2.1 Midpoint of a Line Segment

Student Text Pages
56–69

Suggested Timing
70–90 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad*®
- computer
- Cabri® Jr.
- graphing calculator
- Internet access

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 2–3 Section 2.1 Practice Master
- A–7 Thinking General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- As an introduction, discuss a problem similar to **Example 1**, which involves finding the halfway point between two locations to build a new facility such as a school or hospital. (5 min)

Investigate

- Have students complete the **Investigate** using one of the three methods. Use **T–4** ***The Geometer's Sketchpad*® 3** or **T–5** ***The Geometer's Sketchpad*® 4** to support this activity. Discuss the results as a class, making sure that students understand the midpoint formula $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. (20 min)
- You may want to show explicitly that the midpoint coordinates are the mean of the endpoint coordinates:

$$\begin{aligned}(x, y) &= \left(x_1 + \frac{\text{run}}{2}, y_1 + \frac{\text{rise}}{2}\right)\\ &= \left(x_1 + \frac{x_2 - x_1}{2}, y_1 + \frac{y_2 - y_1}{2}\right)\\ &= \left(\frac{2x_1 + x_2 - x_1}{2}, \frac{2y_1 + y_2 - y_1}{2}\right)\\ &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)\end{aligned}$$

Examples

- Discuss **Examples 1**, **2**, and **3** with the class. (15 min)

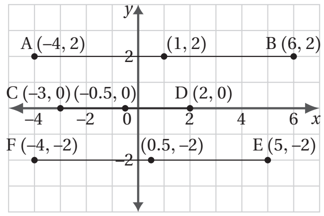
Communicate Your Understanding

- Review the vocabulary in this section (Cartesian grid, midpoint, median, equidistant, right bisector) before discussing the **Communicate Your Understanding** questions. (15 min)
- Use **BLM 2–3 Section 2.1 Practice Master** for remediation or extra practice.

Investigate Answers (pages 56–60)

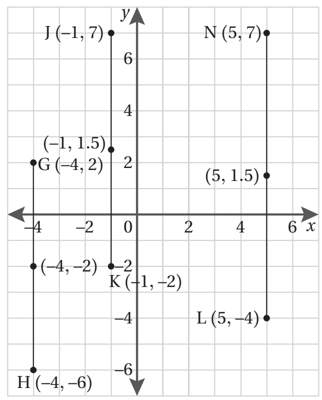
Method 1

1. The line segments are all horizontal.



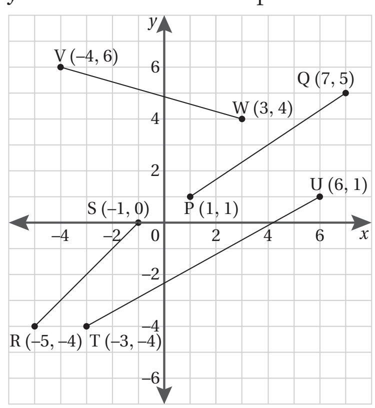
2. The x-coordinate of the midpoint of each horizontal line segment is the mean of the x-coordinates of the endpoints, and the y-coordinate is the same as the y-coordinate of the endpoints.

3. The line segments are all vertical.



4. The x-coordinate of the midpoint of each vertical line segment is the same as the x-coordinate of the endpoints, and the y-coordinate is the mean of the y-coordinates of the endpoints.

5.



6. a) (4, 3) **b)** (–3, –2) **c)** (1.5, –1.5) **d)** (–0.5, 5)

For each line segment, the x-coordinate of the midpoint is the mean of the x-coordinates of the endpoints and the y-coordinate is the mean of the y-coordinates of the endpoints.

7. $M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Method 2

3. Since the line segment is horizontal, students need to estimate just the x-coordinate of the midpoint from the plot. So, most students will predict the actual value, (1, 2).

4. Answers may vary. Most students will find that the coordinates of the midpoint match their prediction.

5. (–0.5, 0); (0.5, –2)

6. The line segments are all horizontal. The x-coordinate of the midpoint of each horizontal line segment is the mean of the x-coordinates of the endpoints, and the y-coordinate is the same as the y-coordinate of the endpoints.

7. a) (–4, –2) **b)** (–1, 2.5) **c)** (5, 1.5)

8. The line segments are all vertical. The x-coordinate of the midpoint of each vertical line segment is the same as the x-coordinates of the endpoints, and the y-coordinate is the mean of the y-coordinates of the endpoints.

9. a) (4, 3) **b)** (–3, –2) **c)** (1.5, –1.5) **d)** (–0.5, 5)

10. For each line segment, the x-coordinate of the midpoint is the mean of the x-coordinates of the endpoints and the y-coordinate is the mean of the y-coordinates of the endpoints.

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Method 3

4. Since the line segment is horizontal, students need to estimate just the x-coordinate of the midpoint from the plot. So, most students will predict the actual value, (2, 2).

5. Answers may vary. Most students will find that the coordinates of the midpoint match their prediction.

6. (–0.5, 0); (1.5, –2)

7. The line segments are all horizontal. The x-coordinate of the midpoint of each horizontal line segment is the mean of the x-coordinates of the endpoints, and the y-coordinate is the same as the y-coordinates of the endpoints.

9. a) (–4, 1) **b)** (–1, 1) **c)** (5, 0.5)

10. The line segments are all vertical. The x-coordinate of the midpoint of each vertical line segment is the same as the x-coordinates of the endpoints, and the y-coordinate is the mean of the y-coordinates of the endpoints.

11. a) (1, 3) **b)** (3, 1.5) **c)** (1, –1.5)

12. For each line segment, the x-coordinate of the midpoint is the mean of the x-coordinates of the endpoints and the y-coordinate is the mean of the y-coordinates of the endpoints.

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Communicate Your Understanding Responses (page 65)

C1. The midpoint of a line segment can be found by adding half of the run to the x-coordinate of the first endpoint and adding half of the rise to the y-coordinate of the endpoint.

The formula $M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ can also be used.

C2. Answers may vary. For example: Find the coordinates of the midpoint M of side BC. Then, find the slope of the median AM. Substitute this slope and the coordinates of either point A or point M into $y - y_1 = m(x - x_1)$ and simplify. Or, substitute the values into $y = mx + b$ and solve for the y-intercept, b.

C3. Find the coordinates of the midpoint M of the line segment PQ. Then, find the slope of PQ. The negative reciprocal of this slope is the slope of the right bisector. Now, substitute the coordinates of the midpoint and the slope of the right bisector into either $y - y_1 = m(x - x_1)$ or $y = mx + b$.

Practise

- **Question 1** provides visuals. Point out to students that a diagram is helpful, but not always necessary.
- In **question 2** students should determine the midpoint by using the formula. They could draw a diagram to check the reasonableness of their answers, but a diagram is not necessary.
- **Question 3** involves rational coordinates and coordinates with variables.
- For **question 4**, refer students to **Example 2**. However, this question only requires the calculation of the slope of each median.
- **Question 13** can be done by counting squares on a grid. The rise and run from C to the midpoint M are the same as the rise and run from M to the unknown endpoint, D. Some students may find the endpoint algebraically. Let the unknown endpoint be (x, y). Substitute the known values into the midpoint formula: $(4, 2) = \left(\frac{x-6}{2}, \frac{y+5}{2}\right)$.

  This leads to the equation $4 = \frac{x-6}{2}$, which can be solved for x, and the corresponding equation that can be solved for y. Show students this algebraic method and ensure they are comfortable with either approach. Use **A–7 Thinking General Scoring Rubric** when assessing students.
- **Question 14** is similar to **question 13**, in the context of a circle.
- **Question 15** has two possible answers. Since either endpoint of the radius can be considered the centre of the circle, the radius can be extended in either direction to create a diameter.
- **Questions 16** and **17** are similar to **Example 3**.
- **Question 20** is a good lead-in to **question 21**, the Chapter Problem. There are many web sites with information on Sierpinski's triangle. An extension might be the Sierpinski Carpet. Students can use the Internet Link to find more information about the Sierpinski Triangle and the Sierpinski Carpet.
- **Question 22** requires a clearly labelled diagram. Use the fact that ED is 2 units in length and work backward to determine that AD must be twice as long (4 units), and AC is twice as long again (8 units), leading to BC being 16 units.
- **Question 23** is easier if students draw a diagram on a grid so they can see that the rise of AB is 18 and the run is 9. The rise and run must be cut into thirds, so a vertical move of 6 and a horizontal move of 3 will locate the division points.
- **Question 24** can be completed most easily on a grid. This question can also be solved using algebraic techniques—finding the equations of the lines that contain the sides of △ABC, and then finding the intersection of pairs of these lines to determine the vertices A, B, and C. However, this method requires a lot more work.
- **Question 25** provides an extension of the midpoint formula to three-dimensional space. The formula is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$.
- **Question 26** extends the idea that a point equidistant from two given points must lie on the right bisector of the segment joining the given points. Point out to students that it is only necessary to find the intersection of any two of the right bisectors, since the third right bisector must always pass through that same point. In other words, the three right bisectors are concurrent.
- **Question 27** requires students to realize that the curvature of Earth means that the line joining two points on Earth's surface is not a straight line. Use a globe to give the students a visual understanding of this concept.

Common Errors

- Some students may confuse the formulas for slope and midpoint and have trouble remembering when to add the coordinates and when to subtract them.

R_x Remind students that the coordinates of the midpoint are an "average" of the coordinates of the endpoints. This should help them realize that they need to add the coordinates and divide by two. In contrast, when calculating slope, the horizontal and vertical displacement are the issues, so they need to subtract the coordinates.

- Students may use improper mathematical form when calculating midpoints.

R_x Brackets should enclose the coordinates of a point. Encourage students to keep the brackets throughout the calculation steps. An alternative is to calculate each coordinate separately, then express them together as coordinates of a point enclosed in brackets in the final conclusion.

Accommodations

Visual—Let students use colour-coding to relate the x-values and y-values in ordered pairs, such as $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$, to the formula for the rise, $y_2 - y_1$; the formula for run, $x_2 - x_1$; and the formula for the midpoint of a line segment, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Perceptual—Provide students with opportunities to use an alternative method, such as paper folding, to determine the midpoint of a line segment, to construct the median of a triangle, and to construct the right bisector of a line.

Motor—Allow students to work in groups or with a partner when completing the Investigations in Section 2.1 that require the use of technology.

Memory—Encourage students to use a number line when adding integers such as $-5 + 3$ and $-2 + 3$.

Student Success

Use a Word Wall to keep a visual record of terminology.

Use a jigsaw approach to simultaneously develop knowledge of slope, midpoint, and length.

Refer to the introduction of this Teacher's Resource for more information about how to use a jigsaw strategy.

Literacy Connections

Discuss the two definitions in the margin on page 56, Cartesian grid and midpoint. There is a related Making Connections, which discusses René Descartes, on page 69. As extension or enrichment, have students research Descartes on the Internet or in the library.

Draw attention to the marginal definition that appears on page 62. The word "median" means one thing in this context. Ask students if they can think of another meaning in mathematics for this term. Discuss the median as a measure of central tendency.

On page 63 there are two marginal definitions and a Literacy Connections. Discuss the terms "equidistant" and "right bisector" (perpendicular bisector). Ask students to recall other phrases that mean the same thing.

Create a Word Wall for this chapter. Add "fractal," "Cartesian grid," "midpoint," "median," "equidistant," and "right bisector" to it.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	21, 26, 27, 30, 31
Reasoning and Proving	12, 13, 15, 20, 22–31
Reflecting	7, 11, 13, 19, 20, 24
Selecting Tools and Computational Strategies	1–3, 23, 26, 29, 31
Connecting	5, 6, 8, 10, 14–17, 19–21, 25–27
Representing	7, 9, 11, 12, 18–26, 29
Communicating	7, 9, 11–13, 15, 17, 18, 21, 23, 26–28

Ongoing Assessment

- Chapter Problem question 21 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.