3.1 Investigate Properties of Triangles

Student Text Pages
110–116

Suggested Timing
70–90 min

Tools
- ruler
- cardboard
- scissors
- compasses

Technology Tools
- *The Geometer's Sketchpad*®
- computer
- Cabri® Jr.
- graphing calculator

Related Resources
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 3–3 Section 3.1 Practice Master
- A–10 Observation General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- As an introduction, discuss the concept of centre of mass—the balance point of an object. For example, have at least one cardboard triangle ready and let students come up to the front of the class and try to balance the triangle on the tip of a pencil. (5 min)

Investigate

- Have students complete the **Investigate** using one of the three methods. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support Method 2 of this activity. As a class, discuss the results, making sure that students understand that the centre of mass is at the centroid of the triangle, which is the point where the medians intersect. The point is also called the centre of gravity. (15 min)

Example

- The **Example** can be discussed as a class or in small groups. (5 min)

Key Concepts

- Discuss the **Key Concepts**. (5 min)

Communicate Your Understanding

- Review the vocabulary in this section (concurrent and centroid) before discussing the **Communicate Your Understanding** questions. (5 min)
- Use **A–10 Observation General Scoring Rubric** at any point during this section to assist you in assessing students.
- Use **BLM 3–3 Section 3.1 Practice Master** for remediation or extra practice.

Investigate Answers (pages 110–112)

Method 1

3. Answers may vary. For example: The balance point is located on the fold line.

4. Answers may vary. For example: The two triangles formed by constructing the median have the same height and their bases are equal, since the median joins the vertex and the midpoint of the opposite side. Since the areas of the triangles formed are equal, the median bisects the area of the triangle.

5. The balance point is located at the point where the two medians intersect.

6. The creases from the three folds intersect at the same point. Answers will vary.

7. a) Yes. The medians of the three triangles are concurrent. Answers will vary.

b) The balance point is the point of intersection of the three medians of the triangle.

Method 2

1. Yes, the medians are concurrent.

3. Yes, every triangle has a centroid. Answers will vary.

4. The two triangles that have median AE as their base are equal in area. The altitudes of the two triangles are equal in length.

5. The two triangles that have median BF as their base are equal in area. The altitudes of the two triangles are equal in length. The two triangles that have median CD as their base are equal in area. The altitudes of the two triangles are equal in length.

6. No. The areas of the smaller triangles are equal and the lengths of the altitudes of the smaller triangles are equal.

7. a) The median of a triangle bisects the area of the triangle. Answers will vary.

b) Yes. The median of a triangle bisects the area of the triangle.

c) The balance point of a flat triangular object is located at the centroid, the point where the three medians of the triangle intersect.

Method 3

1. Yes, the medians of the triangle are concurrent.

3. Yes, every triangle has a centroid, a single point where all three medians intersect. Answers will vary.

4. The areas of the two small triangles are equal. The altitudes to the common base are equal in length.

5. The areas of the two triangles that have the second median as their base are equal in area and the altitudes to the common base of the two triangles are equal in length. The areas of the two triangles that have the third median as their base are equal in area and the altitudes to the common base of the two triangles are equal in length.

6. The areas of the smaller triangles are equal. The altitudes of the two triangles are equal in length.

7. a) The median of a triangle bisects the area of the triangle. Answers will vary.

b) Yes; the median of a triangle bisects the area of the triangle.

c) The balance point of a flat triangular object is located at the centroid, the point where the three medians of the triangle intersect.

Communicate Your Understanding Responses (page 113)

C1. Answers may vary. For example: Find the midpoint of a side by folding one vertex onto another. Then, construct a median by joining this midpoint to the opposite vertex. Alternatively, use compasses to find the midpoint of a side by constructing the right bisector of the side. Then, join the midpoint to the opposite vertex.

C2. Answers may vary. For example: PR = QR, PS = QS, and RS is common to $\triangle$PRS and $\triangle$QRS. Therefore, $\triangle$PRS and $\triangle$QRS are congruent (side-side-side) and the three pairs of corresponding angles are equal. Alternative method: RS is a median, so $\angle$PRS = $\angle$QRS. Since $\triangle$PQR is isosceles $\angle$P = $\angle$Q. Since two pairs of angles in $\triangle$PRS and $\triangle$QRS are equal, the third pair, $\angle$PSR and $\angle$QSR, are also equal.

Common Errors

- When students investigate properties of any triangle, they may unknowingly draw a special triangle such as an isosceles or an equilateral triangle.

R_x Encourage students to start with a triangle that is not special in any way if they are investigating whether a property is true for all triangles. Technology is a good way to overcome this tendency. It allows students to experiment with different triangles to determine whether a property is true in all cases.

Accommodations

Gifted and Enrichment—Challenge students to learn more about the golden ratio and the golden triangle.

Motor—Let students work in groups when completing the Investigate and the questions in this section.

Language—Provide students with opportunities to work with a reading buddy who will read the questions aloud to them.

ESL—Allow students to use their dictionaries or translators to understand the new words in this section.

Practise

- **Questions 1** and **2** use the fact that a median bisects the area of a triangle, one of the key concepts.
- **Question 3** and **4** are similar to the **Example** and emphasize the fact that the median to the base of an isosceles triangle is also the perpendicular bisector of the base, and it bisects the angle between the equal sides.
- **Question 10** investigates the property that the medians divide each other into a 2:1 ratio.
- **Question 11** revisits the concept of the line joining the midpoints of two sides of a triangle from Chapter 2. This line segment is parallel to the third side of the triangle and one half the length of that side.
- **Question 12** has students finding the incentre of a triangle. This can be repeated with technology in **question 13**.
- In **question 14**, students investigate the circumcentre. Students should realize that the circumcentre is equidistant from the three vertices of the triangle. Therefore in part c), the distribution centre could be located at the circumcentre of the triangle joining the three cities.
- **Question 16** is an open-ended task. Answers will vary depending on the shape of the triangle used.
- **Question 17** can be completed using technology.
- The Chapter Problem, **question 18**, is an introduction to the golden triangle. In part c), students calculate the ratio using the lengths they measured in part b). It should be approximately the same as the ratio given in part d), which is the exact golden ratio.
- **Questions 19** through **22** can be more easily investigated using technology. For example, in **question 19**, once the angle bisectors have been constructed, the vertices of the triangle can be dragged to alter the shape of the triangle. Acute, obtuse, and right triangles should all be considered in the investigations.

Literacy Connections

Note the definitions in the margin on page 111 for “concurrent” and “centroid.” Discuss the idea of concurrent in life (doing concurrent activities), and the mathematical meaning of meeting at one point. As an extension, have students investigate the centroid of other triangles.

Question 4 uses terminology from Chapter 2. Remind students of the meaning of the terms bisector, altitude, right bisector, and median.

In question 12b) students find the incentre of a triangle, in question 14a) they investigate whether every triangle has a circumcentre, and in question 15 they investigate whether every triangle has an orthocentre. As an extension, have students attempt to find circumcentres and orthocentres that are outside the triangle. This may lead to an interesting discussion of the angle needed to find particular properties.

Create a Word Wall for this chapter. Add “concurrent,” “centroid,” “incentre,” “circumcentre,” and “orthocentre.” From the Word Wall for Chapter 2, leave up “midpoint,” “median,” “equidistant,” and “right bisector.”

Student Success

Construct a Word Wall for properties of triangles.

Have students write a journal entry summarizing the properties of triangles.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 7, 9–12, 17, 21, 23
Reasoning and Proving	3–6, 8, 9, 12–16, 18–20, 22, 23
Reflecting	5, 16, 18
Selecting Tools and Computational Strategies	7, 10, 12, 14, 15, 17, 19–23
Connecting	10, 14, 16, 18
Representing	4–7, 9–13, 15, 16, 18–23
Communicating	7, 8, 10–22

Ongoing Assessment

- Chapter Problem question 18 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.