Task Multiple Midpoints

Student Text Pages
158

Suggested Timing
25–35 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computer
- Cabri® Jr.
- graphing calculator

Related Resources
- G–1 Grid Paper
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 3–15 Task: Multiple Midpoints Rubric

Accommodations

Visual—Encourage the students to create several sets of midpoints with the same and similar characteristics as the given midpoints, and to look for patterns in the vertices of the triangles.

Perceptual—Allow students to work in groups.

Motor—Let students use sheets of grid paper that have been photocopied and enlarged when they work on this Task.

Memory—Provide students with visual and verbal prompts for the formulas for the midpoints of lines and the slopes of lines.

Specific Expectations

Analytic Geometry

Using Linear Systems to Solve Problems

AG1.01 solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination.

Solving Problems Involving Properties of Line Segments

AG2.01 develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software);

AG2.05 solve problems involving the slope, length, and midpoint of a line segment (e.g.,determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Using Analytic Geometry to Verify Geometric Properties

AG3.01 determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle);

AG3.02 verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices);

AG3.03 plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Teaching Suggestions

- Students may need some prompting. Ask students what they know about triangles and the midpoints of their sides, which should lead to the idea that the line joining midpoints is parallel to the third side.
- Some students may try a solution using only grid paper or ruler and compasses. This should be encouraged, as the algebraic solution is not the intended goal for everyone. Other students may have sufficient skill with *The Geometer's Sketchpad®* to attempt solutions using that software. If needed, use **T–4 *The Geometer's Sketchpad®* 3** or **T–5 *The Geometer's Sketchpad®* 4** to support this activity.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:

- Knowledge of midpoints, slope, line equation, and solving systems of equations
- Ability to construct context-specific diagrams

- Evidence of ability to connect the problem context to the underlying mathematical ideas
- Evidence of communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

- Use **BLM 3–15 Task: Multiple Midpoints Rubric** to assess student achievement.

Level 3 Sample Response

a) Recall that the line joining midpoints of two sides of a triangle is parallel to the third side of the triangle (and half its length).

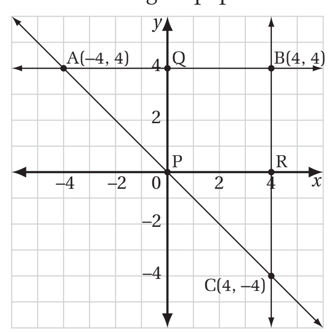
If Q is the midpoint of AB, Q and AB are on the horizontal line $y = 4$, since PR is parallel to AB and PR is the x-axis.

If R is the midpoint of BC, R and BC are on the vertical line $x = 4$, since QP is parallel to BC and QP is the y-axis.

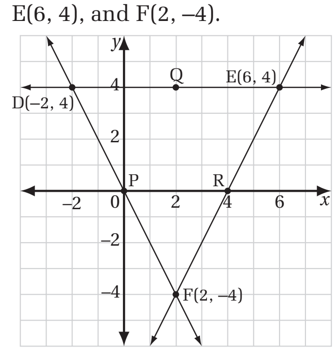
If P is the midpoint of AC, P and AC are on the line $y = -x$, since QR is parallel to AC where QR has slope –1.

Solving each pair of equations using substitution gives the following coordinates of the vertices of △ABC: A(–4, 4), B(4, 4), C(4, –4).

This could also be easily solved using the above ideas, but plotting the points and lines on grid paper.



b) This can also be solved by graphing. The vertices of this △DEF are D(–2, 4), E(6, 4), and F(2, –4).



c) Answers will vary.

d) Students are not expected to carry through the algebra related to this process, but they should be able to describe the steps.

Find the slope of QR: $\dfrac{q-m}{p-k}$.

Find the equation of the line through P, parallel to QR: $y = \left(\dfrac{q-m}{p-k}\right)x$. ①

Find the slope of PQ: $\dfrac{m}{k}$.

Find the equation of the line through R, parallel to PQ: $y = \dfrac{m}{k}x + (q - \dfrac{mp}{k})$. ②

Lastly, find the slope of PR: $\dfrac{q}{p}$.

Find the equation of the line through Q, parallel to PR: $y = \dfrac{q}{p}x + (m - \dfrac{qk}{p})$. ③

The intersection of ① and ② is L($p-k$, $q-m$).
The intersection of ① and ③ is J($k-p$, $m-q$).
The intersection of ② and ③ is K($k+p$, $m+q$).

e) This procedure is always possible provided that the three points are not collinear. The solution in part d) is actually a general one even though it appears that one of the given midpoints is the origin. To see why this is so, suppose none of the midpoints is at the origin. Pick one of the midpoints; translate all three midpoints so this first one is now at the origin; find the triangle using the formulas of part d); and translate the triangle by a translation that is the opposite of the first translation. The triangle that you now have is the required triangle. This translation technique is useful in many other situations in analytic geometry. Clearly, it helps to make algebraic calculations much simpler.

Level 3 Notes

Look for the following:
- Reasonably complete solutions to parts a), b), and c)
- Sufficiently well organized work that allows one to follow the intent of the steps
- Use of good form and correct mathematical notation most of the time
- Clear diagrams with solutions
- Comment on the generality of the method in parts d) and e), but without much reasoning

What Distinguishes Level 2

At this level, look for the following:
- Attempted solutions to parts a) and b), but may have gaps
- Poorly organized work that does not always allow one to follow the intent of the steps
- Inconsistent use of good form and correct mathematical notation
- Incomplete or poorly related diagrams with solutions
- Lack of comment on the generality of the method in parts d) and e)

What Distinguishes Level 4

At this level, look for the following:
- Complete solutions to all parts of the question, including the general case
- Clear and complete steps on how to complete the analytic geometry solutions
- Use of very good form and correct mathematical notation
- Clear, well-labelled diagrams that support the narrative of solutions
- Comment on the generality of the method in parts d) and e), with some logical reasoning

Task A Site for the New Hospital

Student Text Pages
158

Suggested Timing
70 min

Tools
- grid paper

Technology Tools
- *The Geometer's Sketchpad®*
- computer

Related Resources
- G–1 Grid Paper
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 3–16 Task: A Site for the New Hospital Rubric

Accommodations

Visual—Encourage students to use a calculator to find the distances between the towns.

Perceptual—Provide students with a photocopy of the diagram for the Task.

Motor—Allow students to work with a partner when using *The Geometer's Sketchpad®*.

Language—Let students use a computer to prepare a letter to the area's planning council describing how to find a good location.

Specific Expectations

Analytic Geometry

Using Linear Systems to Solve Problems

AG1.01 solve systems of two linear equations involving two variables, using the algebraic method of substitution or elimination;

AG1.02 solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method.

Solving Problems Involving Properties of Line Segments

AG2.01 develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software);

AG2.02 develop the formula for the length of a line segment, and use this formula to solve problems (e.g., determine the lengths of the line segments joining the midpoints of the sides of a triangle, given the coordinates of the vertices of the triangle, and verify using dynamic geometry software);

AG2.05 solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Using Analytic Geometry to Verify Geometric Properties

AG3.01 determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle);

AG3.02 verify, using algebraic techniques and analytic geometry, some characteristics of geometric figures (e.g., verify that two lines are perpendicular, given the coordinates of two points on each line; verify, by determining side length, that a triangle is equilateral, given the coordinates of the vertices);

AG3.03 plan and implement a multi-step strategy that uses analytic geometry and algebraic techniques to verify a geometric property (e.g., given the coordinates of the vertices of a triangle, verify that the line segment joining the midpoints of two sides of the triangle is parallel to the third side and half its length, and check using dynamic geometry software; given the coordinates of the vertices of a rectangle, verify that the diagonals of the rectangle bisect each other).

Teaching Suggestions

- Parts a) and b) are fairly straightforward and involve finding the circumcentre of a triangle.
- Part c) gives students a chance to see that some problems are too difficult to be solved with the available analytic geometry tools. *The Geometer's Sketchpad®* (GSP) allows students to construct an approximate solution that gives a satisfactory answer for practical purposes. Let students attempt the algebraic solution. Then, ask them to explain why they cannot complete it before they attempt a software solution.
- Students may need some assistance with creating a slider in GSP that allows one segment to be twice as long as another.

- Use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:

- Knowledge of analytic geometry and GSP
- Evidence of communication of mathematical thinking and mathematical vocabulary
- Evidence of explanations of methods and justifications
- Ability to construct a GSP sketch
- Ability to apply formulas and techniques (midpoint, slope, perpendicular slope, equation of line, intersection of lines)

Ongoing Assessment

- Use **BLM 3–16 Task: A Site for the New Hospital Rubric** to assess student achievement.

Level 3 Sample Response

a) Find the point of intersection of the three perpendicular bisectors of the sides of the triangle.

Consider side AC. Since it is parallel to the y-axis, it is easy to see that its midpoint is D(10, 7) and its perpendicular bisector has equation $y = 7$.

For side AB, the midpoint is E(6, 11).

The slope of AB is $\frac{1}{2}$. Therefore, the slope of its perpendicular bisector is –2.

The equation of the perpendicular bisector of AB is $y = -2x + 23$.

Now find the point of intersection of these two perpendicular bisectors, (8, 7).
Therefore, the point that is equidistant from the three towns is H(8, 7).

Check the distances AH, BH, and CH.

$\text{AH} = \sqrt{(10-8)^2 + (13-7)^2} = \sqrt{40}$ $\quad$ $\text{BH} = \sqrt{(2-8)^2 + (9-7)^2} = \sqrt{40}$ $\quad$ $\text{CH} = \sqrt{(10-8)^2 + (1-7)^2} = \sqrt{40}$

So, H(8, 7) is equidistant from the three towns.

This location may not be the best site for many reasons. For example,

- lack of good roads nearby
- a land form such as a river may be located at this point
- the land at this point is already used for something
- lack of essential services such as water, sewers, and hydro at this location

b)

To: Area Planning Council
Re: Location for new Hospital

We were contracted to complete a mathematical investigation to find a location that is an equal distance from each of the three towns, Abbott, Banting, and Colton. We have determined that the location (8, 7) is the ideal location. We recommend that you choose a location as close to this point as possible.

A number of other factors will need to be considered. These include

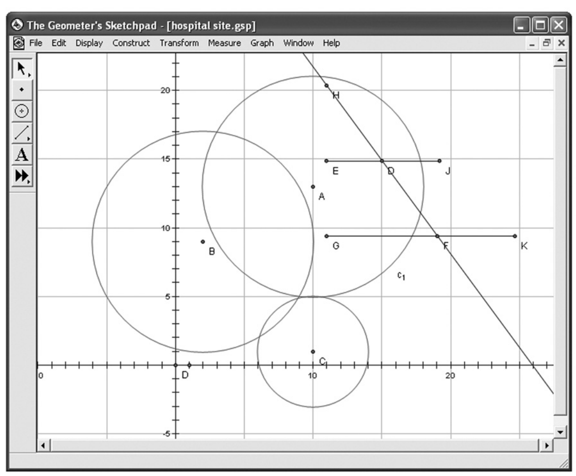
- Are there adequate roads to this site from Abbott, Banting, and Colton?
- Is there land in this area that is available and at a reasonable price?
- Can the infrastructure needs be met at this location? These include hydro, water, sewers.
- Is this site accessible by public transit?
- Is the geology of the site adequate for building a hospital?

If these factors can be resolved favourably, we are sure you will be pleased with the location (8, 7) that is so well situated with respect to Abbott, Banting, and Colton.

Respectfully submitted,
ABC Mathematical Investigations Company

c) A solution can be obtained using *The Geometer's Sketchpad*®, as follows.

- Construct a horizontal segment EJ.
- Construct a point D on segment EJ.
- Construct points H and G two units above and below E (Hint: Use Snap to Grid).
- Construct another horizontal segment GK.
- Construct the point of intersection F of line HD and segment GK.
- GF will always be twice the length of ED.
- Construct a circle, centre A, with radius GF.
- Construct a circle, centre B, with radius GF.
- Construct a circle, centre C, with radius ED.
- Drag point D until the three circles appear to have a common point of intersection.
- This point (X, X) is the location for the hospital. (Note: this point is only approximate. In practice, it would be sufficient to give the required location.)



Level 3 Notes

Look for the following:

- Some explanation of the method used in each part
- Mostly correct mathematical notation
- Construction of the necessary slider, but without explanation
- Generally correct use of all relevant formulas and techniques (midpoint, slope, perpendicular slope, equation of line, intersection of lines), with minor errors in substitution or execution

What Distinguishes Level 2

At this level, look for the following:

- Little explanation of steps taken
- Some use of good form and correct mathematical notation
- Incomplete solution using some of the relevant formulas and techniques (midpoint, slope, perpendicular slope, equation of line, intersection of lines), with some errors in substitution or execution
- Inability to progress with part c) without assistance

What Distinguishes Level 4

At this level, look for the following:

- Clear and complete steps on how to complete the analytic geometry solution
- Very good use of form and correct mathematical notation
- Successful execution of the GSP construction, possibly including detailed instructions on how to complete it
- Clear explanation of how to create the necessary slider and why it works
- Verification that the point found satisfies the conditions (approximately)
- Recognition that the solution is approximate, and perhaps an explanation of why this is the case

Task Pythagoras Park

Student Text Pages
159

Suggested Timing
70 min

Tools
- grid paper
- protractor
- compasses
- ruler

Technology Tools
- *The Geometer's Sketchpad®*
- computer

Related Resources
- G–1 Grid Paper
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 3–17 Task: Pythagoras Park Rubric

Accommodations

Gifted and Enrichment—Challenge students to create more than one Pythagoras Park.

Perceptual—Let students work in groups when completing this Task.

Motor—Let students work in groups or with a partner when completing this Task.

ESL—Let students use their dictionaries or translators when completing this Task.

Specific Expectations

Analytic Geometry

Using Linear Systems to Solve Problems

AG1.02 solve problems that arise from realistic situations described in words or represented by linear systems of two equations involving two variables, by choosing an appropriate algebraic or graphical method.

Solving Problems Involving Properties of Line Segments

AG2.01 develop the formula for the midpoint of a line segment, and use this formula to solve problems (e.g., determine the coordinates of the midpoints of the sides of a triangle, given the coordinates of the vertices, and verify concretely or by using dynamic geometry software);
AG2.02 develop the formula for the length of a line segment, and use this formula to solve problems (e.g., determine the lengths of the line segments joining the midpoints of the sides of a triangle, given the coordinates of the vertices of the triangle, and verify using dynamic geometry software);
AG2.03 develop the equation for a circle with centre (0, 0) and radius r, by applying the formula for the length of a line segment;
AG2.04 determine the radius of a circle with centre (0, 0), given its equation; write the equation of a circle with centre (0, 0), given the radius; and sketch the circle, given the equation in the form $x^2 + y^2 = r^2$;
AG2.05 solve problems involving the slope, length, and midpoint of a line segment (e.g., determine the equation of the right bisector of a line segment, given the coordinates of the endpoints; determine the distance from a given point to a line whose equation is given, and verify using dynamic geometry software).

Using Analytic Geometry to Verify Geometric Properties

AG3.01 determine, through investigation (e.g., using dynamic geometry software, by paper folding), some characteristics and properties of geometric figures (e.g., medians in a triangle, similar figures constructed on the sides of a right triangle).

Teaching Suggestions

- Students will benefit from some class time to brainstorm ideas for this Task. This is best done in small groups so that everyone has an opportunity to contribute ideas. Although each student or group should do original work, they can benefit from the creative ideas of their classmates.
- Prepare a warm-up activity that reviews the definitions of all of the relevant geometrical terms.
- Encourage students to submit this Task in draft form and then prepare a final version based on feedback provided. Feedback could even be structured as a peer activity if the class has sufficient maturity. Students seldom get the opportunity to refine their mathematical solutions based on constructive reflection. Clearly, a revision cycle is time consuming, but this task is ideal for it.
- Encourage students to be creative. They can either improve on ideas actually seen in park design or design a futuristic concept that may be whimsical and not necessarily practical. Some students will want to use computer software including *The Geometer's Sketchpad®*, as well as graphic or design software. Others will want to work by hand, perhaps

using grid paper as a design aid. All formats should be accepted as equally valid. If needed, use **T–4 *The Geometer's Sketchpad*® 3** or **T–5 *The Geometer's Sketchpad*® 4** to support this activity.
- This task is ideal for marking with a rubric. However, it is important to give students individual feedback on their submissions. Feedback should be a blend of recognition of positive aspects of their design together with some suggestions for future improvements. Such feedback is probably more valued by students than a numerical mark.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they represent. When evaluating a response, look for the following:
- Knowledge of geometry terminology
- A design that contains geometric shapes and properties with practical connections
- A design that incorporates a variety of shapes
- Evidence of ability to connect the problem context to the underlying mathematical ideas
- Evidence of communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

- Use **BLM 3–17 Task: Pythagoras Park Rubric** to assess student achievement.

Level 3 Sample Response

Solutions will vary greatly. Any solution that includes the required components should be accepted.

Level 3 Notes

Look for the following:
- A design that includes five different required geometric shapes
- Chosen shapes serve a purpose in the design
- Use of a variety of shapes, including triangles, quadrilaterals, and circles
- Features with a purpose in the design

What Distinguishes Level 2

At this level, look for the following:
- A design that includes five geometric shapes that may not be different
- Chosen shapes may or may not serve a purpose in the design
- Use of simple shapes such as all triangles or just squares and rectangles
- Five features are present, but they may not have a practical use in the design

What Distinguishes Level 4

At this level, look for the following:
- A design that includes many of the required geometric shapes and may include shapes not listed
- All shapes serve a purpose in the design; many are interrelated in important ways
- Use of a variety of shapes, including triangles, quadrilaterals, and circles
- Features integrated into the design in a meaningful way