2.3 Apply Slope, Midpoint, and Length Formulas

Student Text Pages
80–91

Suggested Timing
60–75 min

Tools
- grid paper
- protractor

Technology Tools
- *The Geometer's Sketchpad®*
- computer
- Cabri® Jr.
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- G–3 Coordinate Grids
- G–4 Protractor
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 2–6 Section 2.3 Practice Master
- BLM 2–7 Section 2.3 Achievement Check Rubric
- A–8 Application General Scoring Rubric
- A–17 Teamwork Self-Assessment

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- Introduce this section with a discussion about power lines and finding the shortest route from a point to an existing line. (5 min)

Investigate

- The **Investigate** can be done using pencil and paper or technology. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. Afterward, have a class discussion to consolidate the idea that the shortest distance from a point to a line is a line segment that is perpendicular to the line. (10 min)

Examples

- Discuss the **Examples**. (30 min)
- **Example 1** incorporates the skills from the last chapter, requiring students to find the equations of two lines and then find their point of intersection algebraically.
- In **Example 2**, students must understand that they need to use analytic methods to prove two lines are perpendicular.
- **Example 3** incorporates the concepts of midpoint and length to prove a property of a triangle—that the median to the hypotenuse is half the length of the hypotenuse. Encourage students to use dynamic geometry software to show that this is true for any right triangle in **question 29**.

Communicate Your Understanding

- Review the vocabulary in this section (altitude) before discussing the **Communicate Your Understanding** questions. (10 min)
- Use **A–8 Application General Scoring Rubric** at any point during this section to assist you in assessing students.
- Use **BLM 2–6 Section 2.3 Practice Master** for remediation or extra practice.

Investigate Answers (pages 80–82)

Method 1

2. The line segment must be perpendicular to the given line. This perpendicular line segment can be constructed using compasses or a protractor.

4. Answers may vary. For example: The sides of △PQR are related by the Pythagorean theorem because ∠PQR is a right angle. PR is the hypotenuse, so it is longer than PQ.

5. The shortest line segment from a point to a line is perpendicular to the given line.

Method 2

2. Construct the perpendicular line from point C to the line that passes through points A and B.

5. Most students will estimate that ∠ADC = 90°.

6. The measurement should be 90° or close to this value. Most students will find that their estimate was accurate.

7. The shortest line segment from a point to a line is perpendicular to the given line.

Method 3

6. Most students will estimate that ∠ADC = 90°. The measurement should be 90° or close to this value.

7. The shortest line segment from a point to a line is perpendicular to the given line.

Communicate Your Understanding Responses (page 88)

C1. The midpoint, M, of line segment AB is a point on the right bisector. Use the midpoint formula to find the coordinates of the midpoint. Then, find the slope of line segment AB. Calculate the negative reciprocal of this slope to find the slope of the right bisector. Substitute the slope of the right bisector and the coordinates of point M into the formula $y = mx + b$, and solve for the y-intercept, b.

C2. Use the length formula to find the lengths of CD, CE, and DE. If two of the lengths are equal, then the triangle is isosceles.

C3. Use the coordinates of vertices G and H to find the equation of the line through these two points. The altitude from F is perpendicular to GH, so the slope of the altitude is the negative reciprocal of the slope of GH. Use the slope of the altitude and the coordinates of vertex F to find the equation of the altitude. The coordinates of point I, where the altitude meets side GH, satisfy both equations. Solve this system of the equations to find these coordinates. Then, substitute the coordinates of point I and vertex F into the length formula to find the length of the altitude from F.

C4. Find the midpoint of each median. Use the coordinates of the vertices and the midpoints to find the equations of any two of the medians. Then, solve these two equations to find the point of intersection of the two medians. (To verify these coordinates, check that they satisfy the equation for the third median.)

Common Errors

- Use of proper mathematical form will be a challenge for some students.

R_x Encourage students to break solutions down into manageable steps. Have them explain what they are doing in each step. This will make solutions easier for others to follow as well as clarify thinking. Discuss model solutions as a class so that students know what is expected.

- Students may have trouble with the multi-step problems.

R_x Emphasize the problem solving steps outlined in this section. Encourage students to reflect on their answers and to look for alternative approaches. Have students work in pairs or small groups on the more challenging questions. Encourage students to communicate their reasoning both verbally and in written form.

Practise

- **Question 1** reviews the skill of finding the equation of a line, which is used in many of the questions.
- **Question 5** proves that the line joining the midpoints of two sides of a triangle is parallel to, and half the length of, the third side. Students can try this with a triangle of their own choice to help them realize that it is true in all cases. Dynamic geometry software can also be used to illustrate the property in general. Stronger students might try proving this property by using general coordinates for the vertices of the triangle, such as (x_1, y_1), (x_2, y_2), and (x_3, y_3).
- **Question 6** could be done by showing that the segment from T to the midpoint of UV is perpendicular to UV. It could also be shown that TU and TV are equal in length, forming an isosceles triangle whose median is a perpendicular bisector. Any point on the perpendicular bisector of UV will be equidistant from U and from V.
- **Questions 10** through **14** involve finding the shortest distance from a point to a line.
- In **question 16** students determine the fourth vertex of a parallelogram. Since the parallelogram's vertices are named in order, there is only one possibility for vertex D. An extension would be to determine a fourth vertex using the given three vertices in any order.

Accommodations

Gifted and Enrichment—Challenge students to show more than one solution to a problem.

Visual—Encourage students to create graphs to understand and solve the problems in this section.

Memory—Remind students to solve problems in small sequential steps.

ESL—Let students use their dictionaries to understand the meanings of the new words in this section.

- **Questions 17**, **18**, and **22** have students determine the equation of a line. Refer students to **question 1** if they need help with this skill. If students need more assistance, the Get Ready section on finding the equation of a line would be useful.
- **Question 20** proves that the diagonals of a rectangle bisect each other, since they share the same midpoint.
- **Questions 23** through **25** involve finding the shortest distance from a point to a line in various applications.
- The use of technology is encouraged throughout the exercises to verify answers. See **questions 15**, **19**, **22**, and **26**.
- **Question 28** involves a fair bit of algebraic work, but is worthwhile to consolidate these skills.
- **Question 30** will appeal to stronger students who might appreciate the extension of the distance concept into three-dimensional space.
- **Question 31** is a multi-step problem that could be used as an assessment item. Students could work in pairs or small groups and use **A–17 Teamwork Self-Assessment**.

Achievement Check Sample Solution, question 27, page 91

Provide students with **BLM 2–7 Section 2.3 Achievement Check Rubric** to help them understand what is expected.

27. a) midpoint D of BC: $\left(\frac{-2+4}{2}, \frac{-3+2}{2}\right) = \left(1, -\frac{1}{2}\right)$

slope of the median: $\frac{-\frac{1}{2}-3}{1-(-7)} = -\frac{7}{16}$

The equation of the median is $y = -\frac{7}{16}x + b$. Substitute the coordinates of A to find b.

$3 = -\frac{7}{16}(-7) + b$

$b = -\frac{1}{16}$

The equation of the median is $y = -\frac{7}{16}x - \frac{1}{16}$.

b) slope of BC: $\frac{2-(-3)}{4-(-2)} = \frac{5}{6}$

The slope of the perpendicular bisector is $-\frac{6}{5}$.

The equation of the perpendicular bisector is $y = -\frac{6}{5}x + b$. Substitute the coordinates of A to find b.

$3 = -\frac{6}{5}(-7) + b$

$b = -\frac{27}{5}$

The equation of the perpendicular bisector is $y = -\frac{6}{5}x - \frac{27}{5}$.

c) The equations in parts a) and b) are different. This is also obvious from the diagram.

d) These two lines will coincide only when the sides adjacent to A are equal (i.e., the triangle is isosceles or equilateral).

Literacy Connections

Draw attention to the marginal definition on page 88. Be sure to point out the difference between the altitude and the right bisector of a triangle.

Question 28b) introduces a new word, "centroid."

Add "altitude" and "centroid" to the Word Wall.

Student Success

Use a placemat activity with a common problem in the centre for problem solving. Use **G–2 Placemat** to support this activity.

Use Think-Pair-Share to engage students in problem solving.

Refer to the introduction of this Teacher's Resource for more information about how to use placemat and Think-Pair-Share strategies.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	20, 21, 25, 30, 31–33
Reasoning and Proving	2, 3, 5–7, 9, 11–14, 20, 23–27, 29–31
Reflecting	10, 13, 16, 19, 22, 26, 27
Selecting Tools and Computational Strategies	6, 7, 11–14, 20, 23–25, 31
Connecting	1, 8, 10–14, 17, 18, 21, 23–25, 27, 28, 30, 31
Representing	3, 7, 10–26, 28–31
Communicating	6, 7, 19, 22, 24, 26, 29, 31

Ongoing Assessment

- Use Achievement Check question 27 to monitor student success. See Achievement Check Answers and **BLM 2–7 Section 2.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.