

The Cosine Law and *The Geometer's Sketchpad*®

How are the side lengths and cosines of angles related in an acute triangle?

1. Open *The Geometer's Sketchpad*® and begin a new sketch.
2. a) Use the **Segment Tool** to construct an acute triangle.
b) Label the vertices as A, B, and C. Label the corresponding sides as a , b , and c .
 - Select a vertex or line segment.
 - Right click and choose **Label Point...** or **Label Segment...**, accordingly from the drop-down menu.
 - Change the label and click on **OK**.
3. a) Measure the lengths a , b , and c .
b) Measure $\angle ACB$.
4. a) Calculate $a^2 + b^2$.
 - From the **Measure** menu, choose **Calculate**.
 - Click on the measure of a . On the calculator, click \wedge , 2, and $+$.
 - Click on the measure of b . Click \wedge , 2, and **OK**.
- b) Calculate c^2 .
- c) For acute $\triangle ABC$, compare $a^2 + b^2$ and c^2 . When would $a^2 + b^2 = c^2$? What is this relationship called?
5. Calculate $2ab(\cos C)$.
 - From the **Measure** menu, choose **Calculate**.
 - Click 2, $*$ the measure of a , $*$, the measure of b , and $*$. From the **Functions** drop-down menu on the calculator, choose **cos**. Then, click the measure of $\angle ACB$ and **OK**.
6. a) Describe any relationship you notice between $a^2 + b^2$, c^2 , and $2ab(\cos C)$.
b) Move vertex C to examine other acute triangles. Does this relationship hold true?
7. Suppose $\angle C$ is 90° .
 - a) What happens to the quantity $2ab(\cos C)$?
 - b) What relationship applies to the three sides of the triangle when this happens?
8. **Reflect**
 - a) Write an equation that relates the cosine of an angle and the three sides of an acute triangle.
 - b) What happens to this relationship when the measure of one of the angles is 90° ?

