Chapter 4 Problem Wrap-Up

Student Text Pages
203

Suggested Timing
30–70 min, depending on whether the Chapter Problem questions were assigned in the regular homework

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 4–15 Chapter 4 Problem Wrap-Up Rubric

Using the Chapter Problem

- Introduce the problem, or re-introduce it if you introduced it at the beginning of the chapter. Clarify any words or contexts that may be confusing to the students. Most of this component relies on students' communication and analytical skills.
- This problem provides students with one of their first opportunities to work with "messy" data in a modelling context. The concept of using mathematical equations to model real-life situations is a very important idea. Students have to realize that some models approximate a situation more closely than others, and this question gives an opportunity to discuss which model works best in the context of this question. In particular, for the last part of the problem, students may present values for b from 1.5 to 2.0, since these will give some values for y for $x = 0$, 1, 2, and 3 that are acceptably close to the actual data values. In addition, others may note that there is stronger evidence for the quadratic model than the exponential model of this form.
- The Chapter Problem Wrap-Up is very dependent on the Chapter Problem development throughout Chapter 4 and, especially, on question 16 in Section 4.4. You may wish to wait to assign this question until this stage to act as a lead-in to the Wrap-Up. Alternatively, review Section 4.1, question 6; Section 4.2, question 7; and Section 4.4, question 16 with the class or in small groups.
- Review the following topics with the class or in small group discussions:
  - curve of best fit
  - linear/quadratic regression and related models
  - a simple exponential model based on $y = 2^x$, e.g., paper folding, cell division
  - the impact of re-use, recycling, new technologies, and/or pick-up limits (e.g., one bag per residence) on the total amount of garbage produced
- Provide the students with a rubric so they know how they will be evaluated. Students may answer the Wrap-Up questions on their own and then compare solutions and responses. This could be done in pairs, in small groups, or in a class discussion.

Summative Assessment

- Use **BLM 4–15 Chapter 4 Problem Wrap-Up Rubric** to assess student achievement.

Level 3 Sample Response

a) From Section 4.4, question 16, my model is accurate for $x \geq 2000$, where x represents the years 2000, 2001, … .

b) I could use a graphing calculator to find the value of y when $x = 2020$ or substitute $x = 2020$ into $y = 14.6(x - 1992.7)^2 - 576.2$.

$$y = 14.6(2020 - 1992.7)^2 - 576.2$$
$$= 14.6(27.3)^2 - 576.2$$
$$\doteq 10\,305$$

The total mass of garbage in 2020 will be approximately 10 305 thousands of tonnes.

This result assumes that the total mass of garbage in the landfill site continues to grow as the model predicts.

Many factors may affect the situation:
- a recycling program should result in less garbage entering the landfill
- population growth may increase the amount beyond the model predictions
- new technologies could lessen the amount (e.g., biodegradable plastic)
- changes in laws relating to re-use/recycling could decrease the amount of garbage

c) Possible effects of a recycling program on the model could be as follows:
- The total mass of garbage would grow at a slower rate. This would mean that the graph would curve upward at a slower rate. The quadratic model equation in the form $y = ax^2 + bx + c$ would reflect this with $a < 14.6$.
- If recycling resulted in the same amount of garbage being added each year, the graph/model would become linear.
- If all garbage were to be recycled, then the graph would become a horizontal line.

d) The total amount of garbage in the landfill site at the end of each year would have to be growing according to an exponential rather than a quadratic model.
I tried different values for b and compared the y-values with the total garbage column of my table for the landfill site.

x	$y = 200 \times 2^x$	$y = 200 \times 1.9^x$	$y = 200 \times 1.8^x$	$y = 200 \times 1.7^x$
0	200	200	200	200
1	400	380	360	340
2	800	722	648	578
3	1600	1371.8	1166.4	982.6
4	3200	2606.42	2099.52	1670.42

I think $b = 1.7$ is a good choice.

Level 3 Notes

Look for the following:

- Clear understanding of properties of quadratic relations and features of $y = 2^x$
- Use of technology to obtain a curve of best fit (linear or quadratic) and the related equation that models the problem
- Planning and thinking in analysing the equation(s) that model the problem
- Ability to find the vertex of a parabola, and the equation in the forms $y = ax^2 + bx + c$ and $y = a(x - h)^2 + k$
- Mostly accurate calculations
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some understanding of properties of quadratic relations and features of $y = 2^x$
- Some use of technology to obtain a curve of best fit (linear or quadratic) and the related equation that models the problem
- Some planning and thinking in analysing the equation(s) that model the problem
- Some ability to find the vertex of a parabola, and the equation in the forms $y = ax^2 + bx + c$ and $y = a(x - h)^2 + k$
- Somewhat accurate calculations
- Some well-constructed mathematical arguments
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Very clear understanding of properties of quadratic relations and features of $y = 2^x$
- Detailed use of technology to obtain a curve of best fit (linear or quadratic) and the related equation that models the problem
- Detailed planning and thinking in analysing the equation(s) that model the problem
- Detailed ability to find the vertex of a parabola, and the equation in the forms $y = ax^2 + bx + c$ and $y = a(x - h)^2 + k$
- Accurate calculations
- Detailed evidence of well-constructed mathematical arguments
- Use of very good form and correct mathematical notation