest tenth of a unit.

6. a) Show that the triangle with vertices A(–4, 0), B(0, 3), and C(4, 0) is isosceles.

b) List the coordinates of the vertices of another isosceles triangle.

7. **a)** Determine the length of the median from vertex X in the triangle with vertices X(–5, 6), Y(–3, 7), and Z(5, –3).
   b) Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to part a).

8. **a)** Determine the area of the right triangle with vertices P(1, 1), Q(–5, –5), and R(7, –5).
   b) Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to part a).

9. A line segment has endpoints D(–5, –7) and E(9, 3).
   a) Find the coordinates of the midpoint of this line segment.
   b) Verify your answer to part a) by determining the distance from the midpoint to each of the endpoints.
   c) Use geometry software to verify your answer to part b).

10. The plans for a new house show a pipe running diagonally under the living room floor from a propane tank to a fireplace. The fireplace connection is at a point 5 m east and 3 m north of the southwest corner of the living room. The propane connection is at a point 13 m east and 18 m north of the southwest corner of the living room. The pipe costs $3.26 per metre, including taxes. How much should the builder budget for the pipe for the fireplace?

11. **a)** Draw a triangle with vertices A(–1, –2), B(7, 3), and C(4, 9).
    b) Determine the coordinates of the midpoints of AB and AC. Label these midpoints D and E.
    c) Show that DE is half the length of BC.
    d) Show that DE is parallel to BC.
    e) Show that the triangle formed by joining the midpoints of the sides of ΔABC is similar to ΔABC.

C

12. The vertices of ΔDEF are D(–3, 2), E(1, 4), and F(–1, –6).
    a) Determine the length of the median from D. Round your answer to the nearest tenth.
    b) Determine the length of the median from E. Round your answer to the nearest tenth.
    c) Determine the length of the median from F.

13. A line segment has endpoints A(–5, 2) and B(3, –6).
    a) Determine the coordinates of the midpoint of AB. Label this point M.
    b) Use the length formula to show that AM is one half of the length of AB.
    c) Use the length formula to show that MB is one half of the length of AB.

14. **Use Technology** Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to question 13 parts b) and c).

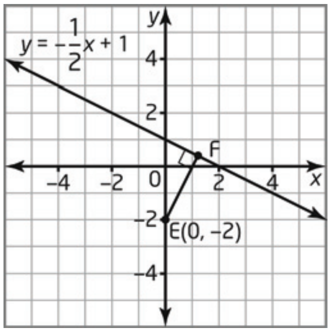
15. The point C(2, y) is four units from the point (6, 3).
    a) Find a possible value for y.
    b) Is this value the only solution? Explain.

2.3 Apply Slope, Midpoint, and Length Formulas

***Principles of Mathematics 10*, pages 80–91**

A

1. Find an equation for the line containing line segment EF.

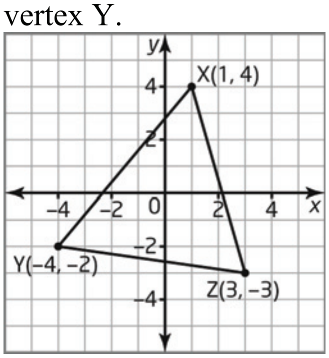


2. A triangle has vertices A(2, 5), B(–1, 3), and C(4, 2).

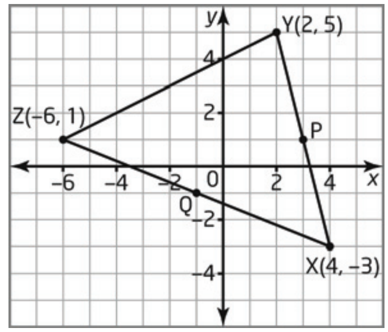
a) Draw ΔABC.

b) Use analytic geometry to verify that ∠A is a right angle.

3. Find the length of the median from vertex Y.



4. In ΔXYZ, P is the midpoint of XY and Q is the midpoint of XZ.



a) Find the coordinates of P and Q.

b) Show that PQ is parallel to YZ.

c) Show that PQ is half the length of YZ.

B

5. A triangle has vertices L(–1, –5), M(–5, 1), and N(–3, 5).

a) Draw ΔLMN.

b) Determine an equation for the median from vertex L.

c) Determine the length of the median from vertex L.

6. A quadrilateral has vertices A(–1, 1), B(4, 2), C(3, –2), and D(–2, –3).

a) Draw quadrilateral ABCD.

b) Determine whether quadrilateral ABCD is a parallelogram.

c) Determine the perimeter of the quadrilateral. Round your answer to the nearest tenth of a unit.

7. Use Technology Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to question 6. Describe the method you used.

8. The endpoints of the diameter of a circle are J(–5, 7) and K(3, –1).
   a) Determine the length of the diameter of the circle. Round your answer to the nearest tenth of a unit.
   b) Determine the coordinates of the centre of the circle.
   c) Determine the length of the radius of the circle. Round your answer to the nearest tenth of a unit.

9. Determine whether the triangle with vertices D(–5, 1), E(–3, –5), and C(1, –1) is isosceles.

10. Determine the shortest distance from the point (5, –1) to the line $y = 2x - 1$. Round your answer to the nearest tenth of a unit.

11. Determine the shortest distance from the point (3, –3) to the line through points G(–4, 1) and H(2, 4). Round your answer to the nearest tenth of a unit. Use a diagram to check your answer.

12. The points A(–3, –3), B(–5, 1), and C(3, 5) are three vertices of a rectangle ABCD.
   a) Find the coordinates of vertex D.
   b) Find the length of the diagonals AC and BD.
   c) Show that the diagonals AC and BD bisect each other.

13. **a)** Draw ΔPQR with vertices P(–5, –4), Q(–3, 2), and R(5, 1). Then, draw the altitude from vertex Q.
   b) Find an equation for the altitude from vertex Q.
   c) Determine the length of the altitude from vertex Q.

14. **Use Technology** Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answers to question 13. Describe the method you used.

C

15. Ron and Mary are hiking on the Rideau Trail in Frontenac Provincial Park. They have reached the point that has coordinates (2, 4) on their map of the trail.
   a) Find the shortest distance from this point to a straight section of a river in the park that joins points (–7, 6) and (–5, –2). Assume each unit on the map represents 1 km.
   b) Explain why the shortest route might not be the best route.

16. **Use Technology** Use geometry software to verify your answer to question 15. Describe the method you used.

17. The endpoint of the radius of a circle with centre C(4, 1) is D(1, 6). Determine
   a) the length of the radius of the circle
   b) the coordinates of the endpoint E of the diameter DE of the circle

18. Determine the shortest distance from the point (–2, 5) to the line $y = \frac{3}{4}x + 3$.

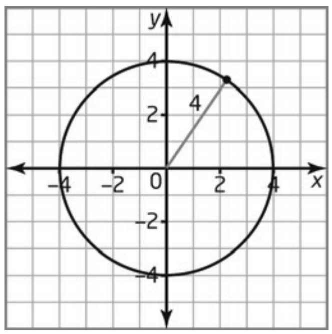
2.4 Equation for a Circle

***Principles of Mathematics 10*, pages 92–99**

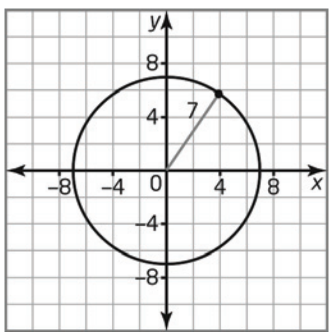
A

1. Determine an equation for each circle.

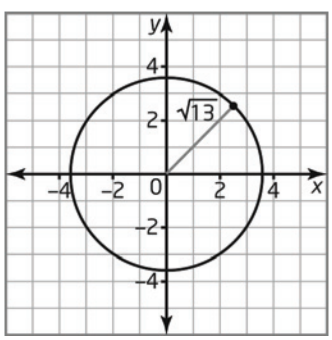
a)



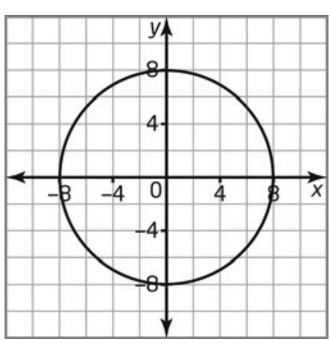
b)



c)



d)



2. For each equation, state the radius of the corresponding circle and give the coordinates of one point on the circle.

a) $x^2 + y^2 = 9$

b) $x^2 + y^2 = 81$

c) $x^2 + y^2 = 40$

d) $x^2 + y^2 = 1.21$

B

3. For each point, find an equation for the circle that is centred at the origin and passes through the point.

a) (4, 3)

b) (–2, 5)

c) (6, –4)

d) (–7, –1)

4. Determine whether each point is on, inside, or outside of the circle defined by $x^2 + y^2 = 18$.

a) (2, 4)

b) $(\sqrt{18}, 0)$

c) (–1, 2)

5. Determine an equation for the circle that has a diameter with endpoints X(–3, 4) and Y(3, –4).

6. The point P(a, –3) lies on the circle defined by $x^2 + y^2 = 25$.

a) Find the possible value(s) of a.

b) Use a graph to show that the point(s) corresponding to the possible value(s) of a are on the circle.

7. A gardener is building a circular flower garden for a client. With distances measured in metres, the flower garden is modelled by the equation $x^2 + y^2 = 25$.

   a) Find the length of fencing required for the flower garden. Round your answer to the nearest tenth of a metre.

   b) Find the area of the flower garden. Round your answer to the nearest tenth of a square metre.

8. **a)** Graph the circle defined by $x^2 + y^2 = 17$.

   b) Verify algebraically that the points D(–4, –1) and E(1, 4) are on the circle.

   c) Find an equation in the form $y = mx + b$ of the right bisector of the chord DE.

   d) Verify that the right bisector in part c) passes through the centre of the circle.

9. **Use Technology** Use *The Geometer's Sketchpad*® or Cabri® Jr. to verify your answer to question 8. Describe the method you used.

10. **a)** Graph the circle defined by $x^2 + y^2 = 100$.

   b) Verify algebraically that the point A(–6, –8) lies on this circle.

   c) Construct radius AO. Determine the slope of the line segment AO.

   d) Draw the line that is perpendicular to the line segment AO through the point A. Determine the slope of this line.

   e) Determine an equation for the tangent line in part d).

C

11. A pebble is dropped into the water and creates a circular ripple. The radius of this ripple increases at a rate of 5 cm/s.

   a) Find an equation for the circle 5 s after the pebble is dropped.

   b) A maple leaf is floating in the water 5 m east and 6 m south of the point where the pebble was dropped. How long does the ripple take to reach the maple leaf?

   c) Describe any assumptions you made for your answers to parts a) and b).

12. **a)** Describe the region defined by the inequality $x^2 + y^2 < 36$.

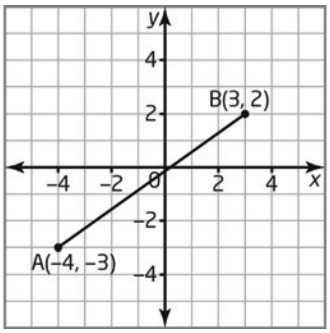
   b) Describe the region defined by the inequality $x^2 + y^2 > 49$.

Chapter 2 Review

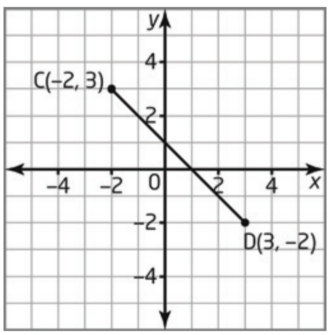
***Principles of Mathematics 10*, pages 100–103**

1. Find the midpoint of each line segment.

a)



b)



2. a) Determine the midpoint of the line segment with endpoints P(–5, –8) and Q(3, 2).

b) Determine the midpoint of the line segment with endpoints X(–4, 3) and Y(7, –8).

3. a) Draw the triangle with vertices D(–3, 4), E(1, –2), and F(5, 5).

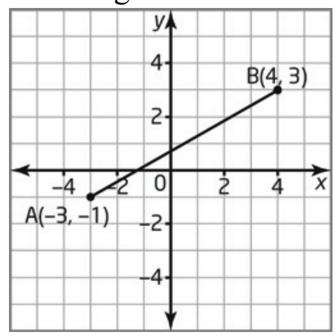
b) Draw the median from vertex D. Then, find an equation in the form $y = mx + b$ for this median.

c) Draw the right bisector of DF. Then find the equation in the form $y = mx + b$ for this right bisector.

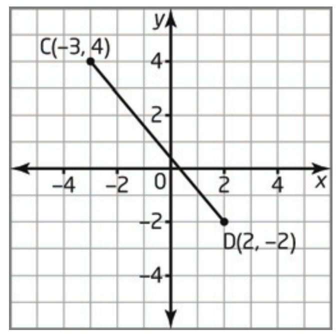
d) Draw the altitude from vertex F. Then, find an equation in the form $y = mx + b$ for this altitude.

4. Find the length of each line segment.

a)



b)

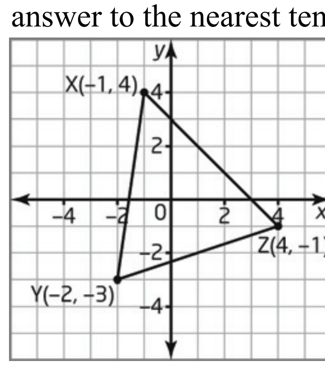


5. Determine the length of the line segment defined by each pair of points. Round answers to the nearest tenth of a unit.

a) A(–3, 5) and B(4, –2)

b) M(–2, 6) and Q(7, –3)

6. a) Determine the length of the median from vertex X of ΔXYZ. Round your answer to the nearest tenth of a unit.

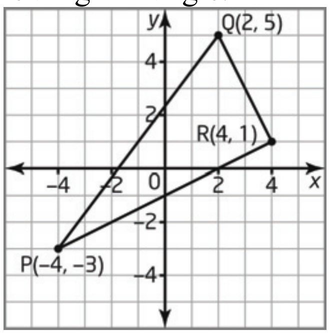


b) Show that ΔXYZ is isosceles.

c) Determine the perimeter of the triangle. Round your answer to the nearest tenth of a unit.

d) Describe how to use geometry software to answer part c).

7. a) Show algebraically that this triangle is a right triangle.



b) Find the midpoint of the hypotenuse.

c) Show that this midpoint is equidistant from each of the vertices.

8. A section of a ski jump on a ski hill is shown as a straight line running from C(20, 35) to D(70, 85) on a map grid.

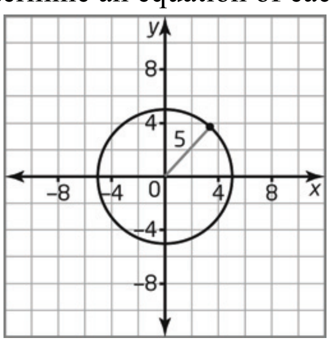
a) How long is the section of the ski jump if each unit on the map grid represents 1 m?

b) Is the point E(40, 55) on the ski jump? Explain your reasoning.

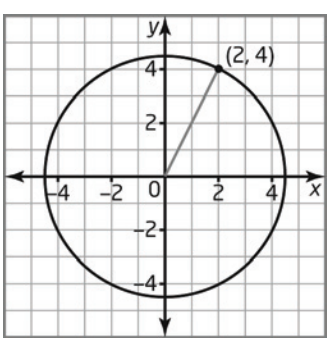
c) Is the point F(50, 35) on the ski jump? Explain your reasoning.

9. Determine an equation of each circle.

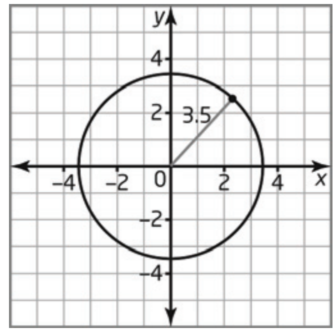
a)



b)



c)



10. Find the equation for the circle that is centred on the origin and

a) has a radius of 5.2

b) has a radius of $\sqrt{18}$

c) has a diameter of 20

d) passes through the point (–2, 3)

11. a) Show that the line segment joining M(–4, –1) and N(4, 1) is a chord of the circle defined by $x^2 + y^2 = 17$.

b) Determine an equation of the right bisector of the chord MN.

c) Show that the line in part b) passes through the centre of the circle.

12. a) Determine whether the point D(–2, 5) lies on the circle defined by $x^2 + y^2 = 29$.

b) Find an equation for the radius from the origin O to point D.

c) Find an equation for the line that passes through D and is perpendicular to OD.