2.2 Length of a Line Segment

Student Text Pages
70–79

Suggested Timing
40–70 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computer
- Cabri® Jr.
- graphing calculator
- Internet access

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 2–4 Section 2.2 Practice Master
- BLM 2–5 Section 2.2 Achievement Check Rubric
- A–4 Presentation Checklist
- A–7 Thinking General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- Start this section with a discussion about coordinates. Most students will have had experience with coordinates in previous grades. Games like Battleship also use a coordinate system that will be familiar to some students. (5 min)

Investigate

- The **Investigate** can be done using a paper-and-pencil approach or by using technology. *The Geometer's Sketchpad*® or a TI-83 Plus or a TI-84 Plus graphing calculator with Cabri® Jr. can be used effectively for this activity. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. (15 min)
- Ensure that students understand the formula for the length of a line segment, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_2)^2}$. Students should understand that this can also be thought of as $d = \sqrt{\text{run}^2 + \text{rise}^2}$. Emphasize that this formula is a result of applying the Pythagorean theorem.

Examples

- Discuss **Examples 1**, **2**, and **3**. (15 min)

Communicate Your Understanding

- Discuss the **Communicate Your Understanding** questions as a class. (5 min)
- Use **A–7 Thinking General Scoring Rubric** at any point during this section to assist you in assessing students.
- Use **BLM 2–4 Section 2.2 Practice Master** for remediation or extra practice.

Investigate Answers (pages 70–73)

Method 1

1. The line segment represents the distance from the start of the canoe trip to the first campsite.
2. The run is the difference between the x-coordinates of the endpoints of line segment AB, and the rise is the difference between the y-coordinates of the endpoints.
3. (6, 2). The x-coordinate of point D is the same as the x-coordinate of point B, and the y-coordinate of point D is the same as the y-coordinate of point A.
4. AD = 4, BD = 3, AB = 5
5. 20 km
6. $\text{AB} = \sqrt{\text{run}^2 + \text{rise}^2}$
   $= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
7. 16.5 km. Answers may vary. For example: No, the square of the run and the rise is always positive.
8. 20.4 km
9. $d = \sqrt{\text{run}^2 + \text{rise}^2}$
   $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Method 2

3. The run is the difference between the x-coordinates of the endpoints of line segment AB, and the rise is the difference between the y-coordinates of the endpoints.
4. (6, 2). The x-coordinate of point D is the same as the x-coordinate of point B, and the y-coordinate of point D is the same as the y-coordinate of point A.
5. AD = 4, BD = 3, AB = 5
6. 20 km
7. $\text{AB} = \sqrt{\text{run}^2 + \text{rise}^2}$
   $= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
8. 16.5 km. Answers may vary. For example: No, the square of the run and the rise is always positive.
9. 20.4 km
10. $d = \sqrt{\text{run}^2 + \text{rise}^2}$
    $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Method 3

6. The run is the difference between the x-coordinates of the endpoints of line segment AB, and the rise is the difference between the y-coordinates of the endpoints.
7. AD = 4, BD = 3, AB = 5
8. 20 km
9. $\text{AB} = \sqrt{\text{run}^2 + \text{rise}^2}$
   $= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
10. 16.5 km. Answers may vary. For example: No, the square of the run and the rise is always positive, so the formula gives a positive value for the length.
11. 20.4 km
12. $d = \sqrt{\text{run}^2 + \text{rise}^2}$
    $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Communicate Your Understanding Responses (page 77)

C1. Answers may vary. For example: The length is the square root of the sum of the squares of run and rise from point A to point B.

C2. Answers may vary. For example: Since the run and rise are both squared in the distance formula, the result is the same regardless of which endpoint is taken as the start of the line segment. For the points A(3, 6) and B(5, 9), the distance between the points can be calculated as

$$\begin{aligned} d &= \sqrt{(5-3)^2 + (9-6)^2} \\ &= \sqrt{2^2 + 3^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned} \qquad \text{or} \qquad \begin{aligned} d &= \sqrt{(3-5)^2 + (6-9)^2} \\ &= \sqrt{(-2)^2 + (-3)^2} \\ &= \sqrt{4+9} \\ &= \sqrt{13} \end{aligned}$$

C3. The square of any difference of coordinates is always greater than or equal to zero.

Common Errors

- Some students may confuse the formulas for length and midpoint and have trouble remembering when to add and when to subtract the coordinates.

Rx Remind students that the length formula results from applying the Pythagorean theorem. Reinforce the role of rise and run, so that students have an understanding of why the length formula works. Even after students are accustomed to the formula, it is worth revisiting questions where a diagram is provided, so that students do not forget that counting squares on a grid is always an excellent strategy.

- Some students may continue to make sign errors when using the length formula, since subtraction of the coordinates is involved.

Rx Mention rise and run. It is the displacement between the coordinates that matters.

- Some students may use improper mathematical form when calculating length of a segment.

Rx Some students may prefer to use the length formula in the form $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Others may prefer $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, taking the square root of both sides as the final step in the calculation. Either form is acceptable, but make sure that students use proper mathematical form in all cases.

Practise

- **Question 1** provides visuals, and students may do these questions by counting squares to determine the rise and the run. This is always an appropriate strategy when the graph is provided.
- **Questions 2** and **3** lead to a more abstract understanding of the coordinates and the formula. It is important that students realize that a graph is not necessary, and that the formula has been developed for cases like this.
- **Question 4** applies a scale to calculate the actual distance between the two stores. Point out that students can simply calculate the distance on the map and then apply the scale to determine the actual distance. In this case the scale is a 1:1 correspondence, so the conversion to kilometres will be easy for most students to comprehend.

Accommodations

Gifted and Enrichment—Encourage students to use the Internet to research fractals and the mathematician Niels Fabian Helge von Koch.

Perceptual—Allow students to determine the length of the rise and the length of the run for a given line segment on a Cartesian grid. Then, apply the Pythagorean theorem to calculate the length of the line segment.

Motor—Provide students with large sheets of grid paper to complete the graphing questions in this section.

Memory—Give students index cards with each step for calculating the length of a line segment on a separate card. Let them put the index cards in the correct order, finding the length of a line segment.

- **Question 5** is very similar to **Example 3**, where a comparison of distances is made. Students may plot the points and visually decide which school is closer, but this should be verified by using the length formula in part b).
- **Questions 6** applies the length formula to answer questions about a triangle. This pulls together earlier concepts involving median and perimeter of a triangle.
- For **question 7b)**, students find coordinates of another equilateral triangle. This would be a good opportunity to discuss similar triangles. Most students will find it natural to multiply the coordinates by a factor in this case. Ask students whether this technique will always produce a similar shape. This could lead to a discussion about dilatations and other transformations.
- **Question 10** applies the concept of length in an area question.
- In **question 11**, it is worthwhile to have students verify their answer using technology, so that they realize the power of the technology. This may be one of the first opportunities for students to use the area function of these software packages.
- **Questions 12** and **13** use the length formula to verify the midpoint of a segment, which was covered in the previous section.
- **Question 16** is a good follow-up to questions 20 and 21 from the previous section, which involve Sierpinski's triangle.
- **Question 18**, the Chapter Problem, introduces the Koch snowflake. Students may wish to research this topic further. Ask students how the perimeter of the snowflake is changing with each step. What would happen if this pattern continued many, many times? What happens to the area of the snowflake? Students can use the Internet Link to find more information about the Koch snowflake.
- **Question 19** is similar to **question 5**, but the description of assumptions in part b) requires students to consider factors such as wind, land form, etc.
- **Question 20** can be better understood by using a diagram. In this case, there is only one point that is 5 units away, since the point (2, 6) is 5 units from the horizontal line $y = 1$. Discuss with students how the number of solutions would change if the distance was increased. How many points of this form would be less than 5 units away? Encourage stronger students to consider an algebraic solution to this problem. Although they do not have experience with quadratic equations at this point, they will likely understand that the only solution to the resulting quadratic, $(x - 2)^2 = 0$, is $x = 2$, resulting in the point (2, 1).
- **Questions 21** and **22** should help consolidate understanding of the length concept.

Student Success

Use a modified four corners strategy (GSP, graphing calculator, formula, scale drawing) followed by an oral presentation in groups of four, one from each corner. Use **A–4 Presentation Checklist** when assessing students.

Refer to the introduction of this Teacher's Resource for more information about how to use a four corners strategy.

Achievement Check Sample Solution, question 19, page 79

Provide students with **BLM 2–5 Section 2.2 Achievement Check Rubric** to help them understand what is expected.

19. a) Find the distances AF and BF.

$$\begin{aligned} AF &= \sqrt{(23-15)^2+(25-19)^2} \\ &= \sqrt{100} \\ &= 10 \end{aligned} \qquad \begin{aligned} BF &= \sqrt{(23-23)^2+(25-17)^2} \\ &= \sqrt{64} \\ &= 8 \end{aligned}$$

Town B is at a greater risk from the fire.

b) You must assume that the fire will travel at the same pace toward each town. Some factors that affect the rate of burn are wind direction, forest type, and existence or lack of fire breaks (e.g., rivers, highways).

c) Find the midpoint, M, between towns A and B.

$$\begin{aligned} M(x, y) &= \left(\frac{15+23}{2}, \frac{19+17}{2}\right) \\ &= (19, 18) \end{aligned}$$

Find the distance to the fire, MF.

$$\begin{aligned} MF &= \sqrt{(23-19)^2+(25-18)^2} \\ &\doteq 8.1 \end{aligned}$$

The camper is further from the fire than town B is, since the distance is shorter.

Literacy Connections

Discuss the Literacy Connections on page 74. Perhaps your Technology department has small machines that use cams. Some older sewing machines in your school with cams for embroidery might be an interesting way to show students an application of the cam.

Add "cam" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	7, 17, 19, 23–25
Reasoning and Proving	6, 7, 10, 15, 18, 20–22, 24, 25
Reflecting	5, 9, 11, 13, 16, 19–21
Selecting Tools and Computational Strategies	7, 22, 23, 25
Connecting	5–8, 10, 12–15, 17, 19, 25
Representing	8–11, 14–16, 19, 20, 23, 25
Communicating	5, 17–21

Ongoing Assessment

- Use Achievement Check question 19 to monitor student success. See Achievement Check Answers and **BLM 2–5 Section 2.2 Achievement Check Rubric**.
- Chapter Problem question 18 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.