3.5 Properties of Circles

Teaching Suggestions
- Have a short class discussion about circles. Ask students to list as many properties as they can. (2 min)

Investigate
- The **Investigate** can be done with or without technology. Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support Method 2 of this activity. (10 min)
- Discuss the fact that the perpendicular bisector of any chord passes through the centre of a circle. Students may need help to determine how this property can be used to locate the centre of a circle given three points.

Examples
- Discuss the **Examples**. (20 min)
- **Example 1** shows how analytical geometry skills can be used to verify this property.
- In **Example 2**, the distance formula is used to verify that the given point is the centre of the circle passing through the three given points. In part b), the fact that there is only one circle that will pass through three given points is clarified.

Communicate Your Understanding
- Review the vocabulary in this section (chord) before discussing the **Communicate Your Understanding** questions. Have students do some paper folding to further their understanding of these concepts. (5 min)
- Use **BLM 3–9 Section 3.5 Practice Master** for remediation or extra practice.

Investigate Answers (pages 145–146)

Method 1

2. Fold the circle in half to create two diagonals for the circle. The point of intersection of the fold lines of these diagonals is the centre of the circle.

3. Answers may vary. For example: The diameter of a circle is a chord of the circle that contains the centre of the circle.

4. The right bisector of a chord of a circle goes through the centre of the circle. Answers will vary.

6. Answers may vary. For example: Construct the right bisector of the chord PQ and the right bisector of chord QR. The point of intersection of the two right bisectors is the centre of the circle that goes through the points P, Q, and R.

7. Answers may vary. For example: The right bisectors of the chords of a circle intersect at the centre of the circle.

Method 2

4. The right bisector goes through the centre of the circle.
5. Answers may vary. For example: The right bisectors of the chords of a circle intersect at the centre of the circle. Construct the right bisector of the chord connecting the first and second points on the circumference of the circle and the right bisector of the chord connecting the second and third points on the circumference of the circle. The point of intersection of the two right bisectors is the centre of the circle that goes through the three points on the circumference.

Method 3

4. The right bisector goes through the centre of the circle.
5. Answers may vary. For example: The right bisectors of the chords of a circle intersect at the centre of the circle. Construct the right bisector of the chord connecting the first and second points on the circumference of the circle and the right bisector of the chord connecting the second and third points on the circumference of the circle. The point of intersection of the two right bisectors is the centre of the circle that goes through the three points on the circumference.

Communicate Your Understanding Responses (page 149)

C1. Answers may vary. For example:
   a) Fold the paper so that point A is on top of point B, and again so that point C is on point B. The intersection of the two fold lines is the location of the point equidistant to all three points.
   b) Make two chords joining point A to point B and point B to point C. Construct the perpendicular bisector for each chord. The point of intersection of the two bisectors is equidistant from all points A, B, and C.
   c) Find an equation for the right bisectors of AB and BC. Solve these two equations as a linear system to find the coordinates of the point of intersection of the right bisectors.

C2. Answers may vary. For example: Find the point of intersection of the right bisectors of any two chords of circular object.

Practise

• In **question 4**, students can show that the centre is equidistant from the endpoints of the chord. Any point that is equidistant from the endpoints must lie on the perpendicular bisector of that line segment. Alternatively, students can determine the equation of the perpendicular bisector of the chord and then show that the origin satisfies this equation and therefore lies on this line.
• **Question 7** leads students to realize that a circle encloses the maximum area for a given perimeter.
• In **question 8**, students determine the equation of two of the perpendicular bisectors of the chords joining two pairs of the given points. This question requires quite a lengthy solution. Use **A–7 Thinking General Scoring Rubric** when assessing students for this question. Have students work in pairs when completing a question of this type for the first time.
• **Question 9** uses paper folding to determine the centre of a circle.
• **Question 11** is similar to **question 8**, but posed in the form of an application question.
• **Question 12** requires students to realize that the triangles are congruent, further consolidating the idea that the centre lies on the perpendicular bisector of a chord of the circle.
• In **question 13**, join L to the centre of the circle (the midpoint of JK) to form two isosceles triangles. Students may need help with labelling the angles.

Visual—Let students use paper folding to check their answers to the questions that require them to determine the centre of a circle.

Perceptual—Provide students with cue cards showing the completed steps for the solution to a question, and have the students arrange the steps in the correct order.

Language—Allow students to work in groups when completing the questions in this section.

Memory—Encourage students with visual cues and prompts when they work through the questions in this section.

Student Success

Have students write a journal entry summarizing the properties of circles.

Use a graffiti strategy to have groups of students verify properties of circles.

Refer to the introduction of this Teacher's Resource for more information about how to use a graffiti strategy.

• **Question 14** has students use technology to verify the results of **question 13**. This is a good question for those students who had trouble with the abstract nature of **question 13**. By measuring the angle at the circumference, they can verify that it is 90°. Point L can be moved along the circumference of the circle so students see that the angle remains 90° in all cases.
• **Question 15** is an extension to **question 11**.
• **Question 16** is a application that is somewhat open-ended, and allows for different student responses.

Literacy Connections

Note the marginal definition on page 145. Students have encountered "chord" before, but remind them of the meaning.

Add "chord" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	8, 14, 16–19
Reasoning and Proving	1–9, 11–14, 16–18
Reflecting	10, 15, 16
Selecting Tools and Computational Strategies	1, 5, 8, 9, 16, 18
Connecting	7, 8, 11–14, 16, 17
Representing	2, 3, 9, 10, 14, 15, 17
Communicating	6, 7, 10–13, 16, 17

Ongoing Assessment

• Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.