6.1 Maxima and Minima

***Principles of Mathematics 10*, pages 264–273**

A

1. Use algebra tiles to rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = x^2 + 8x + 3$

b) $y = x^2 + 10x + 7$

2. Determine the value of c that makes each expression a perfect square.

a) $x^2 + 8x + c$

b) $x^2 + 6x + c$

c) $x^2 - 14x + c$

d) $x^2 - 16x + c$

e) $x^2 + 2x + c$

f) $x^2 - 130x + c$

3. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = x^2 + 8x + 4$

b) $y = x^2 + 4x + 5$

c) $y = x^2 + 6x - 3$

d) $y = x^2 - 2x - 8$

e) $y = x^2 - 10x - 7$

f) $y = x^2 - 4x - 10$

B

4. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

a) $y = -x^2 + 8x - 30$

b) $y = -x^2 + 10x + 20$

c) $y = -x^2 - 4x + 10$

d) $y = -x^2 - 2x - 13$

5. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square.

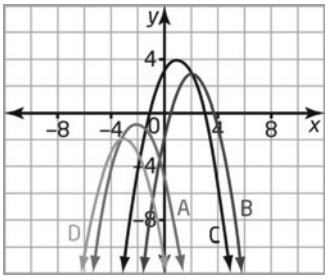
a) $y = 2x^2 + 16x + 3$

b) $y = 3x^2 - 12x - 5$

c) $y = -2x^2 + 8x + 7$

d) $y = -4x^2 - 8x - 1$

6. Match each graph with the appropriate equation.



a) $y = -(x - 2)^2 + 3$

b) $y = -(x - 1)^2 + 4$

c) $y = -(x + 3)^2 - 2$

d) $y = -(x + 2)^2 - 1$

7. Find the vertex of each parabola. Sketch the graph, labelling the vertex, the axis of symmetry, and two other points.

a) $y = x^2 + 8x + 10$

b) $y = x^2 - 10x + 20$

c) $y = -x^2 + 4x - 2$

d) $y = -x^2 - 6x + 3$

8. Find the maximum or minimum point of each parabola by completing the square.

a) $y = x^2 + 4x - 3$

b) $y = x^2 - 12x + 6$

c) $y = -x^2 + 6x - 3$

d) $y = -x^2 - 8x + 5$

9. Find the maximum or minimum point of each parabola by completing the square.

a) $y = 2x^2 + 60x - 3$

b) $y = 3x^2 - 48x + 12$

c) $y = -2x^2 + 28x - 15$

d) $y = -4x^2 - 32x + 9$

10. Use Technology Use a graphing calculator to find the maximum or minimum point of each parabola, rounded to the nearest tenth.

a) $y = 9x^2 + 5x - 4$

b) $y = -11x^2 + 3x - 7$

c) $y = 23x^2 + 7x - 3$

d) $y = -3x^2 - 16x + 5$

e) $y = 0.3x^2 + 2.4x - 4.1$

f) $y = -1.2x^2 - 3.7x + 4.2$

g) $y = \frac{1}{3}x^2 + \frac{3}{5}x - \frac{1}{2}$

h) $y = -\frac{1}{4}x^2 + \frac{2}{9}x + \frac{1}{3}$

C

11. The path of a toy rocket is modelled by the equation $y = -x^2 + 6x + 2$, where x is the horizontal distance, in metres, travelled and y is the height, in metres, of the toy rocket above the ground. What is the maximum height of the toy rocket? At what horizontal distance does the maximum height occur?

12. The cost, in dollars, of operating an appliance per day is given by the formula $C = 2t^2 - 24t + 150$, where t is the time, in months, the appliance is running. What is the minimum cost of running the appliance?

13. Find the two missing values (b, c, and/or h) in each equation.

a) $x^2 + 6x + c = (x + h)^2$

b) $x^2 + bx + 49 = (x + h)^2$

c) $x^2 + bx + c = (x + 2)^2 + 3$

d) $x^2 + bx + c = (x - 5)^2 - 32$

14. Verify that the y-coordinate of the vertex of a parabola of the form $y = ax^2 + bx + c$ is $\frac{4ac - b^2}{4a}$.

6.2 Solve Quadratic Equations

***Principles of Mathematics 10*, pages 274–281**

A

1. Solve.

a) $(x + 3)(x + 4) = 0$

b) $(a + 2)(a - 5) = 0$

c) $(h - 6)(h + 1) = 0$

d) $(n - 7)(n - 8) = 0$

e) $p(p + 10) = 0$

f) $d(d - 15) = 0$

g) $(3e + 5)(2e - 7) = 0$

h) $(4t - 9)(6t + 1) = 0$

2. Solve and check.

a) $x^2 + 5x + 6 = 0$

b) $a^2 + 7a + 12 = 0$

c) $g^2 - 8g + 12 = 0$

d) $w^2 - 9w + 20 = 0$

e) $y^2 + 5y - 24 = 0$

f) $r^2 - 7r - 18 = 0$

g) $m^2 + 5m = 0$

h) $d^2 - 21d = 0$

B

3. Solve and check.

a) $4c^2 - 9 = 0$

b) $25u^2 - 16 = 0$

c) $9m^2 - 49 = 0$

d) $64x^2 - 81 = 0$

4. Solve.

a) $2x^2 + 11x + 12 = 0$

b) $3q^2 + 19q + 20 = 0$

c) $10k^2 - 19k + 6 = 0$

d) $6d^2 - 17d + 5 = 0$

e) $4v^2 + 17v - 15 = 0$

f) $12n^2 - 7n - 10 = 0$

5. Solve.

a) $9k^2 + 24k + 16 = 0$

b) $4z^2 - 20z + 25 = 0$

6. Solve.

a) $x^2 + 3x = -2$

b) $m^2 = -15m - 56$

c) $12g = -g^2 - 27$

d) $s^2 + 15 = 8s$

e) $h^2 - 20 = -8h$

f) $n^2 = 4n + 21$

7. Solve.

a) $6m^2 + 7m = 20$

b) $15p^2 = 8p - 1$

c) $6w^2 = 5w + 6$

d) $7b - 6 = -20b^2$

e) $3c^2 = -10c - 3$

f) $8u^2 + 2u = 21$

8. Solve

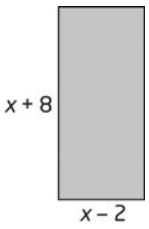
a) $-x^2 - 3x + 10 = 0$

b) $2a^2 + 2a - 24 = 0$

c) $3h^2 - 3h - 60 = 0$

d) $4t^2 - 32t + 48 = 0$

9. A rectangle has dimensions $x + 8$ and $x - 2$. Determine the value of x that gives an area of 24 cm^2.



10. Write a quadratic equation in factored form for each situation.

a) The roots of the equation are 3 and 7.

b) The roots of the equation are 4 and –2.

11. Write a quadratic equation in the form $ax^2 + bx + c = 0$ for each situation.

a) The roots of the equation are –5 and 6.

b) The roots of the equation are –8 and –9.

C

12. Write a quadratic equation in the form $ax^2 + bx + c = 0$, where a, b, and c are integers, for each situation.

a) The roots of the equation are $\frac{1}{2}$ and $\frac{1}{3}$.

b) The roots of the equation are $\frac{3}{5}$ and $-\frac{2}{3}$.

13. The hypotenuse of a right triangle measures 13 cm. One leg is 7 cm shorter than the other. What are the lengths of the legs?

14. As the price of cranberry juice drops at the local food mart, sales increase. On an average day, a 1.89-L bottle of cranberry juice costs \$3.95, and the food mart sells an average of 80 bottles. For each \$0.05 reduction in price of a 1.89-L bottle, sales increase by 10 bottles per day. The price and value of sales can be modelled as follows, where n is the number of \$0.05 price reductions.

Price, in dollars: $3.95 - 0.05n$
Number of bottles: $80 + 10n$

The total revenue is the product of the price and the number of bottles sold. How many price reductions will result in revenue of \$450?

15. Solve for y in terms of x.

a) $y^2 + 8xy + 7x^2 = 0$

b) $y^2 - 6xy + 8x^2 = 0$

c) $y^2 + 3xy - 4x^2 = 0$

d) $y^2 - 10xy - 39x^2 = 0$

6.3 Graph Quadratics Using the *x*-Intercepts

***Principles of Mathematics 10*, pages 282–291**

A

1. Find the *x*-intercepts.

a) $y = x^2 + 8x + 12$

b) $y = x^2 - 12x + 32$

c) $y = x^2 + 7x - 18$

d) $y = x^2 - 9x - 22$

e) $y = x^2 + 6x$

f) $y = x^2 - 11x$

2. Find the *x*-intercepts.

a) $y = 3x^2 + 8x + 5$

b) $y = 12x^2 - 11x + 2$

c) $y = 12x^2 + 9x - 3$

d) $y = 5x^2 - 3x - 8$

e) $y = 6x^2 - 17x + 12$

f) $y = 15x^2 - 11x + 2$

3. Find the *x*-intercepts and the vertex of each parabola. Then, sketch its graph.

a) $y = x^2 + 8x + 15$

b) $y = x^2 - 10x + 24$

c) $y = -x^2 - 2x + 8$

d) $y = -x^2 + 4x + 21$

B

4. Find the *x*-intercepts and the vertex of each parabola. Then, sketch its graph.

a) $y = x^2 + 6x$

b) $y = x^2 - 8x$

c) $y = x^2 - 4$

d) $y = -x^2 + 25$

5. Find the zeros and the vertex of each parabola. Then, sketch its graph. Check your results with a graphing calculator.

a) $y = 2x^2 + 7x + 3$

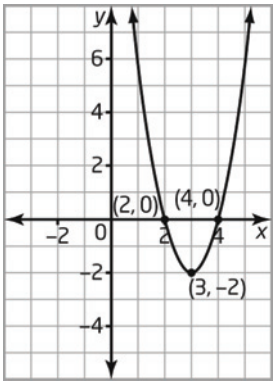
b) $y = 10x^2 + 23x + 12$

c) $y = -3x^2 + 11x + 4$

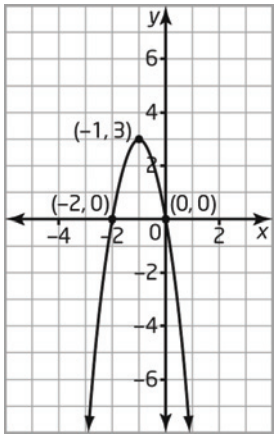
d) $y = -6x^2 - x + 15$

6. Write an equation in the form $y = ax^2 + bx + c$ to represent each parabola.

a)

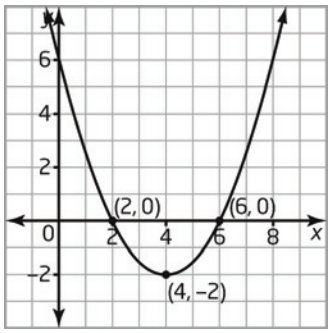


b)



7. Write an equation in the form $y = ax^2 + bx + c$ to represent each parabola.

a)



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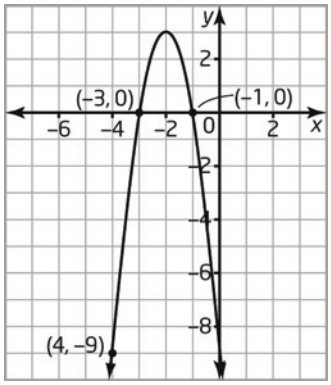
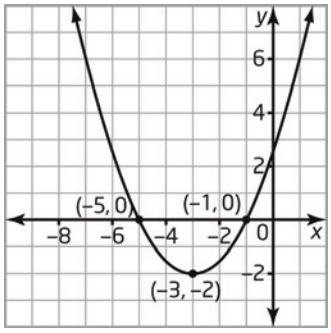
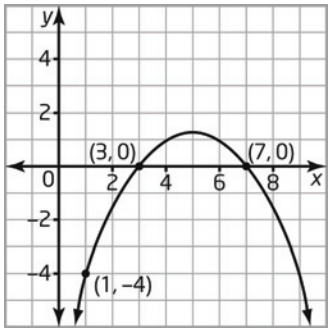


c)



d)



C

8. The parabolic cross section of an arch in front of a museum is modelled by the relation $h = –d^2 + 9$, where h is the height, in metres, above the ground and d is the horizontal distance, in metres, from the centre of the arch.

a) How wide and how tall is the arch?

b) Sketch a graph to represent the cross section.

c) For what values of d is the relation valid? Explain.

9. Find the value(s) of b so that each quadratic relation has only one zero. Then, sketch its graph.

a) $y = x^2 + bx + 16$

b) $y = 4x^2 + bx + 9$

c) $y = –x^2 + bx – 36$

d) $y = 25x^2 – 30x + b$

10. A parabola has equation $y = 9x^2 + x + h$. One x-intercept is –2. What is the value of h?

6.4 The Quadratic Formula

***Principles of Mathematics 10*, pages 292–303**

A

1. Use the quadratic formula to solve each equation. Express answers as exact roots.

a) $5x^2 - 7x + 2 = 0$

b) $9x^2 - 24x + 16 = 0$

c) $3x^2 + 9x - 2 = 0$

d) $2x^2 + 4x = -1$

e) $5x^2 + 3x = 4$

f) $25x^2 - 40x + 16 = 0$

2. **Use Technology** Use the quadratic formula to solve. Express your answers as exact roots and as approximate roots, rounded to the nearest hundredth. Verify graphically with technology.

a) $2x^2 + 9x + 3 = 0$

b) $7x^2 + 13x + 2 = 0$

c) $3x^2 - 8x - 1 = 0$

d) $12x^2 - 48x - 5 = 0$

e) $-7x^2 + 22x - 2 = 0$

f) $-6x^2 + 30x - 5 = 0$

3. Find the x-intercepts, the vertex, and the axis of symmetry of each quadratic relation. Then, sketch the graph of the parabola.

a) $y = 3x^2 - 14x - 5$

b) $y = 3x^2 - 5x - 8$

c) $y = x^2 + 8x + 16$

d) $y = 4x^2 - 12x + 9$

e) $y = x^2 - 5x + 16$

f) $y = -x^2 - 7x - 15$

B

4. For each quadratic relation, state the coordinates of the vertex and the direction of opening. Then, determine how many x-intercepts the relation has.

a) $y = (x - 2)^2 + 3$

b) $y = (x + 4)^2 - 2$

c) $y = -(x - 5)^2 - 4$

d) $y = -(x + 3)^2 + 5$

e) $y = (x + 6)^2$

f) $y = -(x - 8)^2$

5. The path of a football after it is kicked from a height of 0.3 m above the ground is given by the equation $h = -0.2d^2 + 2d + 0.3$, where h is the height, in metres, above the ground and d is the horizontal distance, in metres.

a) How far has the football travelled horizontally, to the nearest tenth of a metre, when it lands on the ground?

b) Find the horizontal distance when the football is at a height of 1.5 m above the ground. Round your answer to the nearest tenth of a metre.

c) What is the maximum height reached by the football? At what horizontal distance will it reach this height?

6. A baseball is thrown upward at an initial velocity of 9.2 m/s, from a height of 1.6 m above the ground. The height of the baseball, in metres, above the ground after t seconds is modelled by the equation $h = -4.9t^2 + 9.2t + 1.6$.

a) How long does it take the baseball to fall to the ground, rounded to the nearest tenth of a second?

b) Find the times when the baseball is at a height of 4.5 m above the ground. Round your answers to the nearest tenth of a second.

c) What is the maximum height of the baseball? At what time does it reach this height? Round your answers to the nearest tenth.

7. **Use Technology** Write an equation in the form $y = a(x - h)^2 + k$, satisfying each description. Then, write each relation in the form $y = ax^2 + bx + c$. Use graphing technology to verify that your equation satisfies the description.

a) The parabola opens upward and has two x-intercepts.

b) The parabola opens upward and has one x-intercept.

c) The parabola opens upward and has no x-intercepts.

d) The parabola opens downward and has two x-intercepts.

e) The parabola opens downward and has one x-intercept.

f) The parabola opens downward and has no x-intercepts.

C

8. The design of a new bridge can be modelled by the equation $h = -0.0055d^2 + 25.2$, where h is the height of the bridge, in metres, and d is the length of the bridge, in metres.

a) Determine the length of the bridge, to the nearest tenth of a metre.

b) Determine the maximum height of the bridge above the ground, to the nearest tenth of a metre.

c) Determine the height of the bridge 42 m from the centre of the bridge, to the nearest tenth of a metre.

9. Solve. Round your answers to the nearest hundredth, when necessary.

a) $5(x^2 + 3x) + 4(x + 2) = 0$

b) $x(4x + 5) - 2(x + 3) = 0$

c) $x(5x + 8) + 3 = x(2x - 4) - 3$

d) $(x + 5)^2 = -3(x + 2)$

e) $(3x - 4)^2 = (x + 5)(x + 4)$

f) $(2x - 1)^2 = -(x + 3)(x - 3)$

10. Find a quadratic equation with each pair of roots.

a) $\frac{3 \pm \sqrt{5}}{2}$

b) $\frac{-5 \pm \sqrt{137}}{4}$

6.5 Solve Problems Using Quadratic Equations

***Principles of Mathematics 10*, pages 304–315**

A

1. **a)** Create a quadratic model for the height of a ball thrown upward at 8 m/s from a height of 1.2 m.

b) How long will it take the ball to fall to the ground, rounded to the nearest hundredth of a second?

2. A toy rocket is launched upward at an initial velocity of 51 m/s, from a height of 1.3 m above the ground. The height of the toy rocket, in metres, after t seconds is modelled by the equation $h = -4.9t^2 + 51t + 1.3$.

a) How long does it take the rocket to fall to the ground, rounded to the nearest hundredth of a second?

b) Find the times when the toy rocket is at a height of 95.7 m above the ground. Round your answers to the nearest hundredth of a second.

c) What is the maximum height of the toy rocket? At what time does it reach this height? Round your answers to the nearest tenth.

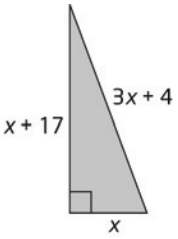
3. The length of a rectangle is 5 cm greater than its width. The area is 104 cm^2. Find the dimensions of the rectangle, to the nearest hundredth of a metre.

B

4. The product of two consecutive integers is 1482. What are the numbers?

5. The sum of the squares of two consecutive integers is 481. What are the integers?

6. The length of one leg of a right triangle is 17 cm more than that of the other leg. The length of the hypotenuse is 4 cm more than triple that of the shorter leg. Find the lengths of each of the three sides.



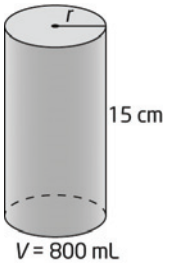
7. **Use Technology** Measurements from the flight path of a softball are recorded.

Horizontal Distance (m)	4.5	6.5	8.5	10.5	12.5
Height (m)	3.11	3.71	3.91	3.71	3.11

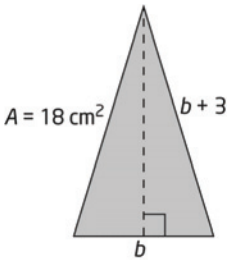
a) Use a graphing calculator or a spreadsheet to create a scatter plot of the data and add a curve of best fit.

b) Determine the equation of the quadratic relation.

8. A cylindrical can with height 15 cm has capacity of 800 mL. What is its radius, to the nearest tenth of a millimetre? [Remember that 1 mL = 1 cm^3.]



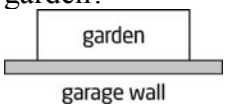
9. The area of a triangle is 18 cm^2, and the altitude is 3 cm greater than the base. Find the length of the base, to the nearest hundredth of a centimetre.



10. The sum of the squares of three consecutive integers is 434. Find the integers.

11. A rectangle has perimeter 28 cm. Its area is 42 cm^2. Determine the dimensions of the rectangle. Include a diagram in your solution. Round your answer to the nearest hundredth of a centimetre.

12. The garden is enclosed on three sides using 60 m of fencing. The remaining side is formed by the wall of a garage. What dimensions enclose 450 m^2 of garden?



C

13. The three sides of a right triangle are consecutive integers. What is the length of each side?

14. A mathematics experiment involves launching a small toy rocket. The following measurements are taken:

Initial height: 0.71 m
Initial vertical velocity: 38.25 m/s

a) Create a quadratic model for the height, in metres, of the toy rocket after a given number of seconds.

b) Determine the time when the rocket will hit the ground. Round your answer to the nearest tenth of a second.

c) Determine the maximum height reached by the toy rocket and the time that the rocket will reach the maximum height. Round your answers to the nearest tenth of a unit.

15. A rectangular floating dock measures 4 m by 5 m. A new dock is to be made by increasing each side length by the same amount. The area of the new dock is to be 42 m^2. Find the dimensions of the new floating dock.

16. Bryanna is selling T-shirts. Her regular price is $20 per T-shirt and she usually sells about 15 T-shirts. Anne finds that, for each reduction in price of $1, she can sell an additional 2 T-shirts.

a) Create an algebraic model to represent Bryanna's total sales revenue.

b) Determine the maximum revenue and the price at which this maximum revenue will occur.

Chapter 6 Review

***Principles of Mathematics 10*, pages 316–317**

1. Rewrite each relation in the form $y = a(x - h)^2 + k$ by completing the square. Use algebra tiles or a diagram to support your solution.

a) $y = x^2 + 6x - 3$

b) $y = x^2 + 4x + 5$

c) $y = x^2 + 10x + 18$

d) $y = x^2 + 12x + 26$

2. Find the vertex of each parabola. Sketch the graph, labelling the vertex, the axis of symmetry, and two other points.

a) $y = x^2 + 10x + 15$

b) $y = x^2 - 8x + 4$

c) $y = -x^2 + 6x - 4$

d) $y = -x^2 - 4x + 5$

3. **Use Technology** Use a graphing calculator to find the maximum or minimum point of each parabola rounded to the nearest tenth.

a) $y = 2.7x^2 + 1.2x + 1.5$

b) $y = -1.1x^2 - 0.8x + 1.3$

c) $y = 3.1x^2 + 5.2x - 2.3$

d) $y = -\frac{1}{2}x^2 - \frac{2}{3}x + \frac{3}{4}$

4. Solve by factoring.

a) $x^2 + 11x + 24 = 0$

b) $y^2 + 5y - 36 = 0$

c) $u^2 - 7u + 6 = 0$

d) $q^2 - 16q + 64 = 0$

e) $k^2 - 36 = 0$

f) $2m^2 - m - 21 = 0$

5. Solve.

a) $y^2 = 7y - 12$

b) $a^2 + 10a = -24$

c) $8m^2 = 3 - 2m$

d) $6p^2 + 20 = 23p$

e) $8r^2 = 2r + 21$

f) $2x^2 - x = 6$

6. The length of the hypotenuse of a right triangle is 3 cm more than twice that of the shorter leg. The length of the longer leg is 2 cm more than twice that of the shorter leg. Find the lengths of the three sides of the triangle.

7. Find the x-intercepts and the vertex of each parabola. Then, sketch its graph.

a) $y = x^2 + 12x + 32$

b) $y = x^2 - 8x + 12$

c) $y = -x^2 + 2x + 15$

d) $y = -x^2 + 8x - 7$

e) $y = x^2 + 6x$

f) $y = x^2 - 16$

8. The parabolic cross section of a bridge in a park is modelled by the relation $h = -d^2 + 4$, where h is the height, in metres, above the ground and d is the horizontal distance, in metres, from the centre of the arch.

a) How wide and how tall is the bridge?

b) Sketch a graph to represent the cross section.

c) For what values of d is the relation valid? Explain.

9. Find the value(s) of m so that each quadratic relation has only one zero. Then, sketch its graph.

a) $y = x^2 + mx + 49$

b) $y = 9x^2 + 24x + m$

c) $y = x^2 + mx + 144$

d) $y = mx^2 + 12x + 9$

10. Use the quadratic formula to solve each equation. Express your answers as exact roots.

a) $4x^2 - 5x - 6 = 0$

b) $4x^2 - 12x + 9 = 0$

c) $2x^2 + 9x + 3 = 0$

d) $9x^2 - 17x + 3 = 0$

e) $5x^2 + 7x - 3 = 0$

f) $16x^2 + 40x + 25 = 0$

11. A ball is thrown upward at an initial velocity of 8.4 m/s, from a height of 1.2 m above the ground. The height of the ball, in metres, above the ground after t seconds is modelled by the equation $h = -4.9t^2 + 8.4t + 1.2$.

a) How long will it take the ball to fall to the ground, rounded to the nearest tenth of a second?

b) What is the maximum height of the ball? At what time will it reach this height? Round your answers to the nearest tenth.

12. The sum of the squares of two consecutive integers is 685. What are the integers?

13. A rectangle has side lengths 4 cm and 8 cm. Each of the side lengths is to be increased by the same amount so that the rectangle has an area of 50 cm^2. Determine the lengths of the sides of the new rectangle. Round your answers to the nearest tenth of a centimetre.