3.3 Investigate Properties of Quadrilaterals

Student Text Pages
128–136

Suggested Timing
60–75 min

Tools
- grid paper
- ruler
- compasses
- protractor

Technology Tools
- *The Geometer's Sketchpad*®
- computer
- Cabri® Jr.
- graphing calculator
- Internet access

Related Resources
- G–1 Grid Paper
- G–4 Protractor
- T–4 *The Geometer's Sketchpad*® 3
- T–5 *The Geometer's Sketchpad*® 4
- BLM 3–6 Section 3.3 Practice Master
- BLM 3–7 Section 3.3 Achievement Check Rubric
- A–10 Observation General Scoring Rubric
- A–22 Report Checklist

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Use the photo in the introduction to initiate a discussion about parallelograms.

Investigate

- The **Investigate** can be done using pencil and paper or technology. Use **T–4** ***The Geometer's Sketchpad*® 3** or **T–5** ***The Geometer's Sketchpad*® 4** to support Method 2 of this activity. Have a class discussion to consolidate the idea that the quadrilateral formed by joining the midpoints of the sides of a parallelogram is a parallelogram. (15 min)

Examples

- Discuss the **Examples**. (30 min)
- Ask students if they think the figure formed by joining the midpoints of the sides of any quadrilateral will have special properties. This is a natural extension of the **Investigate** and leads into **Example 1**.
- **Example 2** has students join the midpoints of the non-parallel sides of a trapezoid. This can be done by using analytic geometry or by paper folding. It is important that the students be exposed to a variety of techniques.

Communicate Your Understanding

- Discuss the **Communicate Your Understanding** questions. (5 min)
- Use **A–10 Observation General Scoring Rubric** at any point during this section to assist you in assessing students.
- Use **BLM 3–6 Section 3.3 Practice Master** for remediation or extra practice.

Investigate Answers (pages 128–131)

Method 1

1. Answers will vary.
2. Answers may vary. For example: The point where the diagonals intersect appears to divide each diagonal in half.
3. Conjecture: The point of intersection of the diagonals divides each diagonal of the parallelogram in half and is located at the midpoint of each diagonal.
4. Answers will vary.
5. The point of intersection of the diagonals divides each diagonal of the parallelogram in half and is located at the midpoint of each diagonal. Answers may vary. For example: Yes, this conclusion applies to all parallelograms. Answers will vary.
6. Answers will vary. For example: The new quadrilateral appears to have sides that are parallel and equal in length.
7. Conjecture: The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram.
8. Answers will vary.
9. The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram. Answers may vary.

Method 2

5. Answers may vary. For example: Quadrilateral ABCD is a parallelogram because the opposite sides have been constructed to be parallel and equal in length.

6. The length of CE equals the length of AE, and the length of DE equals the length of BE.

7. Yes, ABCD remains a parallelogram. The length of CE equals the length of AE, and the length of DE equals the length of BE.

8. The diagonals of a parallelogram intersect at the midpoint of each diagonal.

9. Answers may vary. For example: The quadrilateral formed by joining the midpoints of the adjacent sides of the parallelogram appears to have sides that are parallel and equal in length.

10. Conjecture: The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram.

11. Answers will vary.

12. No, the relationships among the measurements do not change.

13. The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram.

Method 3

5. The length of CE equals the length of AE, and the length of DE equals the length of BE.

6. Yes, ABCD remains a parallelogram. The length of CE equals the length of AE, and the length of DE equals the length of BE.

7. The point of intersection of the diagonals divides each diagonal of the parallelogram in half and is located at the midpoint of each diagonal.

9. Answers will vary. For example: The new quadrilateral appears to have sides that are parallel and equal in length.

10. Conjecture: The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram.

11. Answers will vary.

12. Answers may vary. For example: The lengths of the sides for quadrilateral FGHI change, but the relationships among the measurements do not change. The opposite sides of quadrilateral FGHI remain parallel and equal in length.

13. The quadrilateral formed by joining the midpoints of adjacent sides of a parallelogram is a parallelogram.

Communicate Your Understanding Responses (page 134)

C1. Answers may vary. For example: Fold opposite vertices onto each other to show the diagonals bisect each other.

C2. Answers may vary. For example: Construct a rectangle, its diagonals, and their point of intersection. Measure and compare the lengths of the diagonals. Also, compare the lengths of the line segments from each vertex to the point of intersection of the diagonals.

Practise

- Choose questions that will suit different methods so that students come to appreciate the variety of approaches.
- **Question 1** reinforces the concept that the diagonals of a parallelogram bisect each other.
- **Questions 2** and **3** use the concept from **Example 1**: the quadrilateral formed by joining the midpoints of any quadrilateral is a parallelogram. Students are required to pick out segments in the diagram that are equal and parallel.

Common Errors

- When investigating the properties of any quadrilateral, students tend to consider special quadrilaterals, such as a square.

R_x Encourage students to begin with a quadrilateral that is not special in any way. For a general quadrilateral, draw a diagram where no sides are equal or parallel. This should prevent students from arriving at inappropriate conclusions.

Accommodations

Gifted and Enrichment—Challenge students to research and create new tiling patterns.

Spatial—Let students work with a partner to complete questions that require drawing diagrams.

Motor—Provide students with opportunities to give oral responses to the questions in this section.

Language—Allow students to complete the questions in the Language Lab, where the questions are read to them.

- **Questions 4** and **5** are a follow-up to **Example 2**. Parts a) and b) of **question 4** use the fact that the line segment joining the midpoints of the non-parallel sides of a trapezoid is parallel to the other two sides and has a length that is the mean of the lengths of the parallel sides. For **question 4** part c), the Pythagorean theorem must be applied.
- **Question 6** is a special case of the **Investigate**. Most students will have an intuitive understanding that the diagonals of a square are equal in length and intersect each other at right angles. Paper folding is a quick and effective way of allowing students to see these properties. Students could verify these properties by drawing a square on a grid and using analytic geometry skills.
- **Question 7** asks how software could be used, but you can have students actually do this investigation. Have students hypothesize about the results, and then determine if their hypothesis is true.
- In **question 8**, students construct a rhombus by starting with the diagonals and forming the rhombus around these line segments. Joining the midpoints of the rhombus will create a square.
- **Question 10** requires students to realize that the diagonals of a rectangle, and of a square, bisect each other and are equal in length.
- **Question 11** leads students to realize that the diagonals of a rectangle are equal in length and bisect each other. Since a rhombus has the same properties as a kite and a parallelogram, students see that the diagonals bisect each other and meet at a right angle.
- **Question 12** revisits the concept of the centre of mass from Section 3.1. Students may intuitively think of the point of intersection of the diagonals of a rectangle as the centre of the rectangle. Students can demonstrate the balance point for a rectangle by balancing a cardboard rectangle on the tip of a pencil.
- **Question 13** revisits question 16 from Section 3.1, finding the centre of Canada. If that question was assigned earlier, students can compare the results of the two different approaches.
- **Question 15** has students look at all cases of the Varignon parallelogram. Encourage students to summarize their findings in a report. Use **A–22 Report Checklist** when assessing students.
- **Question 18**, which is about Penrose tilings, could be assigned as a project.

Achievement Check Sample Solution, question 14, page 136

Provide students with **BLM 3–7 Section 3.3 Achievement Check Rubric** to help them understand what is expected.

14. a) Label the midpoints of sides JM, ML, LK, and KJ as A, B, C, and D respectively.

$$\begin{aligned} A(x, y) &= \left(\frac{-2+4}{2}, \frac{3+1}{2}\right) \\ &= (1, 2) \end{aligned}$$

Similarly, the other midpoints are B(5, –1), C(1, –4), and D(–3,–1).

To determine if it is a rhombus, check the lengths of the sides of quadrilateral ABCD.

$$\begin{aligned} AB &= \sqrt{(5-1)^2+(-1-2)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned} \qquad \begin{aligned} BC &= \sqrt{(1-5)^2+(-4-(-1))^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} CD &= \sqrt{(-3-1)^2+(-1-(-4))^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned} \qquad \begin{aligned} DA &= \sqrt{(1-(-3))^2+(2-(-1))^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

Since all sides are equal length, the quadrilateral is a rhombus.

Student Success

Use Frayer models to build a library of properties of quadrilaterals.

Have students write a journal entry summarizing the properties of quadrilaterals.

Refer to the introduction of this Teacher's Resource for more information about how to use a Frayer model strategy.

b) To show that the diagonals bisect each other, find the midpoints of each diagonal.

Midpoint AC: $\left(\frac{1+1}{2}, \frac{2+(-4)}{2}\right) = (1, -1)$

Midpoint BD: $\left(\frac{5+(-3)}{2}, \frac{-1+(-1)}{2}\right) = (1, -1)$

Since the midpoints are identical, the diagonals bisect each other.

Use slopes to check if the diagonals meet at right angles.

$m_{AC} = \frac{-4-2}{1-1}$ $= \frac{-6}{0}$ $=$ undefined

$m_{BD} = \frac{-1-(-1)}{-3-5}$ $= \frac{0}{-8}$ $= 0$

This shows that AC is vertical and BD is horizontal. So, these diagonals are perpendicular.

Literacy Connections

Draw attention to the Did You Know? on page 132. Have students search the Internet for other items dealt with by Varignon. Question 18 introduces Penrose tiles. Have students add these to the list of items that are named after famous mathematicians.

Add "Varignon parallelogram" and "Penrose tiles" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	6, 9, 11, 15, 18
Reasoning and Proving	1–5, 8–19
Reflecting	13
Selecting Tools and Computational Strategies	4, 14, 18
Connecting	4, 9, 12–14, 16–18
Representing	8, 10, 13, 16
Communicating	6–13, 15–18

Ongoing Assessment

- Use Achievement Check question 14 to monitor student success. See Achievement Check Answers and **BLM 3–7 Section 3.3 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.