Chapter 5 Problem Wrap-Up

Using the Chapter Problem
• Introduce the problem. Suggest that students draw separate diagrams for each layer of the pedestal in part c).
• Let the students know that their communication skills are important in parts b), c), and d), and they should provide detailed explanations with appropriate mathematical justification.
• This problem can be done individually (preferably) or in groups.
• Review techniques for factoring. A short matching activity with examples of factoring and the names of the factoring types would be useful. Students also need to be able to distinguish between the concepts of surface area, exposed surface area, and volume. A sample box with marked algebraic dimensions would be a good starting point as a warm-up activity.
• Parts a) and b) are essentially review questions. If students did not complete these questions earlier in the unit, then they will be new work.
• Some students may approach part c) using only a numerical solution (i.e., substituting $x = 5$ immediately). Encourage them to consider an algebraic solution, since this type of solution is more efficient when there are multiple calculations to be made.
• Provide sufficient time for the revision process. Students will create higher quality work if they have an opportunity for constructive feedback and time to incorporate suggestions and additional ideas into their work. Part d) will particularly benefit from this process. Encourage students to provide as complete an answer as possible to this part.

Summative Assessment
• Use **BLM 5–12 Chapter 5 Problem Wrap-Up Rubric** to assess student achievement.

Level 3 Sample Response

a) The surface area expressions from Section 5.2, question 13 were as follows:

Prism	Side Surface Area	Top Surface Area	Exposed Top Surface Area
bottom	$x(2x + 5)$	$(2x + 5)^2$	$(2x + 5)^2 - (2x + 2)^2$
middle	$x(2x + 2)$	$(2x + 2)^2$	$(2x + 2)^2 - (2x - 1)^2$
top	$x(2x - 1)$	$(2x - 1)^2$	

Use difference of squares factoring.

$$(2x + 5)^2 - (2x + 2)^2$$
$$= [(2x + 5) + (2x + 2)][(2x + 5) - (2x + 2)]$$
$$= (4x + 7)(3)$$
$$= 3(4x + 7)$$

$$(2x + 2)^2 - (2x - 1)^2$$
$$= [(2x + 2) + (2x - 1)][(2x + 2) - (2x - 1)]$$
$$= (4x + 1)(3)$$
$$= 3(4x + 1)$$

b) These results match those of Section 5.6, question 13.

c) The expression for the volume for the pedestal is

$$x(2x + 5)^2 + x(2x + 2)^2 + x(2x - 1)^2$$
$$= x(4x^2 + 20x + 25) + x(4x^2 + 8x + 4) + x(4x^2 - 4x + 1)$$
$$= 4x^3 + 20x^2 + 25x + 4x^3 + 8x^2 + 4x + 4x^3 - 4x^2 + x$$
$$= 12x^3 + 24x^2 + 30x$$

If $x = 5$, the volume of wood is $12(5)^3 + 24(5)^2 + 30(5)$, or 2250 cm^3.

If $x = 10$, the volume of wood is $12(10)^3 + 24(10)^2 + 30(10)$, or $14\,700$ cm^3.

The amount of wood needed is greater in the second case.

d) The volume of wood did not double; it was more than 6 times greater. This seems reasonable. If the figure had been a cube and the sides had been x and $2x$, then the volume would have grown from x^3 to $8x^3$ times larger.

Note: A level 4 response might go further. The expression for the volume contains an x^3-term, which would create a volume 8 times greater. The x^2-term would lead to a volume that is 4 times greater, and the x-term would lead to a volume that is 2 times greater. The resulting volume will be an average of these three factors, so 6.533 times the volume is very reasonable.

Level 3 Notes
Look for the following:
- Correct solutions for parts a), b), and c)
- An attempt to explain part d)
- Mostly accurate calculations
- Explanations for algebraic steps
- Mention that the volume of a cube is 8 times larger when the sides are doubled
- Diagram labelled more completely than the one provided

What Distinguishes Level 2
At this level, look for the following:
- Partial solutions for parts a) and b)
- Solution with errors in computation or algebra for part c)
- No attempt at part d)
- Some significant errors in calculations
- Little or no explanation for algebraic steps
- Calculation of the ratio of the volumes from part c) without comment on its reasonableness
- Diagram labelled with less detail than the one provided

What Distinguishes Level 4
At this level, look for the following:
- Correct solutions for all parts of the question
- Comment that the solutions in part b) will be identical
- Accurate calculations with all necessary steps included
- Detailed explanations
- Mention that the volume of a cube is 8 times larger when the sides are doubled and the discrepancy of the magnification factor for part d)
- Comprehensively labelled diagram