Student Text Pages
153

Suggested Timing
70–140 min

Tools
• ruler

Technology Tools
• Internet access

Related Resources
• BLM 3–11 Chapter 3 Problem
  Wrap-Up Rubric

Using the Chapter Problem

• If the Chapter Problem was introduced earlier, students might have already begun their research into the golden ratio. If the questions involving the golden triangle, golden rectangle, and golden spiral were not previously assigned, devote some class time to allow students the opportunity to begin their research in class.

• This problem can be assigned with variable timelines. With a list of references and Internet access, students could complete the answers in one or two periods. Additional time could be used for a presentation on research skills by the school's librarian. Ideally, give students a substantial period of time to complete their research and compile a complete set of responses with a one-week deadline.

• Students can use the Internet Link to find a list of resources that they could use to get started.

• This problem is quite open-ended. Students can provide simple solutions or very complex ones. Encourage students to reflect on their work and consider revising it to provide a more sophisticated set of solutions. This would be a good project to include in a portfolio. Students could also include some data that they obtained from personal measurements suggested by their research.

• Part a) can involve the use of the quadratic formula, which is not studied until Chapter 6. There are alternative explanations of the irrationality of phi that do not involve this mathematics but they are more challenging to understand. Present a mini-lesson on the quadratic formula before students start.

• Consider providing more specific directions to make the submissions more uniform. However, this may limit the creativity and breadth of research. Some students will be truly fascinated by the wealth of available material related to this topic.

• Provide sufficient time for the revision process. Students will create higher quality work if they have an opportunity for constructive feedback and time to incorporate suggestions and additional ideas into their project.

Summative Assessment

• Use **BLM 3–11 Chapter 3 Problem Wrap-Up Rubric** to assess student achievement.

Level 3 Sample Response

Answers will vary. Sample answers are provided.

a) The golden ratio, phi (φ), is not a rational number.

Phi is the positive solution to the quadratic equation $x^2 - x - 1 = 0$.

$$\varphi = \frac{1 + \sqrt{5}}{2}$$
$$\doteq 1.618\ 033\ 989$$

Since $\sqrt{5}$ is an irrational number, the expression $\frac{1 + \sqrt{5}}{2}$ is also irrational.

b) In a regular pentagon, the ratio of the lengths of any diagonal to any side is φ. In addition, if all of the diagonals of a regular pentagon are drawn and then the pentagon sides removed, the resulting figure is a five-pointed star, called a pentagram. The pentagram can also be constructed by starting with a regular pentagon and extending its sides until they intersect each other. In the pentagram each line is divided into three pieces. A number of the ratios formed from these pieces equal φ also.

c) The golden ratio appears to be related to many objects in nature, particularly through the Fibonacci sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, … . In the Fibonacci sequence, the ratios of successive terms get closer and closer to the golden ratio as the numbers increase. In nature, the Fibonacci sequence can be found in (students will provide one of the following ideas)
  – phyllotactic ratios (related to how many leaves grow on a growing branch)
  – the number of petals in flowers (most flowers have a Fibonacci number of petals)
  – pine cone spirals (the number of gradual spirals, and the number of steep spirals)
  – artichoke and pineapple spirals
  – sunflower head spirals
  – many artists have used golden ratio proportions in their masterpieces (e.g., da Vinci, Mondrian, Dali, etc.)
  – the piano keyboard has 13 keys in an octave, 8 white and 5 black, positioned in groups of 2 and 3—all Fibonacci numbers

Researchers may not agree that these are examples of the golden ratio because none of the examples is exact. All examples exhibit Fibonacci numbers but there are always exceptions. To confirm the golden ratio, researchers examine the limiting value of the ratios as the numbers get larger and larger. The numbers never become sufficiently large to satisfy some researchers' desire for exactness.

d) These claims seem valid. There are references to the golden ratio in early Greek writings, including Euclid's *Elements*, so we know that ancient architects were aware of the ratio. Researchers today have measured various dimensions of the Parthenon and the Great Pyramid of Giza and found many ratios close to the golden ratio. There is some controversy about whether these researchers chose appropriate places to make their measurements, since there are no records of the guiding principles used by these ancient architects.

e) The cover dimensions are 261 mm by 210 mm. The ratio of these dimensions is 1:24.

The dimensions of a two-page spread inside are 385 mm by 254 mm. This ratio is 1:29.

Neither of these ratios is particularly close to the golden ratio.

Level 3 Notes

Look for the following:
- Solutions for all parts of the problem
- Most facts supported by references to literature or Internet sites
- An explanation for part a) that may be missing details
- At least one correct example from nature for part c)
- Detailed justification for agreeing (or disagreeing) with the claim in part d)
- For part e), a decision that the golden ratio is not present in measurements of the cover of a book

What Distinguishes Level 2

At this level, look for the following:
- Solutions for most parts of the problem
- Few facts supported by references to literature or Internet sites
- An affirmative answer for part a) but either no explanation or one missing important details
- An answer from nature that is only partially clear or correct for part c)
- Partial justification for agreeing with the claim in part d)
- For part e), no decision about whether the golden ratio is involved in measurements of the cover of a book

What Distinguishes Level 4

At this level, look for the following:
- Complete solutions for all parts of the problem
- All facts supported by references to literature or Internet sites
- A complete explanation for part a), perhaps including reference to the quadratic formula, which will be studied later in the course
- Multiple correct examples from nature for part c)
- Detailed justification that includes arguments for both sides of the issue in part d)
- For part e), a decision that the golden ratio is not present in most measurements of a textbook (one ratio may be close; e.g., dimensions of a photo or some object in a photo)