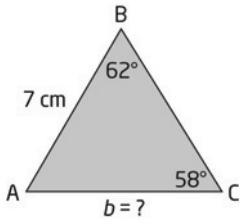
8.1 The Sine Law

***Principles of Mathematics 10*, pages 396–404**

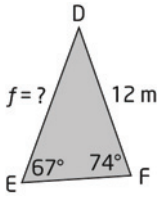
A

1. Find the length of the indicated side in each triangle, to the nearest unit.

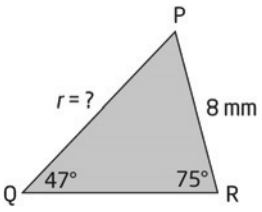
a)



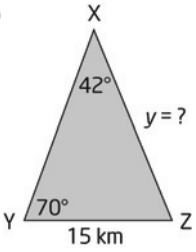
b)



c)

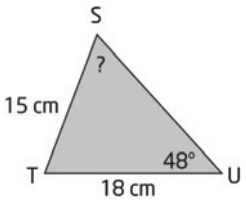


d)

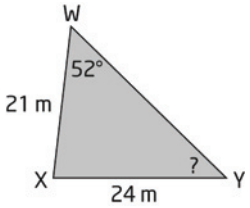


2. Find the measure of the indicated angle in each triangle, to the nearest degree.

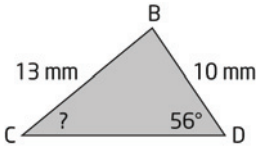
a)



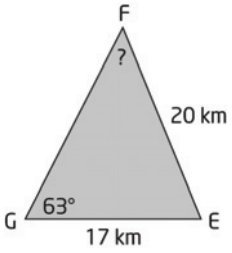
b)



c)



d)



B

3. Draw a diagram and label the given information. Then, find the measure of the indicated side in each triangle, to the nearest tenth of a unit.

a) In acute ΔDEF, ∠D = 64°, ∠E = 78°, and d = 15 cm. Find side e.

b) In acute ΔGHI, ∠G = 80°, ∠I = 37°, and g = 10 m. Find side i.

c) In acute ΔPQR, ∠P = 68°, ∠R = 51°, and r = 18.2 m. Find side p.

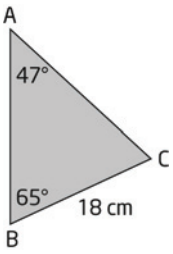
4. Draw a diagram and label the given information. Then, find the measure of the indicated angle in each triangle, to the nearest degree.

a) In acute ΔXYZ, ∠X = 64°, x = 10.4 cm, and y = 7.1 cm. Find ∠Y.

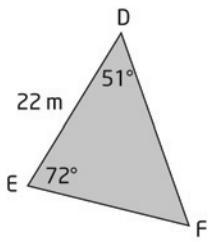
b) In acute ΔMNO, ∠M = 68°, m = 18.2 m, and n = 15.3 m. Find ∠N.

5. Solve each triangle. Round answers to the nearest unit, if necessary.

a)

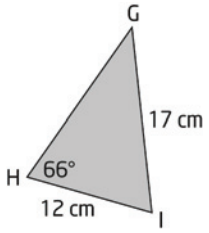


b)

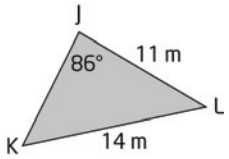


6. Solve each triangle. Round answers to the nearest unit, if necessary.

a)



b)



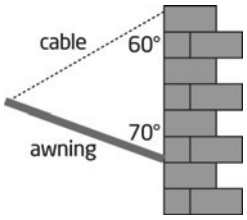
7. Draw a diagram and label the given information. Then, solve each triangle. Round answers to the nearest unit, if necessary.

a) In ΔPSV, $\angle S = 72°$, $\angle V = 25°$, and $p = 18$ cm.

b) In ΔKPR, $\angle K = 63°$, $\angle P = 71°$, and $r = 13$ m.

8. **Use Technology** Check your answers to question 7 using dynamic geometry software.

9. An awning over a window is supported by a cable attached to a building, as shown. The cable is attached to the building 2.1 m above the base of the awning.



a) Find the length of the cable, to the nearest tenth of a metre.

b) Find the length of the awning, to the nearest tenth of a metre.

C

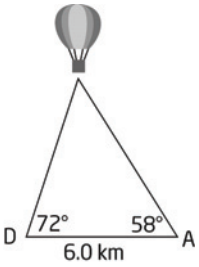
10. A hydro pole is supported by a 20-m guy wire that makes an angle of 48° with the horizontal ground. A 16-m guy wire is to be fastened on the other side of the pole for reinforcement. Both wires attach to the pole at its top.

a) What angle should the second wire make with the ground? Round your answer to the nearest degree.

b) How tall is the hydro pole? Round your answer to the nearest tenth of a metre.

c) How far is the base of the 20-m guy wire from the base of the 16-m guy wire? Round your answer to the nearest tenth of a metre.

11. Domenic and Adriana are observing a hot-air balloon from two tracking stations on the ground. The tracking stations are 6.0 km apart. From Domenic's point of view, the hot-air balloon is at an angle of elevation of 72°. From Adriana's point of view, the angle of elevation is 58°.



a) Determine the distance that Domenic is from the hot-air balloon, to the nearest tenth of a kilometre.

b) Determine the distance that Adriana is from the hot-air balloon, to the nearest tenth of a kilometre.

c) Determine the altitude of the hot-air balloon, to the nearest tenth of a kilometre.

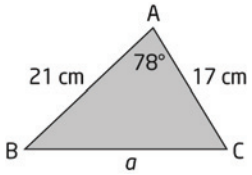
8.2 The Cosine Law

***Principles of Mathematics 10*, pages 405–411**

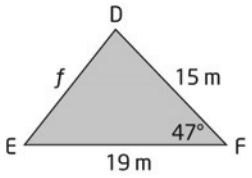
A

1. Find the length of the indicated side in each triangle, to the nearest unit.

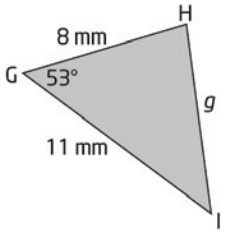
a)



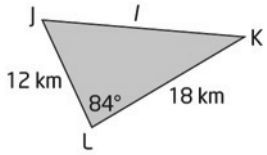
b)



c)



d)



2. Sketch each triangle and use the given information to find the missing side length, to the nearest tenth of a unit.

a) In acute ΔMNO, $m = 4.8$ cm, $o = 5.9$ cm, and $\angle N = 63°$.

b) In acute ΔPQR, $p = 1.8$ m, $q = 2.1$ m, and $\angle R = 73°$.

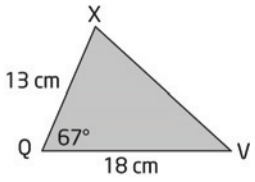
c) In acute ΔSTU, $s = 1.3$ mm, $u = 1.6$ mm, and $\angle T = 49°$.

d) In acute ΔVWX, $w = 7.2$ km, $x = 5.3$ km, and $\angle V = 52°$.

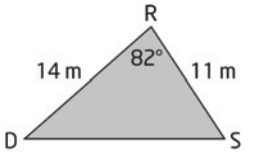
B

3. Solve each triangle. Round side lengths to the nearest tenth of a unit and angles to the nearest degree.

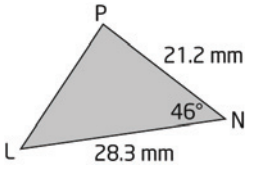
a)



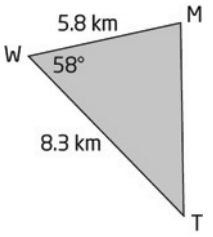
b)



c)



d)



4. Sketch each triangle and label the given information. Then, solve the triangle. Round side lengths to the nearest tenth of a unit and angles to the nearest degree.

a) In ΔTYH, $\angle T = 61°$, $y = 14$ cm, and $h = 19$ cm.

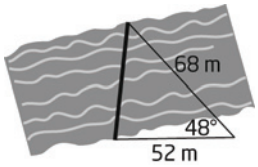
b) In ΔBXR, $\angle X = 54°$, $b = 17$ m, and $r = 12$ m.

c) In ΔJRC, $\angle R = 48°$, $j = 15.2$ mm, and $c = 17.3$ mm.

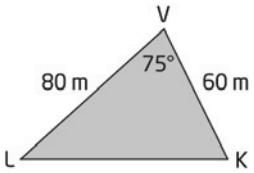
d) In ΔFYZ, $\angle Y = 61°$, $f = 8.3$ km, and $z = 5.4$ km.

5. **Use Technology** Check your answers to question 4 using dynamic geometry software.

6. Find the length of the walkway across the river, to the nearest metre.



7. Vicki is standing on an observation deck in the forest. From where she is standing she can see her daughters Leslie and Kelly. She estimates how far away they are from her and the angle separating their lines of sight, as shown.



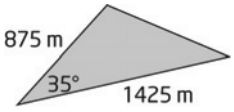
Use Vicki's estimated measures.

a) How far apart are the Leslie and Kelly, to the nearest tenth of a metre?

b) At what angle of elevation does Vicki appear to Leslie, to the nearest degree?

c) At what angle of elevation does Vicki appear to Kelly, to the nearest degree?

8. The lengths of two runways at an airport are 1425 m and 875 m and the beginnings of the runways meet at an angle of 35°, as shown. How far apart are the ends of the runways, to the nearest metre?



C

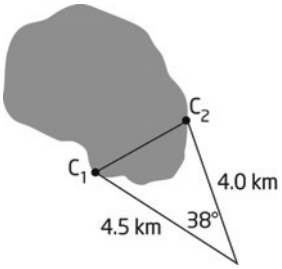
9. Two sailboats leave a dock at the same time, travelling at the same speed of 2 knots. Doug's sailboat sails east, while Mary's sailboat sails northeast. (1 knot = 1.852 km/h)

a) How far apart are the two sailboats after 1 h? Round your answer to the nearest tenth of a kilometre.

b) How far apart are the two sailboats after 2 h? Round your answer to the nearest tenth of a kilometre.

c) Do these answers change if Doug's boat travels half as fast? Justify your answer.

10. Find the distance between the two cottages on opposite sides of the lake, to the nearest tenth of a kilometre.



11. a) Create a problem involving the cosine law for which the answer is 28.

b) Solve the problem.

c) Trade with a partner and solve each other's problem. Check your solutions.

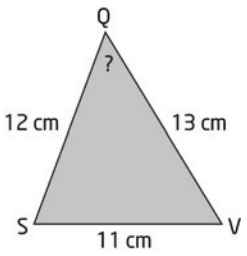
8.3 Find Angles Using the Cosine Law

***Principles of Mathematics 10*, pages 412–423**

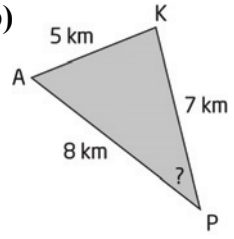
A

1. Find the indicated angle in each triangle, to the nearest degree.

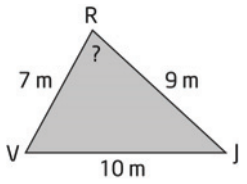
a)



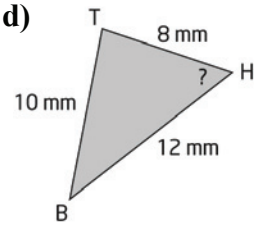
b)



c)



d)



2. Sketch each triangle and label the given information. Then, use the given information to find the indicated angle, to the nearest degree.

a) In acute ΔABC, $a = 3.5$ cm, $b = 4.1$ cm, and $c = 5.4$ cm. Find $\angle A$.

b) In acute ΔPQR, $p = 11.3$ m, $q = 12.4$ m, and $r = 13.2$ m. Find $\angle Q$.

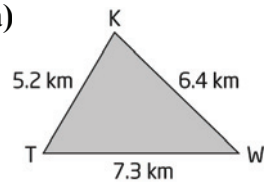
c) In acute ΔXYZ, $x = 21.6$ mm, $y = 23.4$ mm, and $z = 24.5$ mm. Find $\angle X$.

d) In acute ΔDEF, $d = 14.1$ km, $e = 15.2$ km, and $f = 16.3$ km. Find $\angle D$.

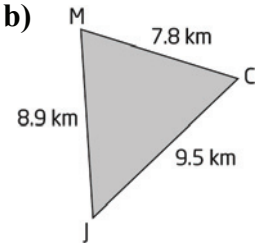
B

3. Solve each triangle. Round your answers to the nearest tenth of a degree.

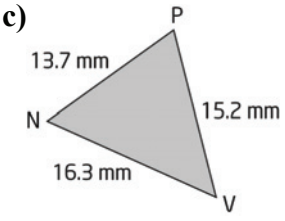
a)



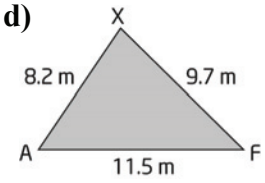
b)



c)



d)



4. Sketch each triangle and label it with the given information. Then, use the given information to solve the triangle. Round your answers to the nearest tenth of a degree.

a) In acute ΔLWG, $l = 8.2$ cm, $w = 9.3$ cm, and $g = 10.4$ cm.

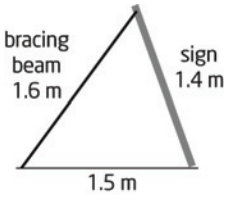
b) In acute ΔKQV, $k = 15.7$ m, $q = 16.8$ m, and $v = 17.3$ m.

c) In acute ΔDJP, $d = 21.3$ mm, $j = 24.6$ mm, and $p = 23.7$ mm.

d) In acute ΔNRX, $n = 7.4$ km, $r = 8.3$ km, and $x = 9.2$ km.

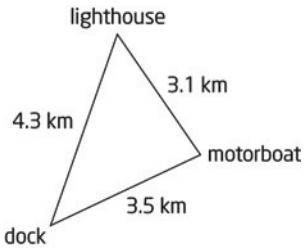
5. **Use Technology** Check your answers to question 5 using dynamic geometry software.

6. A sign for the front of a restaurant is braced, as shown. The sign is 1.4 m long and the bracing beam is 1.6 m long. The foot of the beam is placed 1.5 m from the base of the sign.



a) Determine the angle that the sign makes with the ground, to the nearest degree.
b) Determine the angle that the bracing beam makes with the ground, to the nearest degree.
c) Determine the angle that the sign makes with the bracing beam, to the nearest degree.

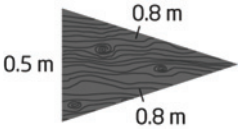
7. A motorboat is located 3.1 km from a lighthouse and 3.5 km from a dock. The lighthouse and the dock are 4.3 km apart.



a) Find the angle between the line from the dock to the lighthouse and the dock to the motorboat, to the nearest degree.
b) Find the angle between the line from the motorboat to the lighthouse and the motorboat to the dock, to the nearest degree.
c) Find the angle between the line from the lighthouse to the dock and the lighthouse to the motorboat, to the nearest degree.

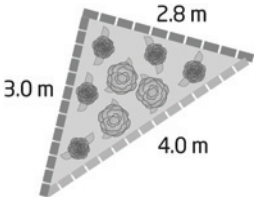
C

8. Michael is designing a coffee table in the shape of an isosceles triangle for his living room.



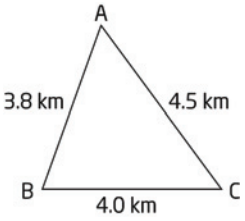
Find the interior angles of the coffee table, to the nearest degree.

9. Danielle is designing a flower garden, in the shape of a triangle, for her backyard.



Find the interior angles of the flower garden, to the nearest tenth of a degree.

10. Three checkpoints, A, B, and C, have been set up in a park for a fundraising relay run.



Find the measure of each of the angles, to the nearest degree, inside the triangle that connects the three checkpoints.

11. a) Acute ΔABC is scalene with $b = 2a$.
Show that $\cos C = \frac{5}{4} - \frac{c^2}{4a^2}$.

b) Acute ΔABC is scalene with $c = 3b$.
Show that $\cos A = \frac{5}{3} - \frac{a^2}{6b^2}$.

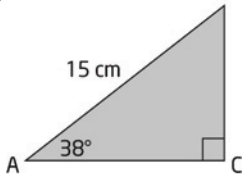
8.4 Solve Problems Using Trigonometry

***Principles of Mathematics 10*, pages 424–429**

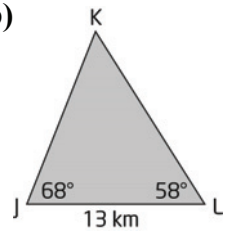
A

1. Determine whether the primary trigonometric ratios, the sine law, or the cosine law should be used first to solve each triangle.

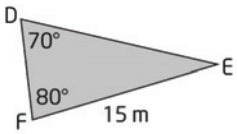
a)



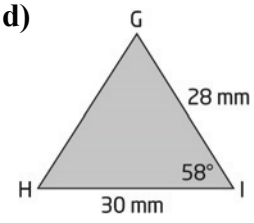
b)



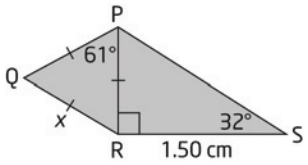
b)



d)

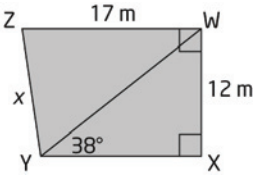


2. **a)** Find x to the nearest hundredth of a metre.



b) Find x using a different method.

3. **a)** Find x to the nearest tenth of a metre.



b) Find x using a different method.

B

4. Iffat is running in a cross-country race. In the first part of the race, she runs 2.3 km from the school to checkpoint A in the park. Then, she turns, making a 68° angle with the first part of the race, and runs 2.5 km to a second checkpoint in the park, checkpoint B. The final turn leads back to the starting point of the race at the school and the finish line.

a) What is the distance from checkpoint B to the finish line, to the nearest tenth of a kilometre?

b) What is the total length of the race?

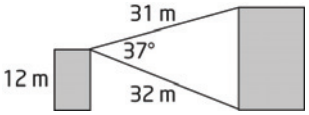
c) Find the remaining angles in the triangle formed by the starting line/finish line and the two checkpoints. Round your answers to the nearest degree.

5. From the top of a cliff the angle of elevation of a biplane is 15°, and the angle of depression of a wooden boat out in the lake, directly below the biplane, is 48°. The distance from the top of the cliff to the biplane is 2.1 km and the distance from the top of the cliff to the wooden boat is 2.5 km.

a) Draw a diagram and label the known information.

b) What is the vertical distance from the biplane to the small boat? Round your answer to the nearest tenth of a kilometre.

6. Jocine is standing on an outdoor stage that is 12 m above the ground. From where she is standing, the angle of elevation of the top of the nearby building is 15°, and the angle of depression to the bottom of the nearby building is 22°. From where she is standing, the distance to the top of the building is 31 m, and the distance to the bottom of the building is 32 m.



a) What is the height of the nearby building, to the nearest metre?

b) How far is the stage from the nearby building, to the nearest metre?

7. Ankit is standing in the playing field at the back of his school. From where he is standing, the angle of elevation of the bottom of a flagpole on the roof of the school is 35°, while the angle of elevation of the top of the flagpole is 40°. The distance from Ankit to the bottom of the flagpole is 37 m, and the distance from Ankit to the top of the flagpole is 40 m.

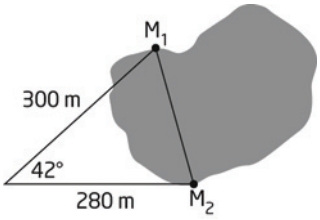
a) Draw a diagram and label the known information.

b) What is the height of the flagpole, to the nearest tenth of a metre?

c) How far along the level ground is Ankit from the school, to the nearest tenth of a metre?

C

8. Two marinas are located across from each other on a lake, as shown. What is the distance between the two marinas? Round your answer to the nearest metre.

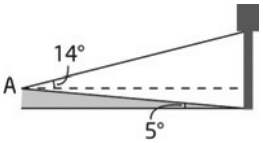


9. Nguyen is walking in the park. On the first section of his walk, he walks 400 m from the parking lot to the hockey rink. Then he turns and walks 500 m to a park bench, making a 54° angle from the first section of his walk. He then turns and walks back to his starting point in the parking lot.

a) What is the distance from the park bench to the parking lot, to the nearest metre?

b) What are the measures of the two remaining angles in the triangular area formed by Nguyen's walk?

10. To measure the height of an observation tower in a forest, Taylor walks 0.25 km from the base of the observation tower along an inclined path to point A. From A, the base of the observation deck appears at an angle of elevation of 14°.



a) Find the height of the observation tower, to the nearest hundredth of a kilometre.

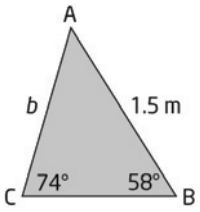
b) Find the distance from point A to the base of the observation deck, to the nearest hundredth of a metre.

Chapter 8 Review

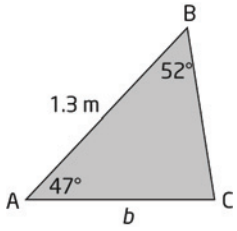
***Principles of Mathematics 10*, pages 430–431**

1. Find the length of b, to the nearest tenth of a metre.

a)

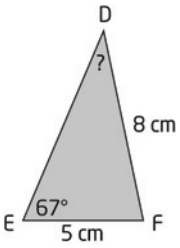


b)

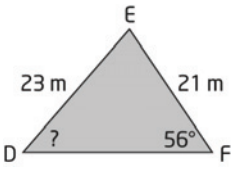


2. Find the measure of $\angle D$, to the nearest degree.

a)

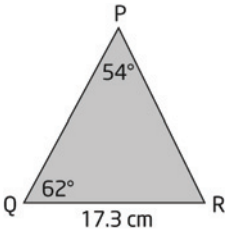


b)



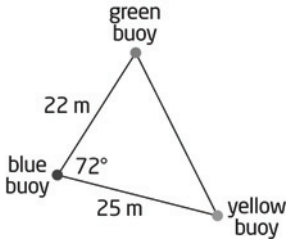
3. Solve each triangle. Round answers to the nearest tenth of a unit, if necessary.

a)



b) In ΔXYZ, $\angle X = 71°$, $\angle Y = 57°$, and $x = 10.3$ m.

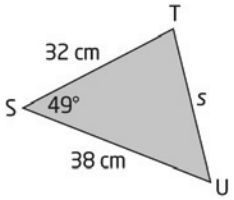
4. Three buoys are positioned in the swimming area at the local park, as shown. The blue buoy is 22 m from the green buoy and 25 m from the yellow buoy. The line from the blue buoy to the green buoy and the line from the blue buoy to the yellow buoy form an angle of 72°.



a) How far apart are the green buoy and the yellow buoy, to the nearest metre?

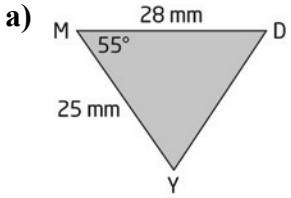
b) Find the measures of the other angles in the triangle formed by these three buoys, to the nearest degree.

5. Find the length of s, to the nearest centimetre.



6. Solve each triangle. Round answers to the nearest unit, if necessary.

a)



b) In acute ΔRDK, $\angle R = 62°$, $k = 15$ cm, and $d = 18$ cm.

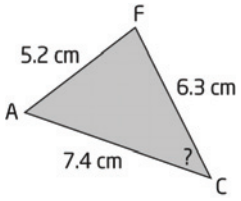
7. Sharon is standing 16 m from the point where Scott is standing and 12 m from the point where Michael is standing. From her point of view Scott and Michael appear to be 43° apart.

a) Draw a diagram and label the known information.

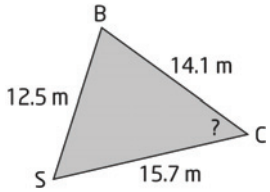
b) How far apart are Scott and Michael, to the nearest metre?

8. Find the measure of ∠C, to the nearest degree.

a)

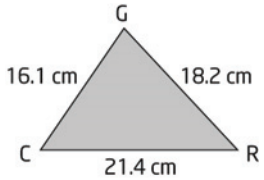


b)



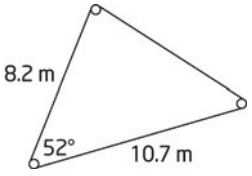
9. Solve each triangle. Round answers to the nearest tenth of a degree.

a)



b) In acute ΔFHM, $f = 3.7$ m, $h = 4.2$ m, and $m = 5.3$ m.

10. A fence is constructed around three posts, as shown.



Find the total length of the fence, to the nearest tenth of a metre. Ignore the curved sections.

11. Ahsan is standing on the level ground directly between two buildings. The top of the first building is 23 m away from Ahsan, at an angle of elevation of 46°, while the top of the second building is 27 m away from Ahsan at an angle of elevation of 52°.

a) Draw a diagram and label the known information.

b) How far apart are the tops of the two buildings, to the nearest metre?

c) How far is Ahsan from the bottom of each building, to the nearest metre?

d) What is the height of each building, to the nearest metre?

12. Sidra is a passenger in a small aircraft. At the same time, she spots a hydro tower that is 5.3 km away to the south and a water tower that is 6.5 km away to the north. The hydro tower and the water tower are separated by a distance of 7.2 km along the flat ground. Find the angles at which Sidra is viewing the hydro tower and the water tower, to the nearest degree.