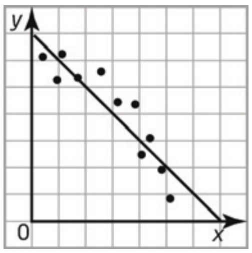
4.1 Investigate Non-Linear Relations

***Principles of Mathematics 10*, pages 164–167**

A

1. State whether each line or curve of best fit is a good model for the data. Justify your answer.

a)



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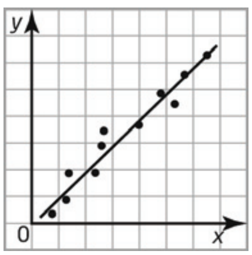

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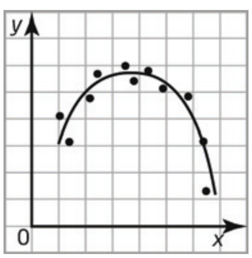

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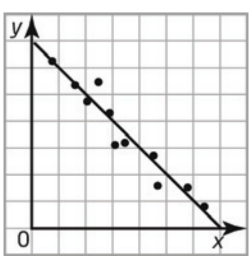
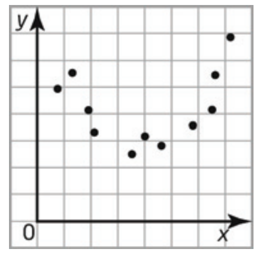


B

2. Which scatter plot(s) could be modelled using a curve instead of a line of best fit? Explain.

a)



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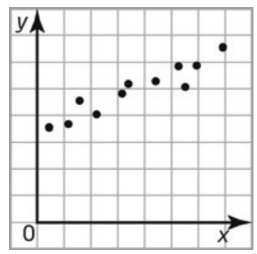

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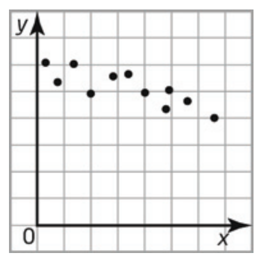
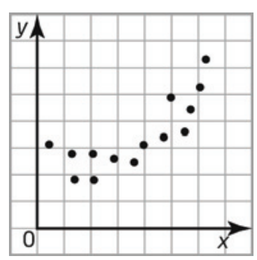
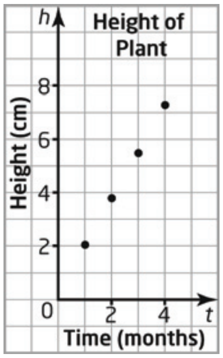


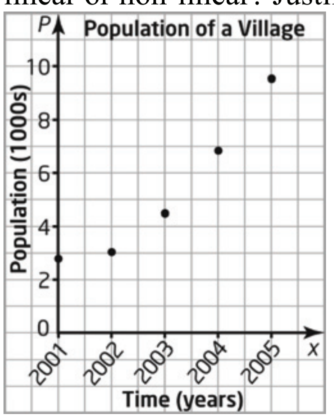
d)



3. The scatter plot shows the relationship between time, in one-month intervals, and the height of a plant. Is the relation linear or non-linear? Justify your answer.



4. The scatter plot shows the relationship between time, in one-year intervals, and the population of a village. Is the relation linear or non-linear? Justify your answer.



5. The table shows the distance that a ball travels when dropped from the top of a 4-m ladder.

Time (s)	Distance (m)
0.1	0.08
0.2	0.21
0.3	0.42
0.4	0.83
0.5	1.25
0.6	1.73
0.7	2.41
0.8	3.24

a) Make a scatter plot of the data.
b) Describe the relation.
c) Draw a curve of best fit.

C

6. The table shows the distance travelled by an SUV from 0 s to 14 s.

Time (s)	0	2	4	6	8	10	12	14
Distance (m)	0	8	26	53	92	145	190	237

a) Make a scatter plot of the data.
b) Describe the relation.
c) Draw a curve of best fit.
d) Use your model to predict the distance travelled by the SUV after 16 s.

7. The table shows the average fuel economy of a truck at a test track.

Speed (km/h)	Fuel Economy (L/100 km)
10	15.36
20	13.75
30	11.52
40	10.47
50	10.82
60	10.98
70	11.68
80	12.51
90	14.34
100	16.86
120	18.43

a) Make a scatter plot of the data.
b) Describe the relation.
c) Draw a curve of best fit.
d) Use your model to predict the fuel economy at 130 km/h.

4.2 Quadratic Relations

***Principles of Mathematics 10*, pages 168–173**

A

1. The table shows the path of a baseball, where x is the horizontal distance, in metres, and h is the height, in metres, above the ground.

x	h
0	2
1	11
2	18
3	23
4	26
5	27
6	26
7	23
8	18
9	11
10	2

a) Sketch a graph of the quadratic relation.

b) Describe the flight path of the baseball. Identify the axis of symmetry and the vertex.

c) What is the maximum height that the baseball reached?

d) Verify that $h = -x^2 + 10x + 2$ can be used to model the flight path of the baseball.

2. The entrance to a garden is an arch that can be approximated by the relation $y = -0.2x^2 + 3.2$, where y is the height, in metres, above the ground and x is the width, in metres, from the centre of the bridge.

a) Graph the quadratic relation.

b) Describe the shape of the arch.

c) How tall and how wide is the arch?

B

3. Use finite differences to determine whether each relation is linear, quadratic, or neither.

a)

x	y
0	1
1	3
2	5
3	7
4	9

b)

x	y
0	2
1	3
2	6
3	11
4	18

c)

x	y
1	4
3	12
5	28
7	52
9	84

d)

x	y
–1	–3
2	0
5	28
8	252
11	2044

4. The parabolic shape of a new bridge in Calgary can be approximated by the equation $h = -\frac{1}{25}x^2 + \frac{16}{5}x$, where x is the horizontal distance, in metres, from one end and h is the height, in metres, above the water.

   a) Graph the quadratic relation with or without technology.

   b) What is the height of the bridge 15 m horizontally from one end?

   c) How wide is the bridge at its base?

   d) What is the maximum height of the bridge? At what horizontal distance does it reach that height?

   e) Identify the axis of symmetry of the bridge.

5. **Use Technology** A ball is thrown upward at an initial velocity of 30 m/s, from a height of 2 m. The height, h, in metres, of the ball above the ground after t seconds can be found using the relation $h = -4.9t^2 + 30t + 2$.

   a) Graph this relation using a graphing calculator.

   b) Describe the relationship between time and height.

   c) Repeat parts a) and b) for a ball thrown upward on Mars, with a height defined by the relation $h = -1.8375t^2 + 30t + 2$.

   d) Repeat parts a) and b) for a ball thrown upward on Neptune, with a height defined by the relation $h = -7.007t^2 + 30t + 2$.

   e) Compare the results from the three locations.

C

6. A baseball is tossed into the air and follows the path $h = -2t^2 + 6t$, where t is the time, in seconds, and h is the height of the baseball, in metres.

   a) Find the maximum height of the baseball.

   b) At what time will the baseball reach its maximum height?

7. A soccer ball is kicked into the air and follows the path $h = -2t^2 + 12t$, where t is the time, in seconds, and h is the height of the soccer ball, in metres.

   a) Find the maximum height of the soccer ball.

   b) At what time will the soccer ball reach its maximum height?

   c) How long will the soccer ball be in the air?

   d) How long does it take the soccer ball to reach a height of 16 m?

8. The sum of the first n even natural numbers is a quadratic relation. Determine that relation and verify it for the first six even natural numbers.

4.3 Investigate Transformations of Quadratics

***Principles of Mathematics 10*, pages 174–179**

A

1. Sketch graphs of these four quadratic relations on the same set of axes.

a) $y = 3x^2$

b) $y = \frac{1}{3}x^2$

c) $y = -2x^2$

d) $y = -\frac{1}{2}x^2$

2. Sketch graphs of these four quadratic relations on the same set of axes.

a) $y = (x + 2)^2$

b) $y = (x - 3)^2$

c) $y = -(x + 5)^2$

d) $y = -(x - 4)^2$

3. Sketch graphs of these four quadratic relations on the same set of axes.

a) $y = x^2 + 3$

b) $y = x^2 - 5$

c) $y = -x^2 + 4$

d) $y = -x^2 - 3$

B

4. Sketch the graph of each parabola. Label at least three points on the parabola. Describe the transformation from the graph of $y = x^2$.

a) $y = 5x^2$

b) $y = \frac{3}{4}x^2$

c) $y = -\frac{2}{3}x^2$

5. Sketch the graph of each parabola. Label at least three points on the parabola. Describe the transformation from the graph of $y = x^2$.

a) $y = x^2 - 4$

b) $y = (x + 3)^2$

c) $y = -x^2 + 3$

6. Write an equation for the quadratic relation that results from each transformation.

a) The graph of $y = x^2$ is translated 5 units upward.

b) The graph of $y = x^2$ is translated 3 units downward.

c) The graph of $y = x^2$ is translated 6 units upward.

d) The graph of $y = x^2$ is translated 8 units downward.

7. Write an equation for the quadratic relation that results from each transformation.

a) The graph of $y = x^2$ is translated 4 units to the left.

b) The graph of $y = x^2$ is translated 7 units to the right.

c) The graph of $y = x^2$ is translated 6 units to the left.

d) The graph of $y = x^2$ is translated 2 units to the right.

8. Write an equation for the quadratic relation that results from each transformation.

a) The graph of $y = x^2$ is stretched vertically by a factor of 3.

b) The graph of $y = x^2$ is compressed vertically by a factor of $\frac{1}{4}$.

c) The graph of $y = x^2$ is stretched vertically by a factor of 5.

d) The graph of $y = x^2$ is compressed vertically by a factor of $\frac{1}{6}$.

e) The graph of $y = x^2$ is reflected in the x-axis and then stretched vertically by a factor of 6.

f) The graph of $y = x^2$ is reflected in the x-axis and then compressed vertically by a factor of $\frac{3}{5}$.

C

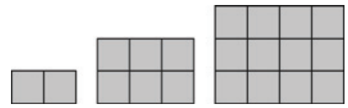
9. The area available for a swimming pool in a backyard is a square with side length 20 m. The square swimming pool is to be placed in the centre of the area. If the side length, in metres, of the swimming pool is x, then the area of the backyard remaining is given by the relation $A = -x^2 + 400$.

a) Graph the relation.

b) Find the intercepts. What do they represent?

c) For what values of x is the equation valid?

10. The first three diagrams in a pattern are shown. Each square has a side length of 1 unit.



a) Determine the number of squares in each of the next two diagrams in the pattern.

b) Make a table comparing the height and area for the five diagrams in your pattern. Use finite differences to determine whether the relation is linear, quadratic, or neither.

c) Determine an equation for the relationship between the height and the area.

d) Compare the graph of $y = x^2$ and the graph of the equation from part c).

4.4 Graph $y = a(x - h)^2 + k$

***Principles of Mathematics 10*, pages 180–188**

A

1. Copy and complete the table for each parabola. Replace the heading for the second column with the equation for the parabola.

Property	$y = a(x - h)^2 + k$
Vertex	
Axis of symmetry	
Stretch or compression factor relative to $y = x^2$	
Direction of opening	
Values x may take	
Values y may take	

a) $y = (x + 3)^2$

b) $y = (x - 4)^2$

c) $y = (x + 2)^2 + 5$

d) $y = (x + 5)^2 - 3$

e) $y = (x - 6)^2 + 7$

f) $y = (x - 1)^2 - 8$

g) $y = -(x + 8)^2 - 4$

h) $y = 3(x + 7)^2 - 2$

i) $y = -2(x + 3)^2 - 6$

j) $y = -\frac{1}{2}(x + 5)^2 - 3$

2. **Use Technology** Graph each parabola in question 1 using a graphing calculator.

B

3. Write an equation for the parabola with vertex (3, 5), opening upward, and with no vertical stretch or compression.

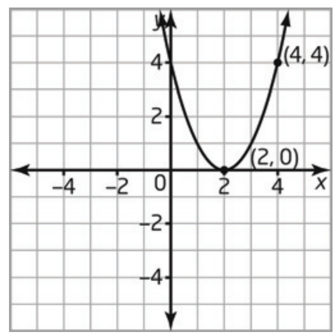
4. Write an equation for the parabola with vertex (6, –2), opening downward, and with no vertical stretch or compression.

5. Write an equation for the parabola with vertex (–4, 5), opening downward, and with a vertical stretch of factor 3.

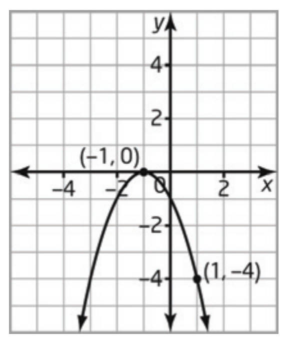
6. Write an equation for the parabola with vertex (–1, –7), opening upward, and with a vertical compression of factor 0.4.

7. Write an equation for each parabola.

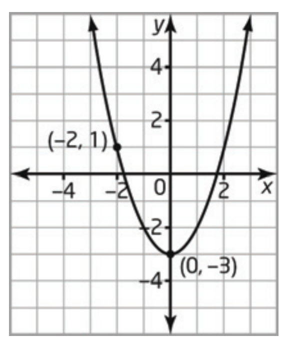
a)



b)

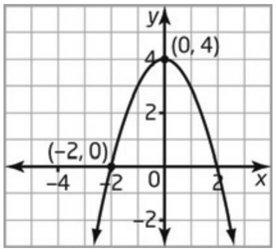


c)

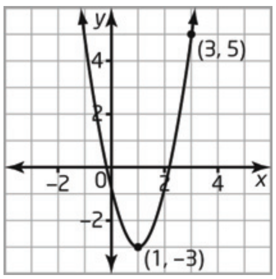


8. Write an equation for each parabola.

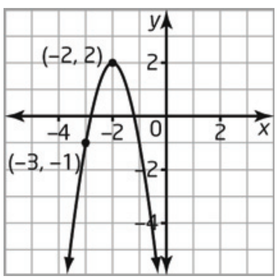
a)



b)



c)



9. a) Find an equation for the parabola with vertex (2, 6) that passes through the point (5, 3).

b) Find an equation for the parabola with vertex (–3, –4) that passes through the point (2, 6).

c) Find an equation for the parabola with vertex (–1, 3) and x-intercept 1.

d) Find an equation for the parabola with vertex (2, 5) and y-intercept –3.

e) Find an equation for the parabola with vertex (–6, –2) that passes through the point (–3, –11).

f) Find an equation for the parabola with vertex (6, 4) that passes through the point (8, 2).

C

10. The path of a football is modelled by the relation $h = -\frac{1}{4}(d - 12)^2 + 36$, where d is the horizontal distance, in metres, after it was kicked, and h is the height, in metres, above the ground.

a) Sketch the path of the football.

b) What is the maximum height of the football?

c) What is the horizontal distance when this occurs?

d) What is the height of the football at a horizontal distance of 10 m?

e) Find another horizontal distance where the height is the same as in part d).

11. A parabola has equation $y = 3(x + 2)^2 + 4$. Write an equation for the parabola after each set of transformations.

a) a reflection in the x-axis

b) a translation 6 units to the right

c) a reflection in the x-axis, followed by a translation of 3 units downward

d) a reflection in the y-axis

12. Find the equation for each of the following circles. Write your answer in the form $(x - h)^2 + (y - k)^2 = r^2$.

a) radius 4, centred at (6, 0)

b) radius 5, centred at (0, –2)

c) radius 3, centred at (–7, 3)

d) radius 6, centred at (–5, –4)

4.5 Quadratic Relations of the Form $y = a(x - r)(x - s)$

***Principles of Mathematics 10*, pages 189–193**

A

1. Sketch graphs of all three relations on the same set of axes. Label the x-intercepts, vertex, and axis of symmetry for each parabola. Then, describe the similarities and differences between the graphs.

a) $y = (x - 3)(x + 1)$

b) $y = 2(x - 3)(x + 1)$

c) $y = -2(x - 3)(x + 1)$

2. Sketch graphs of all three relations on the same set of axes. Label the x-intercepts, vertex, and axis of symmetry for each parabola. Then, describe the similarities and differences between the graphs.

a) $y = (x + 4)(x - 2)$

b) $y = \frac{1}{2}(x + 4)(x - 2)$

c) $y = \frac{1}{4}(x + 4)(x - 2)$

3. Sketch each parabola. Label the x-intercepts and vertex.

a) $y = (x + 2)(x - 4)$

b) $y = -(x + 3)(x - 5)$

c) $y = 3(x - 2)(x + 1)$

d) $y = -2(x + 2)(x - 3)$

B

4. Sketch each parabola. Label the x-intercepts and vertex.

a) $y = 2x(x - 4)$

b) $y = \left(x + \frac{1}{4}\right)\left(x - \frac{1}{2}\right)$

c) $y = -0.1(x - 2)(x + 4)$

5. Sketch each parabola. Label the x-intercepts and vertex.

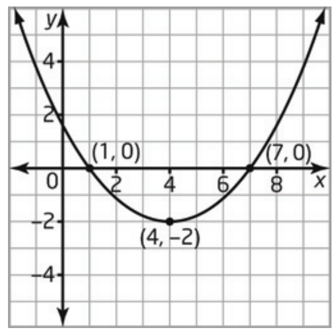
a) $y = (x + 2.5)(x - 2.5)$

b) $y = \frac{1}{2}(x - 3)(x + 5)$

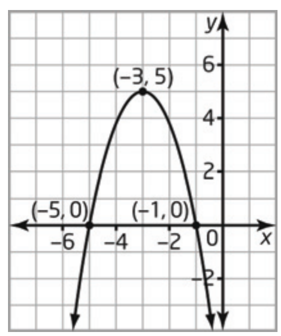
c) $y = -\frac{1}{3}(x + 0.2)(x - 0.4)$

6. Determine an equation in the form $y = a(x - r)(x - s)$ to represent each of the following parabolas.

a)



b)



7. A parabola has equation $y = (x + 3)^2$.

a) State the direction of opening.

b) Write the coordinates of the vertex of the parabola.

c) Write the x-intercept(s) of the parabola.

d) Rewrite the equation in the form $y = a(x - r)(x - s)$.

8. A parabola has equation $y = -(x - 4)^2$.

a) State the direction of opening.

b) Write the coordinates of the vertex of the parabola.

c) Write the x-intercept(s) of the parabola.

d) Rewrite the equation in the form $y = a(x - r)(x - s)$.

9. The predicted flight path of a toy rocket used in a mathematics project is defined by the relation $h = -3(d - 2)(d - 12)$, where d is the horizontal distance, in metres, from a wall, and h is the height, in metres, above the ground.

a) Sketch a graph of the path of the rocket.

b) How far from the wall is the rocket when it is launched?

c) How far from the wall is the rocket when it lands on the ground?

d) What is the maximum height of the rocket, and how far, horizontally, is it from the wall at that moment?

C

10. The revenue, R, from T-shirt sales at a monthly fundraising charity event is calculated as (number of T-shirts sold) × (price of T-shirt). The current price of a T-shirt is $16, and the charity event typically sells 50 T-shirts. For each $2 increase in the price of a T-shirt, five fewer T-shirts are sold. So, the revenue can be modelled using the equation $R = (50 - 5x)(16 + 2x)$, where x represents the number of price increases.

a) Rewrite the equation in the form $R = a(x - r)(x - s)$.

b) Sketch a graph of the relation.

c) What does the R-intercept represent? What do the x-intercepts represent?

d) What does a negative value of x represent?

e) What price maximizes the revenue?

f) What is the maximum revenue?

11. Use Technology Use a graphing calculator to graph each relation. Describe and justify how the shape of the graph relates to the number of factors.

a) $y = 2(x + 1)$

b) $y = 2(x + 1)(x + 2)$

c) $y = 2(x + 1)(x + 2)(x + 3)$

d) $y = 2(x + 1)(x + 2)(x + 3)(x + 4)$

4.6 Negative and Zero Exponents

***Principles of Mathematics 10*, pages 194–201**

A

1. Rewrite each power with a positive exponent.

a) 5^{-3} **b)** 3^{-4}

c) 10^{-5} **d)** 7^{-2}

e) $(-5)^{-1}$ **f)** $(-4)^{-2}$

2. Evaluate.

a) 3^{-2} **b)** 5^{-3}

c) 7^{0} **d)** 8^{-1}

e) 2^{-6} **f)** $(-4)^{-2}$

g) $(-6)^{-3}$ **h)** $-(-10)^{0}$

i) -5^{-2} **j)** -9^{0}

3. Evaluate.

a) $\left(\frac{1}{4}\right)^{-2}$ **b)** $\left(-\frac{1}{2}\right)^{-3}$

c) $\left(\frac{2}{3}\right)^{-1}$ **d)** $\left(\frac{3}{4}\right)^{-2}$

e) $\left(-\frac{5}{7}\right)^{-3}$ **f)** 0^{-4}

g) 2^{-5} **h)** $(-6)^{-2}$

i) $\left(-\frac{3}{5}\right)^{-2}$ **j)** -3^{-2}

B

4. Evaluate using pencil and paper. Check your results using a calculator.

a) $5^{-2}+5^{-2}$ **b)** $(5+5)^{-2}$

c) $3^{-2}+3^{0}$ **d)** $(8-5)^{0}$

e) $2^{-3}+3^{-2}$ **f)** $4-4^{-1}$

5. Iridium-192 is a radioactive element used for therapy in nuclear medicine. It decays to $\frac{1}{2}$, or 2^{-1}, of its original mass after 74 days. After 148 days, it decays to $\frac{1}{4}$, or 2^{-2}, of its original mass.

a) What fraction remains after 222 days?

b) What fraction remains after 296 days?

c) What fraction remains after 370 days?

d) Write each fraction as a power of 2 with a negative exponent.

6. Carbon-14 is a radioactive material that is used for the dating of materials. It decays to $\frac{1}{2}$, or 2^{-1}, of its original mass after every 5730 years. Determine the remaining mass of 0.8 kg of carbon-14 after

a) 11 460 years

b) 17 190 years

7. Caesium-137 is a radioactive element used in medical imaging. Caesium-137 decays to $\frac{1}{32}$ of its mass in 150 years.

a) Write the fraction $\frac{1}{32}$ as a power of 2.

b) What is the remaining mass of 4 mg of caesium-137 after 150 years?

8. Determine the value of x that makes each statement true.

a) $x^{-4} = \frac{1}{16}$ **b)** $x^{-2} = \frac{4}{9}$

c) $x^{-3} = \frac{1}{64}$ **d)** $x^{-1} = \frac{3}{4}$

e) $3^{x} = \frac{1}{81}$ **f)** $\left(\frac{2}{3}\right)^{x} = \frac{27}{8}$

9. The number of bacteria in a bacterial culture is 2000 at 10:00 a.m. and doubles every hour. This can be expressed as $N = 2000 \times 2^{t}$, where N represents the number of bacteria and t represents time, in hours.

a) Find the number of bacteria after 1 h.

b) Find the number of bacteria after 2 h.

c) Find the number of bacteria after 3 h.

d) Find the number of bacteria after 4 h.

e) What does $t = 0$ represent?

f) When were there 250 bacteria? Explain.

C

10. Akina won $50 000. She decides to invest $\frac{1}{2}$, or 2^{-1}, of her winnings in January, then invest half of the remaining amount in February, half again in March, and so on.

a) What fraction of her money remains after four months?

b) What fraction of her money remains after eight months?

c) Write each fraction as a power of 2 with a negative exponent.

d) What amount is remaining at the end of six months?

11. The number of bacteria in a bacterial culture is 500 at 10:00 a.m. and doubles every hour.

a) Write an equation relating time and the bacteria in the bacterial culture.

b) After how many hours will the number of bacteria in the same culture reach 2 048 000?

12. Salmonella is a type of bacteria that is usually found in poultry, eggs, unprocessed milk, meat, and water. It may also be carried by pets like turtles and birds. The approximate time for salmonella to double in number on raw meat, at 15° C, is 2.6 h.

a) How long will it take a bacterial culture of salmonella with an initial count of 800 bacteria to have 25 600 bacteria?

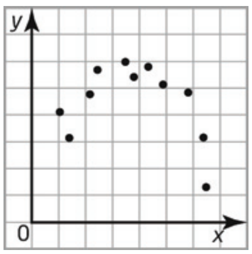
b) When were there 25 bacteria?

Chapter 4 Review

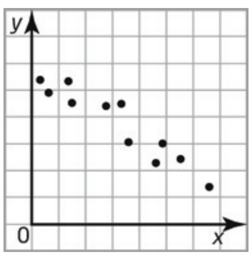
***Principles of Mathematics 10*, pages 202–203**

1. Which scatter plot(s) could be modelled using a curve instead of a line of best fit? Explain.

a)



b)



2. The table shows the operating revenue from dry cleaning and laundry services in Canada for the years from 2000 to 2004 in millions of dollars.

Year	Operating Revenue ($ millions)
2000	1676.0
2001	1770.0
2002	1885.3
2003	1923.3
2004	1930.5

Source: Statistics Canada, CANSIM, table (for fee) 359-0001 and Catalogue no. 63-018-X. Last modified: 2006-09-19.

a) Make a scatter plot of the data. Draw a curve of best fit.

b) Describe the relationship between the year and the operating revenue.

c) Use your curve of best fit to predict the operating revenue in 2005.

3. Use finite differences to determine whether each relation is linear, quadratic, or neither.

a)

x	y
0	–2
1	–1
2	2
3	7
4	14

b)

x	y
0	–2
1	1
2	4
3	7
4	10

c)

x	y
1	7
3	13
5	28
7	–3
9	–8

4. The flight of an aircraft from Toronto to Halifax can be modelled by the relation $h = -3.5t^2 + 210t$, where t is the time, in minutes, and h is the height, in metres.

a) Graph the relation.

b) How long does it take to fly from Toronto to Halifax?

c) What is the maximum height of the aircraft? At what time does the aircraft reach this height?

5. Sketch the graph of each parabola. Describe the transformation from the graph of $y = x^2$.

a) $y = 3x^2$

b) $y = \frac{2}{3}x^2$

c) $y = x^2 - 5$

d) $y = -\frac{1}{5}x^2$

e) $y = (x + 7)^2$

f) $y = -x^2 + 5$

6. Copy and complete the table for each parabola. Replace the heading for the second column with the equation for the parabola.

Property	$y = a(x - h)^2 + k$
Vertex	
Axis of symmetry	
Stretch or compression factor relative to $y = x^2$	
Direction of opening	
Values x may take	
Values y may take	

a) $y = 2(x + 2)^2$

b) $y = -(x - 5)^2$

c) $y = 3x^2 - 4$

d) $y = -5x^2 + 3$

e) $y = 4(x + 5)^2 + 2$

f) $y = -\frac{1}{2}(x - 4)^2 - 5$

7. a) Find an equation for the parabola with vertex (3, –1) that passes through the point (1, 7).

b) Find an equation for the parabola with vertex (–5, –5) that passes through the point (3, 27).

8. Sketch a graph of each quadratic. Label the x-intercepts and the vertex.

a) $y = 2(x + 2)(x - 4)$

b) $y = -\frac{1}{2}(x - 3)(x + 1)$

9. The path of a soccer ball can be modelled by the equation $h = -0.06d(d - 50)$, where h represents the height, in metres, of the soccer ball above the ground and d represents the horizontal distance, in metres, of the soccer ball from the player.

a) Sketch a graph of this relation.

b) At what horizontal distance does the soccer ball land?

c) At what horizontal distance does the soccer ball reach its maximum height? What is its maximum height?

10. Evaluate.

a) 4^{-2}

b) 3^{-5}

c) 5^0

d) $(-3)^{-1}$

e) $\left(\frac{3}{4}\right)^{-3}$

f) $(-7)^0$

11. Andy has $10 000 to invest. He decides to invest $\frac{1}{2}$, or 2^{-1}, of his money in May, then invest half of the remaining amount in June, half again in July, and so on.

a) What fraction of his money remains after five months? after ten months?

b) What amount is remaining at the end of four months?