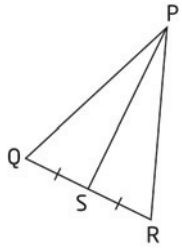
3.1 Investigate Properties of Triangles

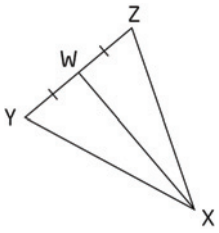
***Principles of Mathematics 10*, pages 110–116**

A

1. The area of ΔPQR is 16 square units. Find the area of ΔPQS.



2. The area of ΔXYW is 20 square units. Find the area of ΔXYZ.

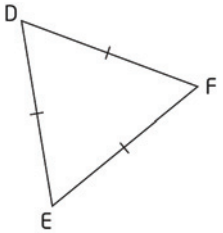


3. **a)** Draw any isosceles right triangle. Then, find and label the midpoint of the hypotenuse.

b) Construct the right bisector of the hypotenuse.

c) Use geometry software to show that the right bisector coincides with the angle bisector of the right angle.

4. Which of the following coincide in ΔDEF?



- the median from vertex D
- the median from vertex E
- the median from vertex F
- the bisector of ∠D
- the bisector of ∠E
- the bisector of ∠F
- the right bisector of side EF
- the right bisector of side DF
- the right bisector of side DE
- the altitude from vertex D
- the altitude from vertex E
- the altitude from vertex F

B

5. **a)** Draw any triangle. Then construct the median from each vertex to the opposite side.

b) Verify that the medians all meet at the same point.

c) State the name of the common point where the medians of each angle meet.

6. **Use Technology** Use geometry software to investigate the properties of the medians of a triangle. Outline your method and describe your findings.

7. **a)** Draw any triangle. Then construct the angle bisector of each vertex.

b) Verify that the angle bisectors of each angle all meet at the same point.

c) State the name of the common point where the angle bisectors of each angle meet.

8. **Use Technology** Use geometry software to investigate the properties of the angle bisectors of a triangle. Outline your method and describe your findings.

9. **a)** Draw any triangle. Then construct the right bisector of each side.

b) Verify that the right bisectors of each side all meet at the same point.

c) State the name of the common point where the right bisectors of each side meet.

10. **Use Technology** Use geometry software to investigate the properties of the right bisectors of a triangle. Outline your method and describe your findings.

11. **a)** Draw any triangle. Then construct the altitude from each vertex.

b) Verify that the altitudes all meet at the same point.

c) State the name of the common point where the altitudes meet.

12. **Use Technology** Use geometry software to investigate the properties of the altitudes of a triangle. Outline your method and describe your findings.

13. List at least six properties of isosceles right triangles. Explain how you know that every isosceles right triangle has each of these properties. Explain using a diagram.

14. Mary claims, "In an isosceles triangle, the angle bisectors and the right bisectors of the sides all meet at the same point." Do you agree with Mary's claim? Explain, using a diagram.

15. Garr says, "If a median of a triangle is also the altitude, the triangle must be equilateral." Is she correct? Justify your answer.

C

16. Marnie says, "If the midpoint of the hypotenuse of a right triangle is equidistant from each of the vertices of the triangle, the triangle must be an isosceles right triangle." Is she correct? Justify your answer.

17. Steven claims, "If the point of intersection of the right bisectors of the sides of a triangle (the circumcentre) lies on a side of the triangle, then the triangle is a right triangle." Is he correct? Justify your answer.

18. **Use Technology**

a) Construct ΔABC. Then construct the midpoint, M, of side AB and the midpoint, N, of side AC.

b) Show that the line segment MN is parallel to line segment BC.

c) Show that the length of line segment MN is half the length of line segment BC.

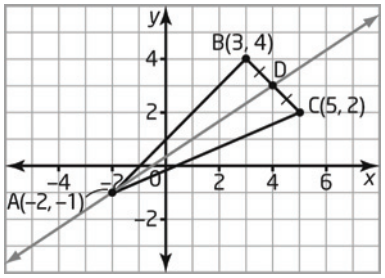
3.2 Verify Properties of Triangles

***Principles of Mathematics 10*, pages 117–127**

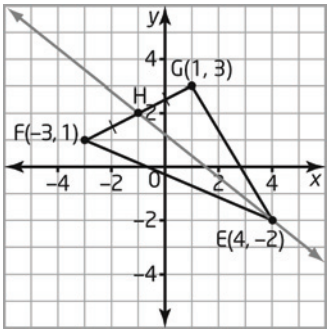
A

1. Determine an equation for the line shown with each triangle.

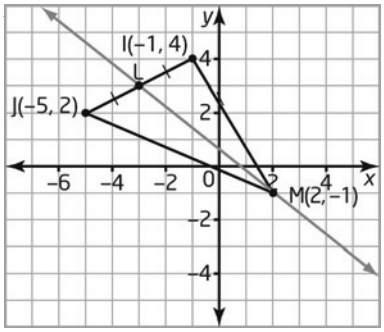
a)



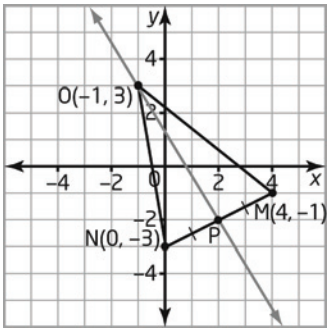
b)



c)

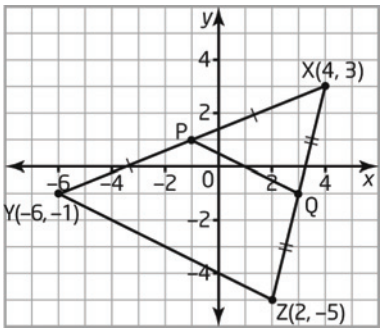


d)



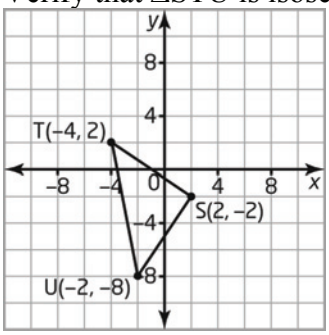
2. a) Verify that PQ and YZ are parallel.

b) Verify that PQ is half the length of YZ.



B

3. a) Verify that ΔSTU is isosceles.



b) Verify that the median from vertex S is also an altitude of the triangle.

4. Use Technology Use geometry software to verify your answers to question 3.

5. a) Draw the triangle with vertices A(–4, –7), B(0, 1), and C(2, –5).

b) Find the lengths of the sides of ΔABC.

c) Find the slopes of the sides of triangle ΔABC.

d) Classify ΔABC. Explain your reasoning.

6. Use Technology Use geometry software to verify your answers to question 5.

7. a) Draw a triangle with vertices P(–14, 6), Q(2, 0), and R(–10, –6).

b) Determine the coordinates of Y, the midpoint of PQ, and Z, the midpoint of PR.

c) Verify that RQ is parallel to YZ.

d) Verify that RQ is twice the length of YZ.

8. a) Draw the triangle with vertices F(5, 5), G(–8, –8), and H(2, –10). What type of triangle does ΔFGH appear to be?

b) Use analytic geometry to verify your classification of ΔFGH.

c) Determine the perimeter of the triangle. Round your answer to the nearest tenth of a unit.

d) Determine the area of the triangle.

9. Use Technology Use geometry software to verify your answers to question 8.

10. a) Draw the triangle with vertices P(5, 6), Q(–3, 4), and R(1, –2). Find the midpoint of each side and label these midpoints S, T, and U.

b) Find the lengths of each side of ΔPQR and ΔSTU.

c) Verify that ΔSTU is similar to ΔPQR. Find the ratio of the lengths of corresponding sides of these triangles.

11. Use Technology Use geometry software to verify your answers to question 10.

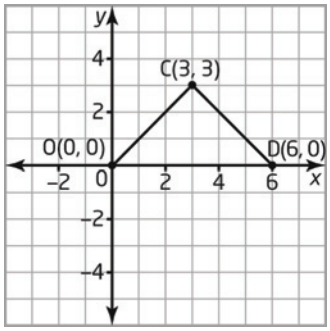
12. a) Verify the triangle with vertices A(–4, 0), B(3, –1), and C(–1, –4) is an isosceles right triangle.

b) Describe another method that you could use to answer part a).

c) Determine the area of the triangle.

C

13. a) Determine the equations of the right bisectors of the sides of ΔOCD.

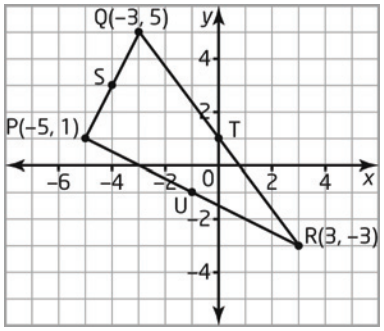


b) Determine the coordinates of the circumcentre, the point of intersection of the right bisectors of the sides.

c) What kind of triangle is ΔOCD? Justify your answer.

d) Describe the location of the circumcentre of this triangle.

14. a) Determine the equations of the medians from each vertex of ΔPQR.



b) Determine the coordinates of the centroid, the point of intersection of the medians from each vertex.

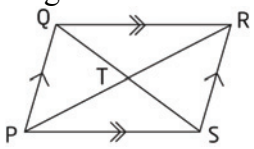
c) Show that the distance from the centroid to each vertex is twice as long as the distance from the centroid to the midpoint of the opposite side.

3.3 Investigate Properties of Quadrilaterals

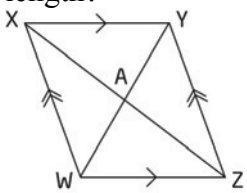
***Principles of Mathematics 10*, pages 128–136**

A

1. a) Which of the line segments inside the parallelogram PQRS are equal in length?

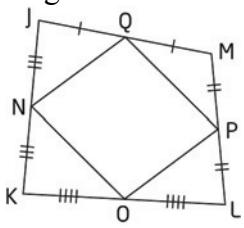


b) Which of the line segments inside parallelogram WXYZ are equal in length?



2. a) Which of the line segments inside quadrilateral JKLM are parallel?

b) Which of the line segments inside quadrilateral JKLM are equal in length?



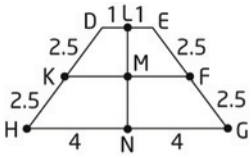
3. Fold a drawing of a parallelogram to investigate the properties of its diagonals. Describe your findings.

B

4. a) Which line segments are parallel in the figure?

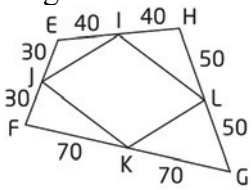
b) Find the length of LN

c) Find the length of KF.



5. a) Which of the line segments inside quadrilateral EFGH are parallel?

b) Which of the line segments inside quadrilateral EFGH are equal in length?



6. a) Show that the diagonals of a square bisect each other at right angles and are equal in length. Explain your reasoning.

b) Show that the diagonals of a rectangle are equal in length. Explain your reasoning.

c) Show that the diagonals of a rhombus bisect each other and are perpendicular. Explain your reasoning.

7. **a)** Describe how you can use geometry software to show that the diagonals of a kite are perpendicular to each other.

b) Describe how you can use geometry software to show that one diagonal of a kite divides the kite into two isosceles triangles, and that the other diagonal divides the kite into two congruent triangles.

8. Greg determines that the diagonals of a particular quadrilateral are perpendicular to each other and that the longer diagonal bisects the shorter diagonal. He concludes that the quadrilateral must be a kite. Is Greg correct? Explain your reasoning.

9. Which of the following are true for each of the following quadrilaterals: square, rectangle, parallelogram, rhombus, and kite?

- the diagonals are equal in length
- the diagonals are not equal in length
- the diagonals bisect each other
- the longer diagonal bisects the shorter diagonal
- the diagonals are perpendicular to each other
- the diagonals are not perpendicular to each other

10. **a)** Draw two line segments, PQ and RS, that bisect each other. Then, draw a quadrilateral that has PQ and RS as its diagonals. Classify the quadrilateral. Justify this classification.

b) Find the midpoints of PR, RQ, QS, and SP, labelling these points A, B, C, and D, respectively. Draw line segments joining the midpoints of adjacent sides. Classify the quadrilateral ABCD. Justify this classification.

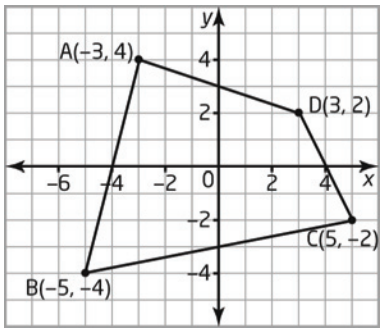
C

11. **a)** Predict the location of the balance point of a flat uniform object in the shape of a parallelogram. Explain your reasoning.

b) Describe how you could determine if your prediction is correct.

12. Use analytic geometry to verify that

a) the quadrilateral formed by joining the midpoints of adjacent sides of quadrilateral ABCD is a rhombus



b) the diagonals of the rhombus bisect each other

c) the diagonals of the rhombus are perpendicular to each other

13. Maurits Cornelis Escher (1898–1972) was schooled in graphic arts. He created woodcuts, prints, and lithographs of enchanted landscapes, spatial illusions, and impossible buildings. He is best known for combining mathematics with art in his fascinating tessellations and patterns.

a) Use a library or the Internet to research Escher and tessellations.

b) Create a tiling pattern or tessellation of your own using a parallelogram.

14. **a)** Use a library or the Internet to research Archimedean tiling patterns.

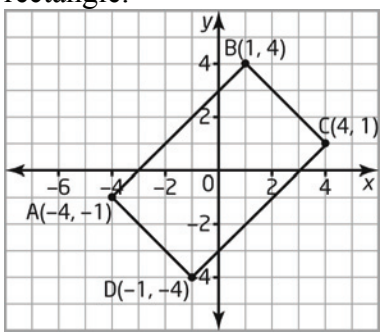
b) Create a tiling pattern or tessellation of your own using equilateral triangles and squares.

3.4 Verify Properties of Quadrilaterals

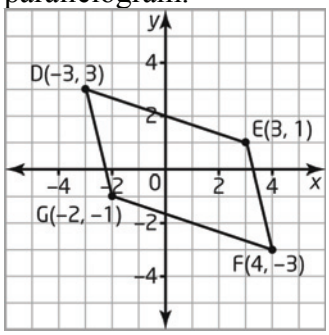
***Principles of Mathematics 10*, pages 137–144**

A

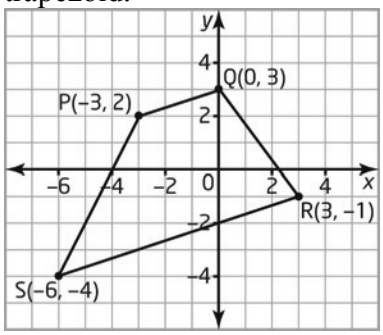
1. Verify that quadrilateral ABCD is a rectangle.



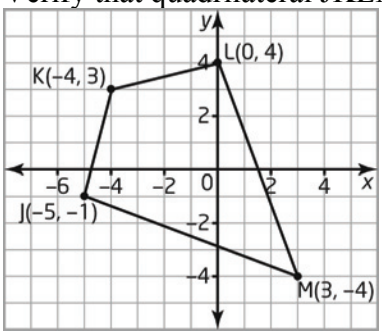
2. Verify that quadrilateral DEFG is a parallelogram.



3. Verify that quadrilateral PQRS is a trapezoid.



4. Verify that quadrilateral JKLM is a kite.

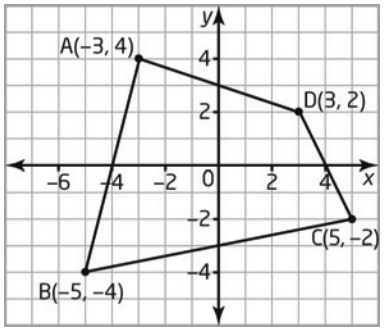


B

5. Consider quadrilateral ABCD.

a) Find the midpoints of AB, BC, CD, and DA, labelling these points as J, K, L, and M, respectively.

b) Show that quadrilateral JKLM is a parallelogram.



6. **a)** Draw the quadrilateral with vertices X(–3, 2), Y(2, 5), Z(7, 2), and W(2, –1).

b) Verify that the quadrilateral XYZW is a rhombus.

c) Verify that the diagonals of XYZW bisect each other.

d) Verify that the diagonals of XYZW are perpendicular to each other.

7. **Use Technology** Use geometry software to answer question 6. Outline your method.

8. **a)** Draw a trapezoid with vertices E(–3, 3), F(2, 4), G(6, –1), and H(–4, –3).

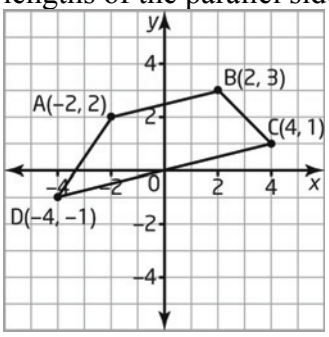
b) Verify that the line segment joining the midpoints, X and Y, of the non-parallel sides of the trapezoid is parallel to the other two sides.

9. **Use Technology** Use geometry software to answer question 8. Outline your method.

10. **a)** Find the midpoint, E, of side AD, and the midpoint, F, of side BC for the trapezoid in the diagram.

b) Verify that the line segment EF is parallel to sides AB and CD.

c) Verify that the length of line segment EF is equal to the mean of the lengths of the parallel sides.



11. **a)** Draw the quadrilateral with vertices S(–1, –5), T(–5, 1), U(3, 3), and V(5, –3).

b) Find the midpoint, J, of side ST; the midpoint, K, of side TU; the midpoint, L, of side UV; and the midpoint, M, of side VS. Join the midpoints of adjacent sides to form a new quadrilateral, JKLM.

c) Verify that the opposite sides of quadrilateral JKLM are parallel.

d) Verify that opposite sides of quadrilateral JKLM are equal in length.

e) What kind of quadrilateral is JKLM? Justify your answer.

12. **Use Technology** Use geometry software to answer question 11. Outline your method.

C

13. **a)** Draw the rhombus with vertices U(–6, 3), V(–2, 4), W(–3, 0), and X(–7, –1).

b) Verify that joining the midpoints, A, B, C, and D, of the adjacent sides of UVWX produces a rectangle.

14. **Use Technology** Use geometry software to answer question 13. Outline your method.

15. Let A(p, q), B(r, s), C(t, u), D(v, w) represent the vertices of a quadrilateral.

a) Determine the coordinates of the midpoints, D, E, F, and G, of sides AB, BC, CD, and DA, respectively.

b) Verify that DEFG is a parallelogram.

16. Use a rectangle to estimate the location of the geographical centre of Ontario.

a) Trace the outline of the map of Ontario on a sheet of paper. Draw a rectangle on your outline to approximate the shape of Ontario as closely as you can.

b) Describe how to find the centre of this area.

c) Find the centre you described in part b). Then, find the city or town closest to this centre. Compare your estimate of the geographical centre of Ontario with those made by your classmates.

17. **a)** Use a library or the Internet to investigate the golden rectangle in art and architecture.

b) Use a library or the Internet to investigate the golden section in art, architecture, poetry, and music.

3.5 Properties of Circles

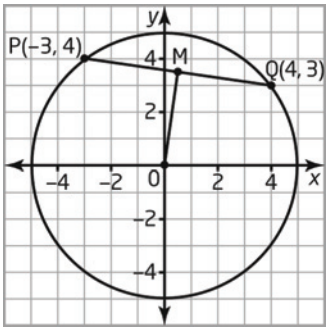
***Principles of Mathematics 10*, pages 145–151**

A

1. a) Find the coordinates of the midpoint, M, of PQ.

b) Find the slope of the chord PQ.

c) Verify that OM is perpendicular to PQ.



2. **a)** Verify that the points D(1, 1), E(2, –6), and F(–6, –6) are equidistant from the point C(–2, –3).

b) Draw the circle that passes through the points D, E, and F.

3. a) Verify that the points P(2, 7), Q(–4, 1), and R(2, –3) lie on a circle with its centre at S(1, 2).

b) Determine the length of the radius of the circle.

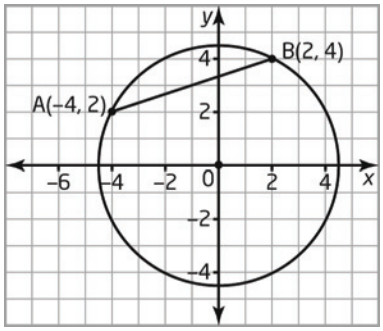
c) Plot points A, B, and C on grid paper, and draw the circle that passes through the points. Use your drawing to check your answers to parts a) and b).

4. Find the centre of the circle that passes through the points D(–5, 4), E(–3, 8), and F(1, 6).

5. Use Technology Use geometry software to answer question 4. Outline your method.

B

6. a) Verify that the centre of this circle lies on the right bisector of the chord AB.



b) State the radius of the circle.

7. a) Explain how you know that the origin is the centre of the circle represented by $x^2 + y^2 = 41$.

b) Verify that the points C(–4, 5) and D(5, –4) lie on the circle.

c) Verify that the line through the origin and the midpoint of the chord CD is perpendicular to the chord.

8. Find the centre and the radius of the circle that passes through the points J(–7, 1), K(–5, 5), and L(–1, 3).

9. A woodworker needs to place a marker in the centre of a circular piece of wood. Describe how the woodworker could mark the correct location for this marker.

10. Three cousins live in Ottawa, Kingston, and Toronto. They are planning to go skiing together and want to find a ski hill that is approximately the same distance from each of their homes. Describe how the cousins could fold a map to help them find a suitable ski hill.

11. You have 9.0 m of fencing to put around a hot tub. Find the maximum area you can enclose if the hot tub is circular.

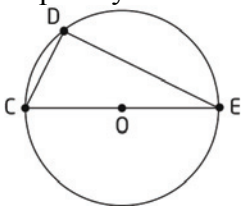
12. On a city map, the coordinates of three libraries are P(3, 12), Q(11, 16), and R(15, 8). The city council is planning to build a new school that is the same distance from all three libraries.

a) Determine the coordinates for the school.

b) What is the advantage of a location at the centre of the smallest enclosing circle?

c) What other factors might prevent the centre from being the best location for the school?

13. For the circle with diameter CE, a point on the circumference of the circle is D. Show that $\angle CDE$ is a right angle. Explain your reasoning.



14. Use Technology Use geometry software to answer question 13. Outline your method.

15. a) Show that A(–3, –4) and B(3, 4) are endpoints of a diameter of the circle $x^2 + y^2 = 25$.

b) State the coordinates of another point C, with integer coordinates, on the circle $x^2 + y^2 = 25$.

c) Show that ΔABC is a right triangle.

C

16. a) Verify that the points P(–4, 7) and Q(–7, 4) lie on the circle with equation $x^2 + y^2 = 65$.

b) Verify that the right bisector of the chord PQ passes through the centre of the circle.

17. a) Draw any circle with centre O.

b) Draw a diameter of the circle and label its endpoints C and D.

c) Let E be any other point on the circumference of the circle.

d) Use analytic geometry to show that EO is a median of ΔCED.

18. Use Technology Use geometry software to answer question 17. Outline your method.

19. a) Draw any circle with centre O.

b) Draw any chord on the circle. Label this chord AB.

c) Find the midpoint, M, of the chord AB.

d) Compare ΔOAM and ΔOBM.

20. Use Technology Use geometry software to answer question 19. Outline your method.

Chapter 3 Review

***Principles of Mathematics 10*, pages 152–153**

1. a) Define the right bisector of a side of a triangle.

b) List tree additional properties of the right bisectors of the sides of a triangle.

c) Outline how you could use geometry software to show that the right bisectors of the sides of all triangles have these properties.

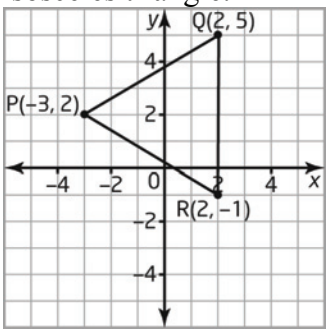
2. a) Show that two of the medians of an isosceles triangle are equal in length.

b) Show that the median from the vertex between the equal sides in an isosceles triangle is also an altitude of the isosceles triangle.

3. a) Which type of triangle has at least one median that is also an altitude?

b) Which type of triangle has more than one median that is also an altitude and an angle bisector of the vertex?

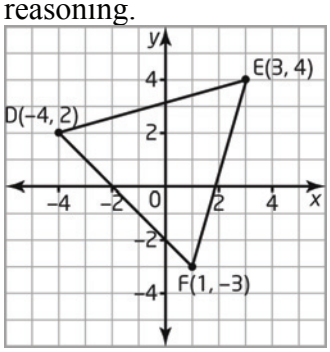
4. a) Verify that the triangle with vertices P(–3, 2), Q(2, 5), and R(2, –1) is an isosceles triangle.



b) Find the midpoint, M, of the side PR and the midpoint, N, of the side PQ.

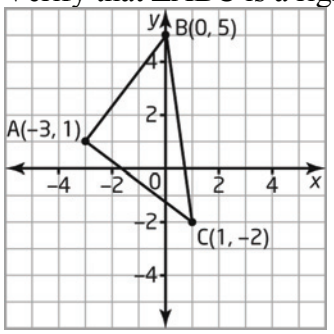
c) Verify that the lengths of the medians of the two equal sides of the isosceles triangle are equal.

5. a) Classify ΔDEF. Explain your reasoning.



b) Verify that the median from vertex E bisects ∠DEF in ΔDEF.

6. a) Verify that ΔABC is a right triangle.



b) Describe another method that you could use to verify that ΔABC is a right triangle.

c) Verify that the midpoint, M, of the hypotenuse of ΔABC is equidistant from all three vertices.

d) Determine the area of ΔABC.

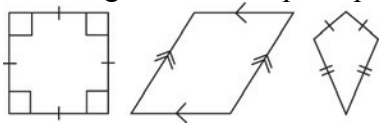
7. List two properties of the diagonals of each of these geometric shapes.

a) rectangle

b) rhombus

8. **a)** Identify each of the quadrilaterals.

b) List two properties of the diagonals of each geometric shape in part a).

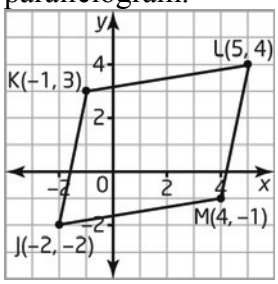


9. **a)** Draw any kite ABCD.

b) Identify the longer diagonal in kite ABCD.

c) Show that the longer diagonal of kite ABCD bisects the area of the kite. Explain your reasoning.

10. Verify that quadrilateral JKLM is a parallelogram.



11. **a)** Draw any trapezoid QRST.

b) Find the midpoints, M and N, of the non-parallel sides of the trapezoid.

c) Show that the line segment joining M and N is parallel to the two parallel sides in the trapezoid.

12. **Use Technology** Use geometry software to answer question 11. Outline your method.

13. **a)** Draw the quadrilateral with vertices W(–2, 2), X(1, 3), Y(5, 0), and Z(–4, –3).

b) Classify the quadrilateral WXYZ. Justify this classification.

c) Verify that the line segment joining the midpoints of the non-parallel sides of WXYZ is parallel to the parallel sides of the quadrilateral.

14. **a)** Verify that A(–24, 7) and B(24, –7) are endpoints of a diameter of the circle $x^2 + y^2 = 625$.

b) State the coordinates of another point with integer coordinates, C, on the circle $x^2 + y^2 = 625$.

c) Verify that ΔABC is a right triangle.

15. **a)** Verify that the points J(–6, 3) and K(3, –6) lie on the circle with equation $x^2 + y^2 = 45$.

b) Verify that the right bisector of the chord JK passes through the centre of the circle.

16. Three friends live in different areas of the same city. They are planning to go to a restaurant for dinner to celebrate a birthday.

a) Describe how the friends could use a city map to find a suitable restaurant.

b) What factors might prevent that restaurant from being the one they select for their dinner?