4.2

Quadratic Relations

Teaching Suggestions
• Discuss the photo and that a mathematical relation can be developed for the shape of the arching ceiling at BCE Place. (5 min)

Investigate
• In **Investigate A**, students make tables of values and plot the points. Students need the time to explore the connection between the algebra and the visuals of the graph. (20 min)
• In **Investigate B**, students learn that finite differences are an important concept in understanding rates of change. With linear relations, the rate of change and the first differences are related. With quadratic relations, the second differences are related to the rate of change of the slope. Have students use **BLM 4–4 Finite Difference Tables** with this Investigate. (15 min)

Example
• Discuss the **Example**, or a similar one, showing the applications of quadratics. This makes an important connection to real-world examples. At this point, students are primarily exploring quadratics, so formal algebraic methods are not important yet. Have students use **BLM 4–5 Model the Roof of the Detroit International Airport** to model a parabolic roof using quadratics. (5 min)
• Review the vocabulary in this section (quadratic relation, parabola, vertex, axis of symmetry, finite differences). (5 min)

Communicate Your Understanding
• Discuss the answers to the questions in this section as a class before assigning homework. (10 min)
• If time permits, have the students try this experiment:
  Materials: clear plastic cylindrical bottle, scissors, ruler, marker, sink or basin, water, stopwatch or timer
  Procedure:
  a) Cut a small hole near the bottom of the bottle.
  b) Mark the location of the hole as 0. Using the ruler, mark off heights of 2 cm, 4 cm, … , 10 cm on the side of the bottle.
  c) Cover the hole and fill the bottle with water to the 10-cm mark.
  d) Uncover the hole and measure the time it takes for the level to drop to 8 cm, 6 cm, and so on. Record the results.
  e) Repeat the experiment two more times.
  f) For each level, calculate the mean of the three trials.
  g) Determine whether the results can be approximated by a quadratic model.
• Use **BLM 4–6 Section 4.2 Practice Master** for remediation or extra practice.
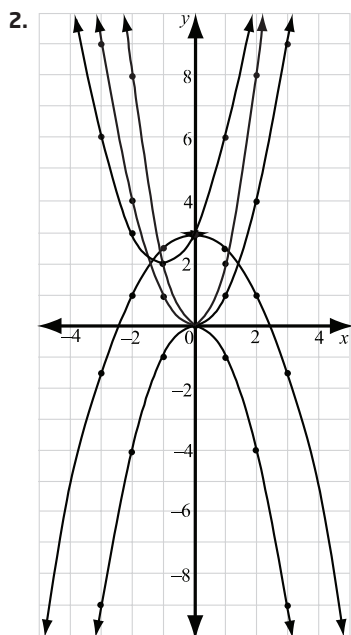
Investigate Answers (pages 168–169)

A

1. a)

x	y
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

b)

x	y
−3	18
−2	8
−1	2
0	0
1	2
2	8
3	18

c)

x	y
−3	6
−2	3
−1	2
0	3
1	6
2	11
3	18

d)

x	y
−3	−9
−2	−4
−1	−1
0	0
1	−1
2	−4
3	−9

e)

x	y
−3	−1.5
−2	1
−1	2.5
0	3
1	2.5
2	1
3	−1.5

2.

3. Answers will vary. For example: Similar: All of the graphs are parabolas. Different: The relations that have a positive number in front of the x^2 open upward, and the relations that have a negative number in front of the x^2 open downward. Some relations have a vertex at (0, 0); some relations do not have a vertex at (0, 0).

B

1. a)

x	y	First Differences
−2	−9	
−1	−7	$-7 - (-9) = 2$
0	−5	$-5 - (-7) = 2$
1	−3	$-3 - (-5) = 2$
2	−1	$-1 - (-3) = 2$

b)

x	y	First Differences
−2	14	
−1	8	$8 - 14 = -6$
0	2	$2 - 8 = -6$
1	−4	$-4 - 2 = -6$
2	−10	$-10 - (-4) = -6$

2. The first differences are constant for linear relations.

3. a)

x	y	First Differences	Second Differences
−2	0		
−1	−3	$-3 - 0 = -3$	
0	−4	$-4 - (-3) = -1$	$-1 - (-3) = 2$
1	−3	$-3 - (-4) = 1$	$1 - (-1) = 2$
2	0	$0 - (-3) = 3$	$3 - 1 = 2$

b)

x	y	First Differences	Second Differences
−2	1		
−1	−2	$-2 - 1 = -3$	
0	−1	$-1 - (-2) = 1$	$1 - (-3) = 4$
1	4	$4 - (-1) = 5$	$5 - 1 = 4$
2	13	$13 - 4 = 9$	$9 - 5 = 4$

4. a) The first differences are not constant for quadratic relations.

b) The second differences are constant for quadratic relations.

5. If a relation has first differences that are constant, then it is linear. If a relation has second differences that are constant, then it is quadratic.

- Students may have trouble getting started, since few of these problems have obvious starting points.

R_x Have students act out the situation. This is especially useful for kinesthetic learners. Other strategies include discussing strategies with other students in the class, or building a model.

Accommodations

Gifted and Enrichment—Challenge students to research and report to the class on famous mathematicians, such as Fermi.

Spatial—Encourage students to use real-life examples when estimating the number of jelly beans in the jar.

Language—Allow students to give verbal responses to the questions in this section.

Memory—Encourage students to use cue cards to memorize formulas.

Communicate Your Understanding Responses (page 171)

C1. Finite differences are differences found from the y-values with evenly spaced x-values. The x-values in the table of values that El-Noor has used are not evenly spaced.

C2. Yes the relation is quadratic, because the points look like they follow a parabola shape.

Practise

- **Questions 1** and **2** provide important contexts for quadratics and encourage communication. At this point, the students should still be graphing by hand, until these skills are consolidated.
- **Questions 4**, **5**, and **8** encourage students to look at the world around them. They may wish to take digital photos of these examples to post in the classroom. Superimposing a grid in order to develop an equation in future sections would add to the context.
- **Question 6** is an interesting study of the effects of gravity. You may wish to discuss the use of the negative coefficient of x^2.
- **Question 7** refers to the Chapter Problem. Students may answer the question here, or wait until the Chapter Problem Wrap-Up.
- **Question 9** is an Achievement Check. It is an ideal formative assessment piece.
- In **question 10**, students need to understand how to work with rates and time.
- In **question 11**, patterning can help develop the relation. Students should add the first two natural numbers, then the first three, and so on.
- Use **A–9 Communication General Scoring Rubric** when assessing your students for questions 4 and 6.

Achievement Check Sample Solution, question 9, page 173

Provide students with **BLM 4–7 Section 4.2 Achievement Check Rubric** to help them understand what is expected.

9. a), **b)**

t	h	First Differences	Second Differences
0	0.4		
		$15.1 - 0.4 = 14.7$	
1	15.1		$4.9 - 14.7 = -9.8$
		$20.0 - 15.1 = 4.9$	
2	20.0		$-4.9 - 4.9 = -9.8$
		$15.1 - 20.0 = -4.9$	
3	15.1		$-14.7 - (-4.9) = -9.8$
		$0.4 - 15.1 = -14.7$	
4	0.4		

c) The path of the rocket is a parabola that opens downward with axis of symmetry $t = 2$ and vertex at $(2, 20)$.

d) When the rocket was fired $t = 0$, and the height above the ground (from the table or graph) is 0.4 m.

Literacy Connections

Add the new terms that appear in this section to the Word Wall.

Photographic Examples of Parabolic Arches: This provides an opportunity for an interesting literacy connection. Have students take one of the photographs and give it a headline. Then ask students to write a short news report. Students should include each of who, what, when, where, and why in their news report. Follow up by having students read their news reports to the class. Use **A–23 News Report Checklist** when assessing your students.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	10, 11
Reasoning and Proving	4, 7–9
Reflecting	6, 11
Selecting Tools and Computational Strategies	10, 11
Connecting	4, 7, 9, 10
Representing	1, 2, 5, 9
Communicating	1, 2, 4, 6, 9

Ongoing Assessment

• Use Achievement Check question 9 to monitor student success. See Achievement Check Answers and **BLM 4–7 Section 4.2 Achievement Check Rubric**.
• Chapter Problem question 7 can be used as an assessment tool.
• Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.