4.3 Investigate Transformations of Quadratics

Student Text Pages
174–179

Suggested Timing
70–140 min, depending on the ability of your students and their skill level with graphing calculators

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 4–8 Section 4.3 Practice Master
- A–7 Thinking General Scoring Rubric
- A–10 Observation General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- Discuss the opening paragraph and point out that not all quadratics are the same. They depend on many factors that result in graphs that are similar in shape but are different sizes and in different locations. (2 min)

Investigate

- In **Part A**, much of the time is taken up by transferring the graphs to paper and by the written descriptions. These are very valuable steps, so do not rush students through them. Summarize the effects of the constant k in $y = x^2 + k$ before moving on to Part B. Explain the importance of using proper mathematical terminology, i.e., a "vertical translation." (30–40 min)
- During **Part B**, students may say that the parabola is skinnier, rather than saying it is taller. Stress the terminology "vertical stretch" and "vertical compression." (20–30 min)
- During **Part C**, students may have difficulty understanding why the horizontal translation seems to be backward. Explain that because this occurs before the squaring, it is a horizontal translation back to the base parabola with vertex (0, 0). (20–30 min)
- It might be a good idea to show the students how to make the graph of the base relation $y = x^2$ appear as a thicker line. That is, arrow back to the left of **Y1** and press **ENTER**. This graph then appears thicker than the others.
- Students can also use the function features of the TI-83 Plus or TI-84 Plus graphing calculator to complete the Investigate activities. For example, enter x^2 into **Y1**. Then, in **Y2** enter **Y1** + 5 for $x^2 + 5$. Similarly, in **Y3** enter 2Y1 for $2x^2$, or in **Y4** enter Y1($x - 4$) for $(x - 4)^2$.
- Another option is to use sliders in either *Fathom*™ or *The Geometer's Sketchpad*®. In *Fathom*™, graph $y = ax^2$, $y = x^2 + k$, and so on, where students have created sliders or teachers have pre-made files, as is common with *The Geometer's Sketchpad*®.

Example

- Spend time going over the **Example** provided or a similar one. It gives a context to the transformations of the parabola. (10 min)

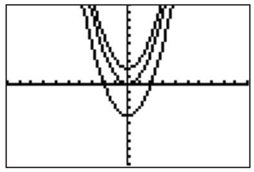
Communicate Your Understanding

- Review the vocabulary term (zero) before discussing student answers for the questions in this section as a class. (10 min)
- Use **BLM 4–8 Section 4.3 Practice Master** for remediation or extra practice.
- Use **A–10 Observation General Scoring Rubric** at any point during this section to assist you in assessing students.

Investigate Answers (pages 174–175)

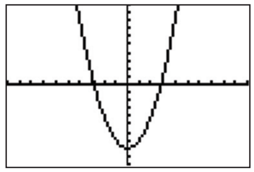
Part A

5. a)



b) The graph of $y = x^2 + 2$ is the graph of $y = x^2$ translated 2 units upward, and the graph of $y = x^2 - 4$ is the graph of $y = x^2$ translated 4 units downward.

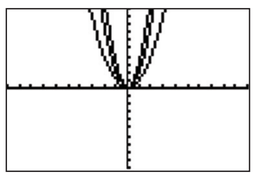
c)



6. If the value of k is positive, the graph of $y = x^2$ will be translated k units upward, and if the value of k is negative, the graph of $y = x^2$ will be translated k units downward.

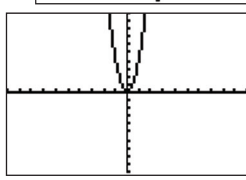
Part B

2. a)

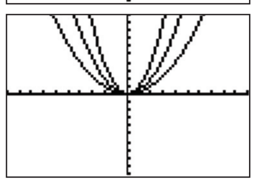


b) The graph of $y = 2x^2$ is the graph of $y = x^2$ stretched vertically by a factor of 2. The graph of $y = 3x^2$ is the graph of $y = x^2$ stretched vertically by a factor of 3.

3.

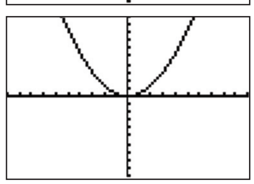


4.

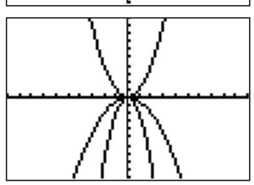


The graph of $y = \frac{1}{2}x^2$ is the graph of $y = x^2$ compressed vertically by a factor of $\frac{1}{2}$. The graph of $y = \frac{1}{3}x^2$ is the graph of $y = x^2$ compressed vertically by a factor of $\frac{1}{3}$.

5.

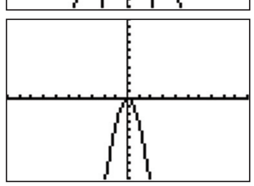


6.



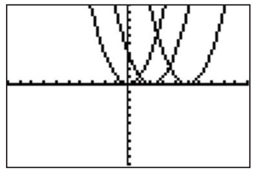
The graphs of $y = -2x^2$ and $y = -0.5x^2$ both open downward because the coefficients are negative. The graph of $y = -2x^2$ is the graph of $y = x^2$ stretched vertically by a factor of 2 and reflected in the x-axis. The graph of $y = -0.5x^2$ is the graph of $y = x^2$ compressed vertically by a factor of 0.5 and reflected in the x-axis.

7.

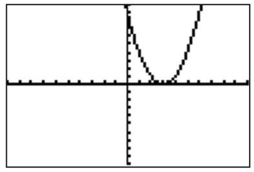


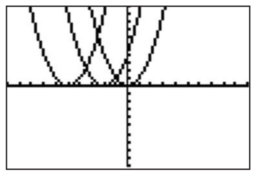
8. If the value of a is positive the parabola will open upward, and if the value of a is negative the parabola will open downward. If $-1 < a < 0$ or $0 < a < 1$, then the parabola will be vertically compressed, and if $a < -1$ or $a > 1$, then the parabola will be vertically stretched.

Part C

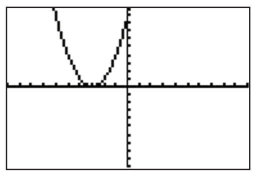
2. a) 

b) The graph of $y = (x - 2)^2$ is the graph of $y = x^2$ translated horizontally 2 units to the right. The graph of $y = (x - 5)^2$ is the graph of $y = x^2$ translated horizontally 5 units to the right.

3. 

4. 

The graph of $y = (x + 2)^2$ is the graph of $y = x^2$ translated horizontally 2 units to the left. The graph of $y = (x + 5)^2$ is the graph of $y = x^2$ translated horizontally 5 units to the left.

5. 

6. If the value of h is positive the parabola will be translated h units to the right, and if the value of h is negative the parabola will be translated h units to the left.

Communicate Your Understanding Responses (page 178)

C1. The graphs of $y = 2x^2$ and $y = -2x^2$ have the same shape, which is stretched vertically by a factor of 2 compared to the graph of $y = x^2$, but the parabola for $y = 2x^2$ opens upward while the parabola for $y = -2x^2$ opens downward.

C2. a) B, because the parabola has been translated 2 units upward.

b) A, because the parabola opens downward and is wider than the parabola for $y = x^2$.

c) D, because (–1, 1), (0, 0), and (1, 1) are three points on this parabola.

d) C, because the parabola has been translated 2 units to the right.

Practise

It is important that students complete part of each Practise question. These should be done without a graphing calculator, and each should include a graph of $y = x^2$ as a reference. Monitor students' work for transformations in the incorrect direction.

- **Questions 6**, **7**, and **8** provide different ways of looking at transformations. Encourage multiple representations of quadratics—words, equations, tables, and graphs.
- In **questions 9**, **10**, and **11**, the students' ability to communicate understanding is the key. Refer students to the **Example** for help. Use **A–7 Thinking General Scoring Rubric** when assessing your students for these questions.
- In **question 12**, students use their understanding of what happened in **Parts B** and **C** of the **Investigate**.
- **Question 13** requires skills with solving systems of equations.

Common Errors

- Some students may shift horizontal translations the wrong way.

R_x Have students plot a few points close to the vertex of $y = (x - 3)^2$ and the equivalent points close to the vertex of $y = x^2$ to see how the parabola has shifted to the right, including the vertex and all other points.

Accommodations

Visual—Encourage students to use technology when completing the questions.

Perceptual—Have students use large sheets of grid paper when completing the questions in this section.

Motor—Allow students to work in pairs to complete the questions.

Language—Encourage students to review the steps required to use the technology in this section.

Memory—Encourage students to create cue cards to remember the transformations on the quadratic relations.

Student Success

Use a jigsaw approach and graphing calculators to have students investigate transformations of quadratics.

Refer to the introduction of this Teacher's Resource for more information about how to use a jigsaw strategy.

Literacy Connections

Add this section's new terms to the Word Wall.

A fun way to deal with the new terminology in this section is to play a card game similar to Go Fish. Have students make up cards with such phrases as "vertical translation up 3" and the equivalent equation, $y = x^2 + 3$. When you have approximately 50 cards for each group, students can play in groups of four and match the cards to form pairs. The person with the most pairs at the end wins. As you work through the chapter you may wish to add a variety of matching pairs to the deck.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	13, 15
Reasoning and Proving	10, 12, 15
Reflecting	5, 10, 14
Selecting Tools and Computational Strategies	13
Connecting	9, 10, 12, 13, 15
Representing	1–4, 9, 11
Communicating	4, 11, 12, 14

Ongoing Assessment

- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.