3.2 Verify Properties of Triangles

Student Text Pages
117–127

Suggested Timing
40–70 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computer
- Cabri® Jr.
- graphing calculator
- Internet access

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 3–4 Section 3.2 Practice Master
- BLM 3–5 Section 3.2 Achievement Check Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Start with a discussion about isosceles triangles. Ask students about the properties of an isosceles triangle. Most of the properties will be intuitive for the students. Ask students how they would go about verifying or proving those properties. Congruent triangles can be used to prove some of the properties. Analytic geometry can also be used to prove some of the properties. (5 min)

Investigate

- The **Investigate** can be done using a pencil-and-paper approach or by using technology. A computer with *The Geometer's Sketchpad*® or Cabri® Jr. on a graphing calculator can both be used effectively for this activity. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support Method 2 of this activity. (15 min)
- Encourage students to write a summary of their findings from the **Investigate**.

Examples

- Discuss the **Examples**. (15 min)
- **Example 1** verifies that the medians of a triangle divide each other in the ratio 2:1. This property will be useful in further questions.
- **Example 2** revisits the concept that the line segment joining the midpoints of the sides of a triangle is parallel to the third side and one half the length of the third side.

Communicate Your Understanding

- Review the vocabulary in this section (collinear) before discussing the **Communicate Your Understanding** questions with the class. (5 min)
- Use **BLM 3–4 Section 3.2 Practice Master** for remediation or extra practice.

Investigate Answers (pages 117–119)

Method 1

1. Since AC = BC, △ABC is isosceles. Also ∠CAB = ∠CBA (angles across from the equal sides).

2. Fold the triangle along the median from vertex C. If the two smaller triangles that are formed are congruent, then the triangle is an isosceles triangle.

3. The midpoint, D, of side AB is $\left(\frac{3}{2}, \frac{7}{2}\right)$. $m_{AB} = 3$, $m_{CD} = -\frac{1}{3}$.
Since $m_{AB} \times m_{CD} = -1$, CD is perpendicular to AB.

4. Answers may vary. For example: D is the midpoint of side AB in △ABC. In △CAD and △CBD, AC = BC (equal sides of an isosceles triangle), AD = BD (D is the midpoint of AB), and CD = CD (common side). Therefore, △CAD is congruent to △CBD (side-side-side). Also ∠CDA = ∠CDB (definition of congruence). ∠CDA + ∠CDB = ∠ADB (supplementary angles). Since ∠ADB = 180°, ∠CDA = ∠CDB = 90°. Therefore CD is perpendicular to AB. Answers will vary.

5. Since △CAD is congruent to △CBD, ∠ACD = ∠BCD (definition of congruence). Since ∠ACD + ∠BCD = ∠ACB, CD bisects ∠ACB.

6. Answers will vary.

7. At vertex C, the median, the altitude, and the angle bisector are the same line segment. The three line segments are also equal to the right bisector of AB. Answers may vary. For example: Yes, all isosceles triangles have these properties. The median of an isosceles triangle bisects the original triangle into two smaller congruent triangles.

Method 2

2. Since AC = BC, △ABC is isosceles. Also ∠CAB = ∠CBA (angles across from the equal sides).

3. The median CD from vertex C is perpendicular to side AB.

4. ∠ACD = ∠BCD.

6. At vertex C, the median, the altitude, and the angle bisector are the same line segment. The three line segments are also equal to the right bisector of AB. Answers will vary. For example: Yes, all isosceles triangles have these properties. The median of an isosceles triangle bisects the original triangle into two smaller congruent triangles.

Method 3

3. Since AC = BC, △ABC is isosceles. Also ∠CAB = ∠CBA (angles across from the equal sides).

4. DC is perpendicular to AB.

5. ∠ACD = ∠BCD.

7. At vertex C, the median, the altitude, and the angle bisector are the same line segment. The three line segments are also equal to the right bisector of AB. Answers will vary. For example: Yes, all isosceles triangles have these properties. The median of an isosceles triangle bisects the original triangle into two smaller congruent triangles.

Communicate Your Understanding Responses (pages 123–124)

C1. Answers may vary. For example: Use the length formula to verify that two of the sides have equal lengths. Use geometry software to plot the triangle and measure the sides.

C2. Use the midpoint formula to find the coordinates of the midpoint of each side. Use the midpoint and vertex coordinates to find equations for two of the medians. Solve this linear system to find the coordinates of the centroid. Then, use the length formula to compare the distances from the centroid to either end of each median.

C3. Answers will vary. For example: Plot the triangle and construct the midpoint of each side. Construct line segments joining each pair of midpoints. Measure and compare the slopes of these line segments and the slopes of the sides of △UVX.

Common Errors

- Some students may think that the properties that they prove for special triangles, such as an isosceles triangle, hold true for all triangles.

R_x Make sure students understand the differences between an altitude, an angle bisector, and a right bisector in all types of triangles. In an equilateral or isosceles triangle, all three can be the same line. However, this is not the case for other types of triangles.

Accommodations

Gifted and Enrichment—Challenge students to research on the Internet to learn more about Giovanni Ceva and to find out more about mathematicians who have published theorems about triangles.

Perceptual—Let students work in groups when completing the questions in this section.

Motor—Provide students with large sheets of grid paper that they can use for the questions that require the drawing of triangles.

Memory—Encourage students to create cue cards with the formulas for the slope of a line segment; the midpoint of a line segment; the equation of a line segment in the slope and *y*-intercept form; and diagrams of a scalene triangle, isosceles triangle, equilateral triangle, and right triangle.

Practise

- **Question 4** is similar to the **Investigate**.
- If technology was not used in the **Investigate**, have students use technology to complete **question 5**.
- **Question 6** uses slopes to verify that a triangle is a right triangle.
- When students graph the triangle in **question 7**, they see that it is a right triangle. This can be verified using slopes once again.
- **Questions 9** and **10** have students join the midpoints of sides of a triangle, similar to **Example 2**. **Question 11** uses technology to examine the same concept.
- **Question 13** requires students to use slope and distance to verify the properties of an isosceles right triangle.
- In **question 14** the right bisectors are easy to draw because of the position of the triangle on the grid. The right bisectors will intersect at the circumcentre of the triangle.
- **Question 15** examines the circumcentre in more detail.
- **Question 16**, the Chapter Problem, looks at the golden triangle by repeatedly bisecting the 72° angle to create the golden spiral. Making Connections on page 127 describes how to do this using *The Geometer's Sketchpad*®.
- **Question 18** is the proof of the general case corresponding to **Example 2**.
- **Question 19** requires students to draw a scalene triangle and use measurements to verify Ceva's theorem. This is another great opportunity to use technology.

Achievement Check Sample Solution, question 17, page 126

Provide students with **BLM 3–5 Section 3.2 Achievement Check Rubric** to help them understand what is expected. This is a challenging question. You could assign only parts a) and b), or perhaps only one part. Many students will have difficulty expressing their reasoning in a clear fashion, or will be unable to express why the calculations work. It may be helpful to provide grid paper for students to make a sketch, but this is not a substitute for the calculations.

17. a) $m_{AB} = \frac{5-1}{8-2} = \frac{4}{6} = \frac{2}{3}$ $m_{BC} = \frac{-1-5}{-1-8} = \frac{-6}{-9} = \frac{2}{3}$ $m_{AC} = \frac{-1-1}{-1-2} = \frac{-2}{-3} = \frac{2}{3}$

Note: It is sufficient to calculate only two of the above slopes since each pair of segments have a point in common.

b) First find the equation of line AB. Use its slope, $\frac{2}{3}$, and a point it passes through, (2, 1).

$$y = mx + b$$
$$1 = \frac{2}{3}(2) + b$$
$$-\frac{1}{3} = b$$

The equation for line AB is $y = \frac{2}{3}x - \frac{1}{3}$.

Now substitute the coordinates of point C(–1, –1) into this equation.

L.S. $= y = -1$ R.S. $= \frac{2}{3}x - \frac{1}{3} = \frac{2}{3}(-1) - \frac{1}{3} = -1$

Therefore point C is on line AB and A, B, and C are collinear.

c) $AB = \sqrt{(5-1)^2 + (8-2)^2} = \sqrt{52} = 2\sqrt{13}$ $BC = \sqrt{(-1-5)^2 + (-1-8)^2} = \sqrt{117} = 3\sqrt{13}$ $AC = \sqrt{(-1-1)^2 + (-1-2)^2} = \sqrt{13}$

Since AB + AC = BC, △ABC has no area and A, B, and C are collinear.

Literacy Connections

Note the marginal definition on page 121. "Collinear" is a word the students have encountered before, but remind them of the meaning.

Draw attention to the word "cevian" in question 19. This type of line segment is named after Giovanni Ceva. If you discussed the Cartesian plane in Chapter 2 and its relation to René Descartes, have students keep a list of items that are named after famous mathematicians.

Add "collinear" and "cevian" to the Word Wall.

Student Success

Use a jigsaw approach to have students verify different properties of triangles and share with their home group.

Use a placemat activity to have students verify the same properties of triangles. Use **G–2 Placemat** to support this activity.

Refer to the introduction of this Teacher's Resource for more information about how to use jigsaw and placemat strategies.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	18, 20, 22
Reasoning and Proving	2–4, 6, 9, 10, 13–17, 19–21
Reflecting	5, 8, 11, 15
Selecting Tools and Computational Strategies	2, 4, 7, 13–15
Connecting	7, 12, 17
Representing	5, 7–12, 14, 16, 18, 19
Communicating	6, 12–14, 16, 19

Ongoing Assessment

- Use Achievement Check question 17 to monitor student success. See Achievement Check Answers and **BLM 3–5 Section 3.2 Achievement Check Rubric**.
- Chapter Problem question 16 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.