2.4 Equation for a Circle

Student Text Pages
92–99

Suggested Timing
40–70 min

Tools
- grid paper
- compasses

Technology Tools
- *The Geometer's Sketchpad®*
- computer

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 2–8 Section 2.4 Practice Master
- A–9 Communication General Scoring Rubric
- A–17 Teamwork Self-Assessment

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- Begin this section with a short discussion about two-way radios. (5 min)

Investigate

- The Investigate can be done using pencil and paper or technology. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity. Afterward, have a class discussion to consolidate the results. In the general case, the resulting equation for a circle should be $x^2 + y^2 = r^2$ or $r = \sqrt{x^2 + y^2}$. (15 min)

Examples

- Discuss **Examples 1** and **2**. (10 min)

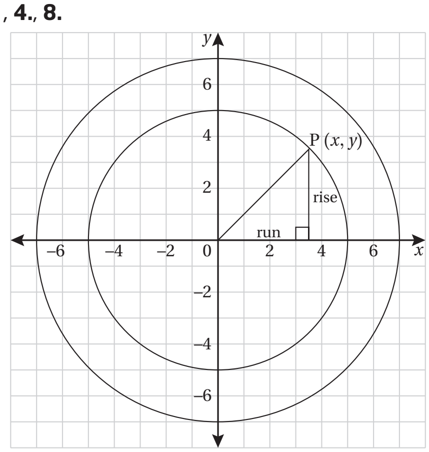
Communicate Your Understanding

- Discuss the **Communicate Your Understanding** questions. (5 min)
- Use **BLM 2–8 Section 2.4 Practice Master** for remediation or extra practice.

Investigate Answers (pages 92–94)

Method 1

1., 4., 8.



2. One coordinate is 0, while the other is either –5 or 5.

3. The other points with integer coordinates have –3 or 3 for one coordinate and –4 or 4 for the other.

5. $x^2 + y^2 = OP^2$; $x^2 + y^2 = 5^2$

7. Answers may vary. For example: Yes, every point on the circle is the same distance from the origin.

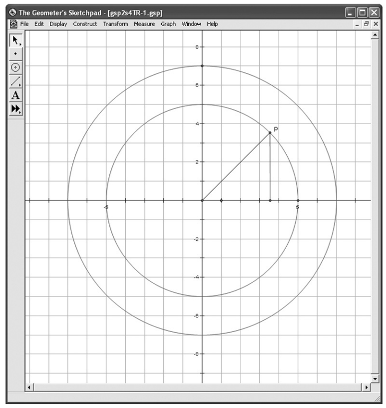
9. $x^2 + y^2 = 7^2$

10. $x^2 + y^2 = r^2$; $r = \sqrt{x^2 + y^2}$

Method 2

3. One coordinate is 0, while the other is either –5 or 5.

4. The other points with integer coordinates have –3 or 3 for one coordinate and –4 or 4 for the other.

6., 10. 

7. $x^2 + y^2 = OP^2$; $x^2 + y^2 = 5^2$

9. Answers may vary. For example: Yes, every point on the circle is the same distance from the origin.

11. $x^2 + y^2 = 7^2$

12. $x^2 + y^2 = r^2$; $r = \sqrt{x^2 + y^2}$

Communicate Your Understanding Responses (page 96)

C1. Substitute $r = 12$ into the formula $x^2 + y^2 = r^2$.

C2. Substitute $x = 3$ and $y = 5$ into the formula $x^2 + y^2 = 35$ to see if these coordinates satisfy the equation. If they do, the point is on the circle.

C3. Determine whether $x^2 + y^2 < 100$ when $x = 8$ and $y = 8$. If so, the point (8, 8) lies inside the circle.

Common Errors

- Some students may confuse r with r^2.

R_x Remind students that the radius of a circle is often a radical. In the case of an equation of the circle, r^2 will be a whole number. For example, if the radius of a circle is $\sqrt{34}$, the equation of the circle will be $x^2 + y^2 = 34$. Students need to have a clear understanding that $(\sqrt{34})^2 = 34$.

- When testing if a point lies on, inside, or outside a circle, students tend to use improper form, substituting the coordinates of the point and keeping the equality sign.

R_x Remind students that they are performing a "test" for which they are not sure of the outcome. Keep the left side and the right side of the equation separate. When the calculations are complete, then make a proper conclusion. If students use the form presented in Example 2, this could prevent such errors in mathematical form.

Practise

- **Question 5** is similar to **Example 2**, but in the context of an application. Students may need a quick review of scientific notation.
- **Question 6** is not difficult if students realize that the origin is the midpoint of the diameter. Reinforce the idea that the general equation $x^2 + y^2 = r^2$ or $r = \sqrt{x^2 + y^2}$ only applies when the circle is centred at the origin.
- **Question 7** provides a good opportunity to discuss the symmetry of a circle. If a point lies on a circle, the image of that point after a reflection in either axis also lies on the circle.
- **Question 8** uses the formulas for the circumference of a circle, $C = \pi d$ or $C = 2\pi r$, and area of a circle, $A = \pi r^2$.
- **Questions 9** through **11** pull together many of the skills developed in this chapter to investigate the properties of the perpendicular bisector of a chord. This line will always pass through the centre of the circle. **Question 12** has students use technology to examine this property.
- For **question 14**, use **A–9 Communication General Scoring Rubric** when assessing students.
- **Question 17** has students determine the radius of the circle by applying the rate of change that is provided. Part b) has them determine the radius from the point that is described. Students need to recognize that in completing this question they are assuming that the rate of change in the radius remains constant. Discuss whether this is a reasonable assumption and how the actual answer is likely to differ from their calculations.

Accommodations

Gifted and Enrichment—Challenge students to learn more about Personal Radio Communications Services.

Spatial—Allow students to use *The Geometer's Sketchpad*® to graph and find the equations of the circles in this section.

Language—Let students complete the questions in this section in the school's language lab, where questions will be read to the students.

Memory—Encourage students to create cue cards to remember the formulas, such as $s = \frac{d}{t}$.

Student Success

In a journal entry, have students explain how to derive the equation for a circle.

Use Circle Battleship to engage students.

Rules for Circle Battleship

1. Each student writes the equation for a circle with centre (0, 0) and constructs a graph on grid paper, on a graphing calculator, or by using GSP.
2. Students play the game in pairs.
3. Each student conceals his/her graph from the other student.
4. Students take turns calling out ordered pairs.
5. The other student replies "hit," "inside," or "outside" their circle.
6. The game ends when a student correctly guesses the equation of the other student's circle.

- **Question 18** introduces the idea of an inequality. This should follow easily after **Example 2** part b), in which students must determine if a point is inside or outside of a circle.
- **Question 19** applies the Pythagorean theorem to find the length of the diagonal of the square, which is the diameter of the larger circle.
- For **question 20**, students find the equation of two lines and then determine the point of intersection. To determine the area of the shaded region in part e), subtract the triangular areas from the quarter sector of the circle.
- **Question 22** considers a circle that is centred at a point other than the origin. Students can follow the same strategy as was used in the **Investigate**, using the point (4, 3) rather than the origin.
- **Question 23** is fairly time consuming, but will consolidate the skills of this chapter. Students could work in pairs or small groups and use **A–17 Teamwork Self-Assessment**.

Literacy Connections

Students should become familiar with the words "chord" (in question 9), "tangent" (in question 13), and "circumcentre" (in question 24).

Add "chord," "tangent," and "circumcentre" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	14, 20
Reasoning and Proving	4, 5, 7, 9–16, 18, 19, 21–26
Reflecting	3, 7, 17
Selecting Tools and Computational Strategies	4, 25, 26
Connecting	5, 8–11, 13–22
Representing	1, 3, 6, 7, 9–13, 17–21, 24
Communicating	7, 9, 13, 14, 17, 24, 26

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.