4.6 Negative and Zero Exponents

Student Text Pages
189–193

Suggested Timing
70 min

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 4–12 Section 4.6 Practice Master
- BLM 4–13 Section 4.6 Achievement Check Rubric
- A–9 Communication General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Although this section may seem out of place, the expectation it addresses is almost a stand-alone one. It fits here as a comparison of quadratics and exponential growth and decay, and other applications of exponents. Discuss the opening paragraph with that in mind. (5 min)

Investigate

- The intention is to complete one part (**A**, **B**, or **C**) in order to learn the concepts appropriately. However, students may complete two or even all three parts, depending on their level of understanding. If your students are doing **Part C**, you could add more examples for weaker students. The key for all three methods is to understand why zero and negative exponents give the results that they do, rather than to memorize a set of rules. (20 min)

Examples

- Work through **Example 1** or similar examples, as needed, to consolidate learning. Encourage alternative solutions using the different methods from the Investigate. Allow students to develop a rule using their own words regarding negative exponents. For example, "if there is a negative exponent upstairs, then you can bring it downstairs if you change the sign of the exponent on the base." This is a very important concept for students to learn. (10 min)
- **Example 2** returns to the situation introduced in the opening paragraph. Examples that include applications are important to give context. (5 min)

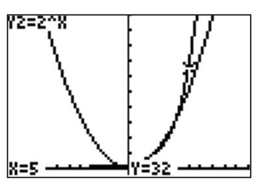
Communicate Your Understanding

- As a class, discuss the questions in this section. They are good assessment tools to determine the students' level of understanding of zero and negative exponents. (5 min)
- Use **BLM 4–12 Section 4.6 Practice Master** for remediation or extra practice.

Investigate Answers (pages 194–196)

Part A

1.



2. Answers will vary. For example: The graphs of both relations are increasing for x-values that are positive and increasing. For x-values that are negative and increasing, the graph of $y = x^2$ is decreasing and the graph of $y = 2^x$ is increasing. When $x = 0$, the graph of $y = x^2$ crosses the y-axis at the point (0, 0) and the graph of $y = 2^x$ crosses the y-axis at the point (0, 1). The graph of $y = x^2$ is a quadratic relation in the shape of a parabola. The graph of $y = 2^x$ is an exponential relation.

3. Answers will vary. For example: For x-values from -3 to 0, the graph of $y = x^2$ decreases quickly, while the graph of $y = 2^x$ increases slowly. When $x = 0$, the graph of $y = x^2$ crosses the y-axis at the point (0, 0) and the graph of $y = 2^x$ crosses the y-axis at the point (0, 1). For x-values from 0 to 3, the graph of $y = x^2$ increases quickly, as does the graph of $y = 2^x$. However, on this interval, the graph of $y = 2^x$ increases faster than the graph of $y = x^2$.

4.

x	y
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8

5. The y-values are reciprocals for x-values that are equal in number, but opposite in sign.

6. When $x = 0$, the y-value is equal to 1.

7. Answers will vary. For example: When powers of a natural number are evaluated, those with negative exponents will have fraction answers and those with an exponent of zero will equal 1.

Part B

1. a) $2^5 = 32, 2^4 = 16, 2^3 = 8, 2^2 = 4, 2^1 = 2$

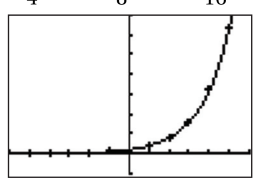
b) $\frac{1}{2}$

c) $2^0 = 1, 2^{-1} = \frac{1}{2}, 2^{-2} = \frac{1}{4}, 2^{-3} = \frac{1}{8}, 2^{-4} = \frac{1}{16}, 2^{-5} = \frac{1}{32}$

2. a)

x	$y = 2^x$
-5	$\frac{1}{32}$
-4	$\frac{1}{16}$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16
5	32

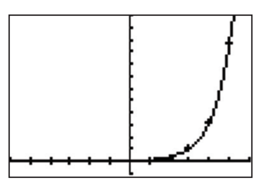
b), d)



c) The graph approaches zero on the left and increases to infinity on the right.

3. $3^5 = 243, 3^4 = 81, 3^3 = 27, 3^2 = 9, 3^1 = 3; \frac{1}{3}; 3^0 = 1, 3^{-1} = \frac{1}{3}, 3^{-2} = \frac{1}{9}, 3^{-3} = \frac{1}{27}, 3^{-4} = \frac{1}{81}, 3–5 = \frac{1}{243}$

x	$y = 3^x$
-5	$\frac{1}{243}$
-4	$\frac{1}{81}$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27
4	81
5	243



The graph approaches zero on the left and increases to infinity on the right.

4. a) $a^0 = 1$ **b)** $a^{-1} = \frac{1}{a}$ **c)** $a^{-2} = \frac{1}{a^2}$ **d)** $a^{-3} = \frac{1}{a^3}$

Part C

1. a)

Expression	Expand and Divide	Exponent Law
$\frac{3^2}{3^5}$	$\frac{3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = \frac{1}{3^3}$	$\frac{3^2}{3^5} = 3^{2-5}$ $= 3^{-3}$
$\frac{4^1}{4^3}$	$\frac{4}{4 \times 4 \times 4} = \frac{1}{4^2}$	$\frac{4^1}{4^3} = 4^{1-3}$ $= 4^{-2}$
$\frac{2^4}{2^7}$	$\frac{2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2} = \frac{1}{2^3}$	$\frac{2^4}{2^7} = 2^{4-7}$ $= 2^{-3}$
$\frac{(-5)^2}{(-5)^3}$	$\frac{(-5) \times (-5)}{(-5) \times (-5) \times (-5)} = \frac{1}{(-5)}$	$\frac{(-5)^2}{(-5)^3} = (-5)^{2-3}$ $= (-5)^{-1}$
$\frac{(-2)^3}{(-2)^5}$	$\frac{(-2) \times (-2) \times (-2)}{(-2) \times (-2) \times (-2) \times (-2) \times (-2)} = \frac{1}{(-2)^2}$	$\frac{(-2)^3}{(-2)^5} = (-2)^{3-5}$ $= (-2)^{-2}$

b) The results in each row are equivalent, and the sign of the base does not affect this result.

2. a)

Expression	Expand and Divide	Exponent Law
$\frac{3^5}{3^5}$	$\frac{3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3} = 1$	$\frac{3^5}{3^5} = 3^{5-5}$ $= 3^0$
$\frac{5^2}{5^2}$	$\frac{5 \times 5}{5 \times 5} = 1$	$\frac{5^2}{5^2} = 5^{2-2}$ $= 5^0$
$\frac{4^3}{4^3}$	$\frac{4 \times 4 \times 4}{4 \times 4 \times 4} = 1$	$\frac{4^3}{4^3} = 4^{3-3}$ $= 4^0$
$\frac{(-3)^4}{(-3)^4}$	$\frac{(-3) \times (-3) \times (-3) \times (-3)}{(-3) \times (-3) \times (-3) \times (-3)} = 1$	$\frac{(-3)^4}{(-3)^4} = (-3)^{4-4}$ $= (-3)^0$
$\frac{(-2)^2}{(-2)^2}$	$\frac{(-2) \times (-2)}{(-2) \times (-2)} = 1$	$\frac{(-2)^2}{(-2)^2} = (-2)^{2-2}$ $= (-2)^0$

b) The results in each row are equivalent, and the sign of the base does not affect this result.

3. a) If a base is raised to a negative exponent, it is equal to the reciprocal of the base raised to the positive of the exponent.

b) If a non-zero base is raised to the exponent 0, the result is 1.

4. a) 1 **b)** $\frac{1}{a}$ **c)** $\frac{1}{a^2}$ **d)** $\frac{1}{a^3}$

Communicate Your Understanding Responses (page 199)

C1. 2^{-3} is not a negative number because it is equal to $\frac{1}{8}$. If a base is raised to a negative exponent, it is equal to the reciprocal of the base raised to the positive of the exponent.

C2. 5^0 can be written as 5^{3-3} which is $\frac{5^3}{5^3} = 1$.

C3. It is easier to find the reciprocal of the fraction and apply the positive exponent and get a fraction answer rather than a decimal answer.

Common Errors

- Some students may make the resulting power negative from a negative exponent.

R_x Have students complete other investigations that are provided to convince them that the sign of the result is independent of the sign on the exponent. You can also have them investigate examples with even versus odd exponents and a negative base.

Accommodations

Gifted and Enrichment—Encourage students to learn more about radioactivity and carbon-14 dating and to present their findings to the class.

Visual—Provide students with photocopies of the tables to be completed in this section.

Perpetual—Have students use patterning to understand the exponents of negative numbers and the exponent of 0.

Memory—Encourage students to review the exponent rules for simplifying expressions involving powers with the same base. Have them review the steps required to use the technology in this section.

Practise

- Assign all of **questions 1** through **3**. Students need practice with negative exponents.
- **Question 4** provides valuable practice at a higher level, and allows practice of other numeracy skills.
- **Questions 5**, **6**, **7**, **11**, and **12** are good applications. Assign a selection of these questions.
- **Question 8** makes students consider the placement of the variable in the base or exponent.
- **Question 13** is an Achievement Check and can be used as an assessment tool.
- **Question 14** requires students to think in a different manner, by placing the variable in the exponent.
- **Questions 16** and **17** are good opportunities to use the regression capabilities of a graphing calculator to examine relationships drawn from data. Students can compare quadratic and exponential curves and make decisions based on the curves of best fit.
- Use **A–9 Communication General Scoring Rubric** when assessing your students for **questions 11**, **12**, and **14**.

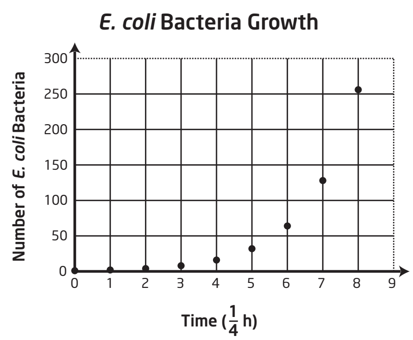
Achievement Check Sample Solution, question 13, page 200

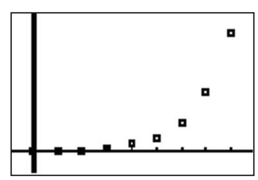
Provide students with **BLM 4–13 Section 4.6 Achievement Check Rubric** to help them understand what is expected.

13. a)

Time ($\frac{1}{4}$ h)	Number of *E. coli* Bacteria
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256

b)





c) The scatter plot and table show exponential growth, as the number of bacteria doubles every 15 min. It is the graph of $y = 2^x$.

d) After 2 h there are 256 bacteria. Extending the pattern gives 512, 1024, 2048, 4096, 9092, 18 184 bacteria after 2.25 h, 2.5 h, 2.75 h, 3 h, 3.25 h, 3.5 h, respectively. So there are 10 000 bacteria just after 3.25 h, or at about 3 h 15 min.

Alternately: $2^{13} = 9092$, so it will take about $13 \times \frac{1}{4}$, or $3\frac{1}{4}$ h, for the mitosis process to reach 10 000 *E. coli* bacteria.

Literacy Connections

Add new terms that appear in this section to the Word Wall.

Questions 11, 12, and 14 are good questions to check whether students are able to explain their reasoning in words. Students should answer in full sentences and be clear in their writing with no grammatical errors.

Student Success

Use concept attainment to help students develop meanings for zero and negative exponents.

Use Blast Off to help students summarize meanings for zero and negative exponents.

Refer to the introduction of this Teacher's Resource for more information about how to use concept attainment and Blast Off strategies.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	13, 15, 20
Reasoning and Proving	9, 10, 12–14, 16, 17
Reflecting	13, 16, 17
Selecting Tools and Computational Strategies	13, 15, 20
Connecting	5–7, 11–13, 15
Representing	12–15
Communicating	11–14, 17

Ongoing Assessment

- Use Achievement Check question 13 to monitor student success. See Achievement Check Answers and **BLM 4–13 Section 4.6 Achievement Check Rubric**.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.