5.2 Special Products

Student Text Pages
220–227

Suggested Timing
70 min

Tools
- algebra tiles

Technology Tools
- TI-89 calculator

Related Resources
- BLM 5–4 Section 5.2 Practice Master
- BLM 5–5 Section 5.2 Achievement Check Rubric
- T–7 The Computer Algebra System (CAS) on the TI-89 Calculator

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- Discuss the opening paragraph by emphasizing that the diagram is a square and that certain binomials have special patterns when expanded. (2 min)

Investigate

- Students learn best when investigating patterns. By doing so, students understand the procedures for expanding special products, rather than simply memorizing them. Both **Parts A** and **B** offer two methods: paper and pencil and CAS. If CAS is available, have the students explore using both methods. Use **T–7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity. (15 min)
- Use algebra tiles or a diagram broken into regions similar to those with algebra tiles to demonstrate the procedures for squaring binomials and the product of a sum and a difference. This is especially useful for visual learners. (10 min)

Examples

- Consolidate learning by having the students expand a few special products, such as those in **Example 1**. If students need to show extra steps, allow them to do so until they see the patterns and understand the procedures. (5 min)
- Present **Example 2** or a similar one. It is important to provide a contextual example so students make a connection to real applications. (5 min)
- Review the vocabulary in this section (perfect square trinomial, difference of squares). (2 min)

Communicate Your Understanding

- Allow students to write answers to these questions before discussing them as a class. The key is to encourage students to use their own words, rather than copying out those used in the textbook. A good supplementary question to ask the students is one that they often ask themselves: "Where does the 2 come from?" This will invariably add some levity as students see themselves asking it. (5 min)
- Some teachers prefer to do the expanding of special polynomials together with factoring special polynomials because they connect logically. If you do so, it is recommended that you wait until completing the other factoring methods so as not to confuse students into thinking that those types of questions are the norm and that other polynomials are the exception to the rule. Present the more general method, and then the special cases, which are just extensions of the general rule.
- Use **BLM 5–4 Section 5.2 Practice Master** for remediation or extra practice.

Investigate Answers (pages 220–222)

Part A

Method 1

1. $(x + 3)^2 = x^2 + 6x + 9$

2. a) $(x + 3)^2 = x^2 + 6x + 9$ **b)** $(x + 2)^2 = x^2 + 4x + 4$

c) $(x - 6)^2 = x^2 - 12x + 36$ **d)** $(x - 4)^2 = x^2 - 8x + 16$

e) $(2x + 5)^2 = 4x^2 + 20x + 25$ **f)** $(3x - 1)^2 = 9x^2 - 6x + 1$

g) $(2x - 5y)^2 = 4x^2 - 20xy + 25y^2$ **h)** $(4x + 7y)^2 = 16x^2 + 56xy + 49y^2$

3. a) The first term in each trinomial is the square of the first term in the binomial.

b) The last term in each trinomial is the square of the last term in the binomial.

c) The middle term in each trinomial is two times the first term times the last term in the binomial.

4. $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$

5. a) $(5x + 3y)^2 = (5x)^2 + 2(5x)(3y) + (3y)^2$
$= 25x^2 + 30xy + 9y^2$

b) $(7c - 4d)^2 = (7c)^2 - 2(7c)(4d) + (4d)^2$
$= 49x^2 - 56xy + 16y^2$

Method 2

2. a) $(x + 1)^2 = x^2 + 2x + 1$ **b)** $(x + 2)^2 = x^2 + 4x + 4$

c) $(x + 3)^2 = x^2 + 6x + 9$ **d)** $(x + 4)^2 = x^2 + 8x + 16$

e) $(x + 5)^2 = x^2 + 10x + 25$

3. Answers may vary. For example: The first term in each trinomial is the square of the first term in the binomial. The last term in each trinomial is the square of the last term in the binomial. The middle term in each trinomial is two times the first term times the last term in the binomial. The sign between the terms of the binomial is positive and the signs in the trinomial are also positive.

4. a) $(2x + 2)^2 = 4x^2 + 8x + 4$ **b)** $(2x + 3)^2 = 4x^2 + 12x + 9$

c) $(2x - 4)^2 = 4x^2 - 16x + 16$ **d)** $(2x - 5)^2 = 4x^2 - 20x + 25$

5. Answers may vary. For example: The first term in each trinomial is the square of the first term in the binomial. The last term in each trinomial is the square of the last term in the binomial. The middle term in each trinomial is the two times the first term times the last term in the binomial. If the sign between the terms of the binomial is positive, then the signs in the trinomial are also positive. If the sign between the terms of the binomial is negative, then the sign of the middle term in the trinomial is negative and the sign of the last term is positive.

6. a) $(3a + 2)^2 = 9a^2 + 12a + 4$ **b)** $(5m - 3)^2 = 25m^2 - 30m + 9$

c) $(4 + 2b)^2 = 4b^2 + 16b + 16$ **d)** $(7 - 3z)^2 = 9z^2 - 42z + 49$

e) $(2x + 3y)^2 = 4x^2 + 12xy + 9y^2$

7. Answers may vary. For example: No, the patterns described in step 5 have not changed.

8. $(a + b)^2 = a^2 + 2ab + b^2$; $(a - b)^2 = a^2 - 2ab + b^2$

Part B

Method 1

1. a) $(x + 3)(x - 3) = x^2 - 9$ **b)** $(2y + 5)(2y - 5) = 4y^2 - 25$

c) $(x - 4)(x + 4) = x^2 - 16$ **d)** $(3k - 7)(3k + 7) = 9k^2 - 49$

2. The binomials are alike: they have the same first term and the same last term. The binomials are different: the sign between the terms is different in each binomial.

3. a) The first term in each binomial is the square of the first term in the binomial expressions.

b) The last term in each binomial is the square of the last term in the binomial expressions.

c) When the binomials are expanded the middle terms are equal in value, but opposite in sign. When simplified, the result of adding the two terms is zero. Therefore, there are only two terms in the simplified expansion.

4. $(a + b)(a - b) = a^2 - b^2$; The rule applies to $(a - b)(a + b) = a^2 - b^2$.

5. a) $(2x + 3y)(2x - 3y) = 4x^2 - 9y^2$ **b)** $(5m + 7)(5m - 7) = 25m^2 - 49$

Method 2

2. a) $(x + 2)(x - 2) = x^2 - 4$
b) $(x - 3)(x + 3) = x^2 - 9$
c) $(2x - 1)(2x + 1) = 4x^2 - 1$
d) $(3x + 4)(3x - 4) = 9x^2 - 16$
e) $(2x + 3y)(2x - 3y) = 4x^2 - 9y^2$
f) $(3m - 4n)(3m + 4n) = 9m^2 - 16n^2$

3. The binomials are alike: they have the same first term and the same last term. The binomials are different: the sign between the terms is different in each binomial.

4. a) The first term in each binomial is the square of the first term in the binomial expressions.
b) The last term in each binomial is the square of the last term in the binomial expressions.
c) When the binomials are expanded the middle terms are equal in value, but opposite in sign. When simplified, the result of adding the two terms is zero. Therefore, there are only two terms in the simplified expansion.

5. $(a + b)(a - b) = a^2 - b^2$; The rule applies to $(a - b)(a + b) = a^2 - b^2$.

Communicate Your Understanding Responses (page 225)

C1. If the binomial has two terms of the same sign, then the middle term will be positive after squaring the binominal. If the binomial has one negative term, then the middle term will be negative after squaring the binominal.

C2. a) Square the first term, find twice the product of the first and last term, and square the last term. The middle term is subtracted since it is a square of a difference. The result is $x^2 - 4x + 4$.
b) Find the product of the first terms and the product of the last terms. The result is $x^2 - 4$.

Common Errors

- Some students may miss the middle term when squaring a binomial.

R_x Have students expand $(a + b)(a + b)$ to see that there are two equal middle terms. Repeat with a few other examples.

Accommodations

Gifted and Enrichment—Challenge students to research digital cameras and the history of photography in the library or on the Internet. Have students present their findings to the class.

Visual—Provide students with the diagrams needed to model the steps required to square binomial expressions.

Spatial—Allow students to use a graphing calculator or CAS to check their answers.

Motor—Let students work in groups when completing the Investigations in this section.

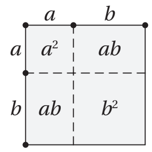
Practise

- Assign a variety from **questions 1** through **6** for skills practice to help students remember the procedures. Look for students missing the middle term, or not squaring the second term, of a squared binomial.
- In **question 7**, have students use more than one method of verifying their results. Explain the importance of verifying answers, i.e., to ensure that they have done the procedure correctly.
- **Question 10** makes a valuable connection to Chapter 4 by investigating a parabola.
- **Question 13** refers to the Chapter Problem. Suggest to students that they draw a top view and label it fully. Students can answer this question now or save it for the Chapter Problem Wrap-Up.
- In **question 16**, remind students to think about how a translation to the right affects a quadratic relation.
- **Question 17** is an interesting, non-traditional application of binomial expansions.
- For **question 20**, refer students to **Example 2** part b).

Achievement Check Sample Solution, question 18, page 227

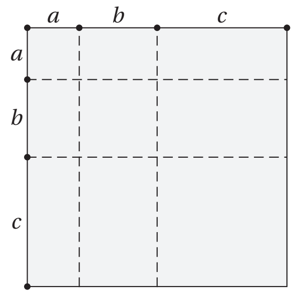
Provide students with **BLM 5–5 Section 5.2 Achievement Check Rubric** to help them understand what is expected.

18. a)



$(a + b)^2 = a^2 + 2ab + b^2$

b)



$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

c) The area is $(x + 2)(9 - 4x) = -4x^2 + x + 18$, which must be positive. (Note: This step may be omitted)

The length and width must be positive values.

From the length $(x + 2)$, $x = -1, 0, 1, 2, 3, \ldots$

From the width $(9 - 4x)$, $x = \ldots, -1, 0, 1, 2$.

Since x must have the same values for both the length and the width, then $x = -1, 0, 1, 2$.

The possible area values are 13, 18, 15, and 4 square units.

Literacy Connections

Discuss the Literacy Connections on page 223 with students. Have students look up "conjugate" in a dictionary and discuss the idea of reciprocal relationships as well. Another Literacy Connections on page 227 looks at "mega" as a prefix. Have students investigate the origin of "mega" (from the Greek *megas*, meaning great). As an extension/enrichment, have students find other prefixes used in mathematics that are derived from Greek.

Question 20 uses the example of kinetic energy—again the root is Greek.

Add "conjugate," "perfect square trinomial," and "difference of squares" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	n/a
Reasoning and Proving	n/a
Reflecting	7, 10, 12
Selecting Tools and Computational Strategies	18
Connecting	8, 9, 11, 12, 14–16, 18, 20
Representing	1, 8, 9, 11–13, 16–18, 20
Communicating	15

Student Success

Use a concept attainment strategy to have students develop patterns for special products.

Refer to the introduction of this Teacher's Resource for more information about how to use a concept attainment strategy.

Ongoing Assessment

- Use Achievement Check question 18 to monitor student success. See Achievement Check Answers and **BLM 5–5 Section 5.2 Achievement Check Rubric**.
- Chapter Problem question 13 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.