1.3 Equivalent Linear Relations and Equivalent Linear Systems

Student Text Pages
29–33

Suggested Timing
60–80 min

Tools
- grid paper

Technology Tools
- graphing calculator
- geometry software

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 1–6 Section 1.3 Practice Master

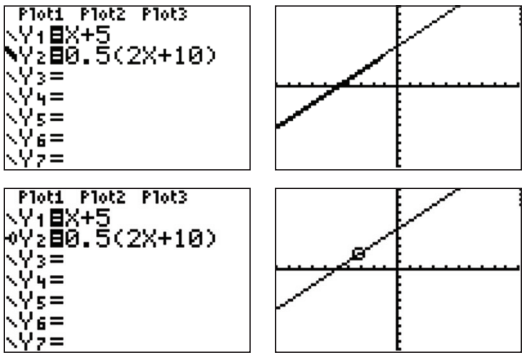
TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

Investigate

- **Investigate A** is best completed by graphing by hand or using *The Geometer's Sketchpad*® (GSP). If students are graphing by hand, encourage them to use the most efficient way to graph each line: use the slope and *y*-intercept or *x*- and *y*-intercepts, depending in the form of the equation. They could use a different colour for each line. With GSP, students can enter the equation in any form. If students use a graphing calculator, they will need to rearrange equations into the form $y =$ first. (10 min)
- **Investigate B** can be completed using any of the three tools. (15 min)
- Use a graphing calculator to determine if two equations are identical using styles as shown:



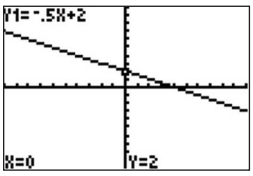
The graphing was stopped by pressing **ENTER** part way through the operation so that the original function can be seen in its entirety while the second function overlaps only part of the original. The first set used a heavier style for **Y2**; the second set used a "trace" style, where the function is graphed but at a much slower rate, as the circle moves along the curve.

Communicate Your Understanding

- Review the vocabulary of this section (equivalent linear equations, equivalent linear system) before discussing the **Communicate Your Understanding** questions. (15 min)
- Use **BLM 1–6 Section 1.3 Practice Master** for remediation or extra practice.

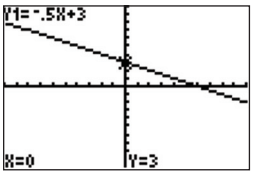
Investigate Answers (pages 29–31)

A

1. a) 

b) They are the same line.

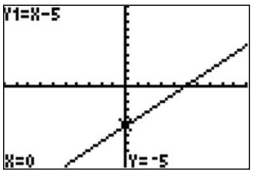
c) If you multiply each term of the first equation by 2, you obtain the second equation.

2. a) 

b) They are the same line.

c) The second equation can be rearranged into the $y = mx + b$ form, to give the first equation.

3. a) $y - x + 5 = 0$ and $2y = 2x - 10$

b) 

4. $2y - x - 4 = 0$

5. Answers will vary. Examples:

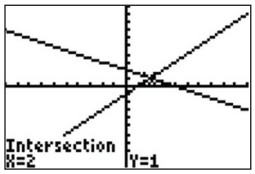
a) $y = -1.5x + 6$ and $6x + 4y = 24$ **b)** $y = -x + 4$ and $2x + 2y = 8$

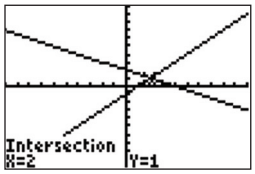
c) $2x - 3y + 3 = 0$ and $4x - 6y = -6$

6. a) Rearrange the equation and/or multiply the equation by any constant.

b) an infinite number

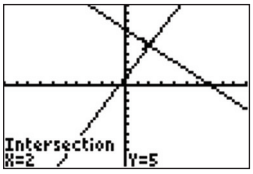
B

1. 

2. 

3. a) The point of intersection is the same for both linear systems.

b) The equations are an equivalent pair of equations. Equations ① and ② from step 1 have been multiplied by 2 and rearranged.

4. a) 

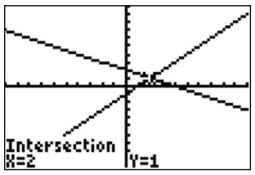
b) Answers may vary. For example: $2y = 4x + 2$. This equation is equivalent to the first equation in part a).

c) Answers may vary. For example: $3y + 3x = 21$. This equation is equivalent to the second equation in part a).

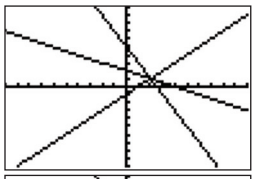
d) The point of intersection should be the same as for the lines in part a).

5. Answers will vary. For example: Multiply either or both of the equations by any number to obtain an equivalent equation for each. The two new forms of the equations will have the same point of intersection.

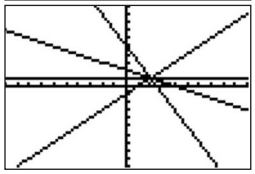
6. a)



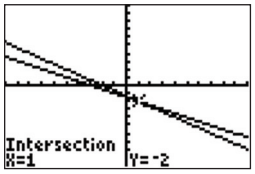
b) The line $2x + y = 5$ passes though the point of intersection of the first two lines. The three lines have a common solution.



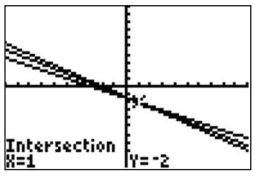
c) The line $3y = 3$ passes though the point of intersection of the first two lines. All four lines have a common solution.



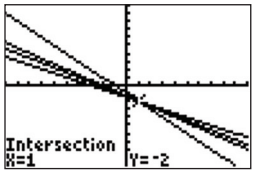
7. a)



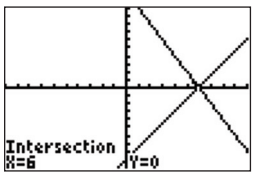
b) The line $3x + 5y = -7$ passes though the point of intersection of the first two lines. The three lines have a common solution. The equation $3x + 5y = -7$ is obtained by adding the left sides and the right sides of the original two equations.



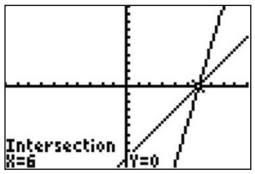
c) The line $x + y = -1$ passes though the point of intersection of the first two lines. All four lines have a common solution. The equation $x + y = -1$ is obtained by subtracting the left sides and the right sides of the original two equations.



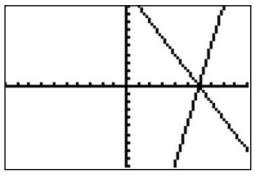
8. a)



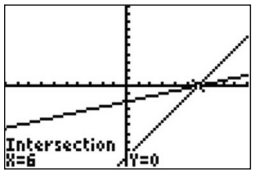
b) $5x - y = 30$ ③; equations ① and ③ will have the same point of intersection as the two lines in part a).

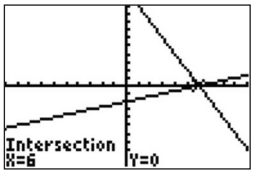


c) Equations ② and ③ will have the same point of intersection as the two lines in part a).



d) $x - 3y = 6$ ④; equations ① and ④ will have the same point of intersection as the two lines in part a). Equations ② and ④ will have the same point of intersection as the two lines in part a).





Common Errors

- Some students may have difficulty articulating their understanding of equivalent linear systems.

R_x Have students explain in words to their friends in class, and have another student scribe the explanation.

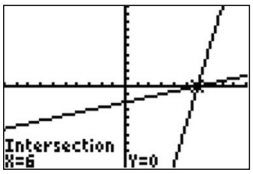
Accommodations

Perceptual—Let students use a graphing calculator to determine if the systems of equations are equivalent.

Motor—Provide students with large sheets of grid paper to complete the questions in this section.

Memory—Encourage students to show that the systems of equations are equivalent using more than one method. For example: showing steps using pencil and paper, and using a graphing calculator.

e) Yes, equations ③ and ④ should have the same point of intersection.



9. Answers will vary. For example: To obtain an equivalent linear system you can write equivalent equations for one or both of the equations by
- multiplying either or both of the original equations by a constant
- adding the left sides and the right sides of the original equations
- subtracting the left sides and the right sides of the original equations

Communicate Your Understanding Responses (page 32)

C1. Yes, because the equations are equivalent: the second equation is four times the first equation.

C2. $k = 6$

C3. Yes, they are equivalent. In system B, the second equation is double the second equation in system A. You could graph all the equations and see if they all have the same point of intersection.

C4. For $x = -1$, subtract ② from ①; then solve for x. For $y = 3$, add ① and ②; then solve for y.

Practise

- Stress that students should not try to decide if equations are equivalent just by looking at them. They should actually rewrite each equation so that they are all in the same configuration. For **questions 1** and **2**, this is all as "$y =$."
- In **question 3** students make up equivalent equations. Suggest that they could multiply every term by two or divide every term by three, as long as they remember to multiply or divide each term by the same value.
- **Questions 4** and **5** may pose more difficulty than the rest of the Connect and Apply questions. Remind students of Section 1.1 and the translating from words to algebra.
- **Question 9** allows students to really stretch their thinking. Once they have created an example of their own, they will see more clearly how all the equivalent equations relate to one another.

Literacy Connections

Question 10 states that Indian mathematician Srinvasa Ramanujan discovered more than 3000 theorems. Have students find out the meaning of "theorem" by using a dictionary. This may spark an interest in the origin of this term and the difference between a theorem and other mathematical terms.

Add the new terms that appear in this section to the Word Wall.

Student Success

Use a timed retell strategy to have students explain the concept of equivalent systems.

Refer to the introduction of this Teacher's Resource for more information about how to use a timed retell strategy.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	10, 11
Reasoning and Proving	7, 8
Reflecting	6, 9
Selecting Tools and Computational Strategies	10, 11
Connecting	4, 5
Representing	4, 5, 7
Communicating	6–8

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.