3.4 Verify Properties of Quadrilaterals

Student Text Pages
137–144

Suggested Timing
40–70 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computer
- Cabri® Jr.
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 3–8 Section 3.4 Practice Master

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Discuss the difference between investigating a property and verifying that a property is true in a particular case. In Section 3.3, students investigated the properties of quadrilaterals. In this section, students use their analytic geometry skills to verify similar properties of quadrilaterals in specific cases.
- The photo in the introduction could be used as a lead-in to a discussion about the properties of a parallelogram and how to verify those properties.

Investigate

- The **Investigate** can be done with or without technology. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support Method 2 of this activity. (15 min)

Examples

- Present the **Examples**. (20 min)
- **Example 1** revisits the Varignon parallelogram, proving its properties in a particular case.
- **Example 2** proves the properties for a rhombus that were investigated in the previous section. The example uses analytical geometry skills to verify those properties for a particular case, which should help students consolidate their understanding.

Communicate Your Understanding

- Discuss the **Communicate Your Understanding** questions. (5 min)
- Use **BLM 3–8 Section 3.4 Practice Master** for remediation or extra practice.

Investigate Answers (pages 137–139)

Method 1

1. $m_{AB} = -4$, $m_{BC} = \frac{2}{5}$, $m_{CD} = -4$, and $m_{AD} = \frac{2}{5}$. Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, quadrilateral ABCD is a parallelogram.
2. AB = $\sqrt{17}$, BC = $\sqrt{29}$, CD = $\sqrt{17}$, and AD = $\sqrt{29}$. Since AB = CD and BC = AD, quadrilateral ABCD is a parallelogram.
3. Answers may vary. For example: If ∠DAB = ∠BCD, and ∠CDA = ∠ABC, then quadrilateral ABCD is a parallelogram.
4. Answers may vary. For example: If the slopes of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram. If the lengths of opposite sides of a quadrilateral are equal, then it is a parallelogram. If the opposite angles of a quadrilateral are equal in measure, then the quadrilateral is a parallelogram.
5. The midpoint of AC is M(1, 0). The midpoint of BD is M(1, 0). Since the midpoint of AC is the same as the midpoint of BD, the diagonals of ABCD bisect each other.
6. Answers may vary. For example: The midpoint of the diagonals of ABCD is the point M. If △MAD and △MCB are congruent, then MA = MC, MD = MB, and the diagonals of ABCD bisect each other.
7. Yes, all quadrilaterals with diagonals that bisect each other are parallelograms. Answers will vary.

Method 2

2. Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, quadrilateral ABCD is a parallelogram.
3. Since AB = CD and BC = AD, quadrilateral ABCD is a parallelogram.
4. Answers may vary. For example: If ∠DAB = ∠BCD, and ∠CDA = ∠ABC, then quadrilateral ABCD is a parallelogram.
5. Answers may vary. For example: If the slopes of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram. If the lengths of opposite sides of a quadrilateral are equal, then it is a parallelogram. If the opposite angles of a quadrilateral are equal in measure, then the quadrilateral is a parallelogram.
6. The midpoint of AC is M(1, 0). The midpoint of BD is M(1, 0). Since the midpoint of AC is the same as the midpoint of BD, the diagonals of ABCD bisect each other.
7. Yes, all quadrilaterals with diagonals that bisect each other are parallelograms. Answers will vary.

Method 3

3. Since $m_{AB} = m_{CD}$ and $m_{BC} = m_{AD}$, quadrilateral ABCD is a parallelogram.
4. Since AB = CD and BC = AD, quadrilateral ABCD is a parallelogram.
5. Answers may vary. For example: If ∠DAB = ∠BCD, and ∠CDA = ∠ABC, then quadrilateral ABCD is a parallelogram.
6. Answers may vary. For example: If the slopes of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram. If the lengths of opposite sides of a quadrilateral are equal, then it is a parallelogram. If the opposite angles of a quadrilateral are equal in measure, then the quadrilateral is a parallelogram.
7. The midpoint of AC is M(1, 0). The midpoint of BD is M(1, 0). Since the midpoint of AC is the same as the midpoint of BD, the diagonals of ABCD bisect each other.
8. Yes, all quadrilaterals with diagonals that bisect each other are parallelograms. Answers will vary.

Communicate Your Understanding Responses (page 142)

C1. Answers may vary. For example: Calculate and compare slopes to check if a pair of sides are parallel.

C2. Answers may vary. For example: Find the point of intersection of the diagonals. Compare the lengths from each vertex to the midpoint and check to see that the lengths are equal for only one of the diagonals.

C3. Answers may vary. For example: Calculate and compare slopes of each side to check if there are two pairs of parallel sides, or check and compare the lengths of the opposite sides to determine if opposite sides have equal length. The slope calculations are simpler.

Common Errors

- Some students may need help in presenting clear solutions to multi-step problems.

R_x Encourage students to describe, in words, what they are doing at each step of their verification. For example: "To show that the sides are parallel, consider the slopes." Follow this with the slope calculations and the appropriate conclusion. Emphasize that descriptions of this type make it easier for another person to follow their reasoning and consolidates their own problem solving process. This is all part of clear mathematical communication.

Accommodations

Gifted and Enrichment—Challenge students to use the Internet to research the golden rectangle and its use in architecture and art.

Motor—Provide students with enlarged photocopies of the graphs from the questions in this section.

Memory—Encourage the students to complete the questions in this section in small sequential steps.

ESL—Let students use their dictionaries to understand the new words in this section.

Student Success

Use a jigsaw approach to have students verify different properties of quadrilaterals and share with their home group.

Use a placemat activity to have students verify the same properties of quadrilaterals. Use **G–2 Placemat** to support this activity.

Refer to the introduction of this Teacher's Resource for more information about how to use jigsaw and placemat strategies.

Practise

- **Question 1** is similar to **Example 1** in that students use slopes to prove that one pair of sides is parallel.
- In **question 4**, students must show that the slopes of adjacent sides are negative reciprocals of each other. For part b), refer students to **Example 2**.
- **Questions 5** and **6** look at the Varignon parallelogram, with and without technology.
- In **questions 7** and **8**, students verify the properties of the line segment joining the midpoints of the non-parallel sides of a trapezoid, with and without technology.
- **Question 10** proves a property of a kite and is a good follow-up to question 11b) in Section 3.3.
- **Question 14** verifies that the Varignon parallelogram inside a rhombus is a rectangle. Students worked with the Varignon parallelograms in Section 3.3, question 15.
- The Chapter Problem, **question 16**, looks at the golden rectangle, which is an extension to the golden triangle.
- **Question 17** proves the general case for the Varignon parallelogram.

Literacy Connections

Draw attention to the use of the nested rectangles in the Chapter Problem, question 16f). When two things are nested, it means that one is completely inside the other.

Add "nested" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	17–19
Reasoning and Proving	1–5, 7, 9–12, 14, 17, 18
Reflecting	6, 8, 13, 15, 16
Selecting Tools and Computational Strategies	1–4, 9–11, 18
Connecting	16, 18
Representing	5–9, 11–17
Communicating	6, 8, 9, 11–13, 15, 16

Ongoing Assessment

- Chapter Problem question 16 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.