# 5.3 Common Factors

**Student Text Pages**
228–235

**Suggested Timing**
70 min

**Tools**
- algebra tiles

**Technology Tools**
- TI-89 calculator

**Related Resources**
- BLM 5–6 Section 5.3 Practice Master
- T–7 The Computer Algebra System (CAS) on the TI-89 Calculator
- A–9 Communication General Scoring Rubric

**TI-Navigator™**

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

## Teaching Suggestions

- The opening paragraph gives a hint about the concept of common factoring. A quick discussion of the paragraph invites students to go through the investigations. (2 min)

### Investigate

- In **Investigate A**, algebra tiles or a diagram provide a visual approach to learning that many students need. Students can manipulate the tiles and can "see" the factors. (5 min)
- **Investigate B** offers two methods: paper and pencil and CAS. If students understand the concept of the greatest common factor of an algebraic list, they can understand common factoring more easily. Ensure they understand the first four steps before moving on. If you have CAS available, it can be used as a patterning approach to learning common factoring. Use **T–7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support this activity. (5 min)

### Examples

- Present the **Examples**. (20 min)
- For **Example 1**, if only one set of algebra tiles is available, or because of time constraints, demonstrate with the tiles on an overhead projector. Do a few examples such as this one.
- In **Example 2**, parts a), b), and c) are scaffolded so students can see the division concept of factoring. Students should progress to minimal steps as in part d). Part e) is important to show that not all polynomials are factorable.
- **Example 3** is important as a lead-in to Section 5.5 Factoring Quadratic Expressions of the Form $ax^2 + bx + c$. Some students may understand the following technique:

  Let $y + 1 = A$.

  $3x(y + 1) + 7z(y + 1)$

  $= 3xA + 7zA$

  $= A(3x + 7z)$

  $= (y + 1)(3x + 7z)$

  Most students will be confused by it, however, because introducing another variable, $A$, seems to add to the complexity. If you use the method in the textbook, ask the students what expression they see twice. That is the common factor. Writing it down as the first factor helps understanding, but they need to recognize that the order of the factors is not important.
- **Example 4** is an extension of **Example 3**. It is also important as a lead-in to Section 5.5.

### Communicate Your Understanding

- Allow the students to write down their responses to the Communicate Your Understanding questions before discussing them as a class. Students may misinterpret question C2 as being correct statements. Caution them that they are looking for the errors. (10 min)
- Use **BLM 5–6 Section 5.3 Practice Master** for remediation or extra practice.

## Investigate Answers (pages 228–230)

### A

**1. a)** 

**b)** $2x + 4 = 2(x + 2)$

**2.** $6x + 18 = 1(6x + 18)$, $6x + 18 = 2(3x + 9)$, $6x + 18 = 3(2x + 6)$, $6x + 18 = 6(x + 3)$; 4 rectangles

**3.** $x^2 + 2x = x(x + 2)$

**4.** $2x^2 + 4x = 2(x^2 + 2x)$, $2x^2 + 4x = x(2x + 2)$, $2x^2 + 4x = 2x(x + 2)$

**5. a)** $3x + 3 = 3(x + 1)$

**b)** $4x + 10 = 2(2x + 5)$

**c)** $x^2 + 4x = x(x + 4)$

**d)** $2x^2 + 6x = 2x(x + 3)$

**e)** $2x + 5$ does not factor. The expression does not have a common factor.

**f)** $4x^2 + 10x = 2x(2x + 5)$

**6.** Answers will vary. For example: Think of the polynomial as a rectangular area, then it can be expressed as a product of the length and width.

### B

#### Method 1

**1. a)** $12 = 2 \times 2 \times 3$
$8 = 2 \times 2 \times 2$
The greatest common factor is $2 \times 2 = 4$.

**b)** $15 = 3 \times 5$
$25 = 5 \times 5$
The greatest common factor is 5.

**c)** $4 = 2 \times 2$
$10 = 2 \times 5$
$6 = 2 \times 3$
The greatest common factor is 2.

**d)** $6 = 2 \times 3$
$18 = 2 \times 3 \times 3$
$24 = 2 \times 2 \times 2 \times 3$
The greatest common factor is $2 \times 3 = 6$.

**2. a)** $12 = 2 \times 2 \times 3$
$9 = 3 \times 3$
The greatest common factor is 3.

**b)** $12 = 3 \times 6$, $9 = 3 \times 3$; Division was used to obtain the second factor.

**3. a)** $7^3 = 7 \times 7 \times 7$
$7^2 = 7 \times 7$
The greatest common factor is $7 \times 7 = 7^2$.

**b)** $5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$
$5^4 = 5 \times 5 \times 5 \times 5$
The greatest common factor is $5 \times 5 \times 5 \times 5 = 5^4$.

**c)** $x^2 = x \times x$
$x = x$
The greatest common factor is $x$.

**d)** $x^3 = x \times x \times x$
$x^4 = x \times x \times x \times x$
The greatest common factor is $x \times x \times x = x^3$.

**4. a)** $x^6 = x \times x \times x \times x \times x \times x$
$x^4 = x \times x \times x \times x$
The greatest common factor is $x \times x \times x \times x = x^4$.

**b)** $x^6 = x^4 \times x^2$, $x^4 = x^4 \times 1$; Division was used to obtain the second factor.

**5. a)** $2x$

**b)** $2x^2 + 4x = 2x \times x + 2x \times 2$

**c)** $2x^2 + 4x = 2x(x + 2)$;
The polynomial $x + 2$ is the second factor.

**d)** $2x(x + 2) = 2x^2 + 4x$

**6. a)** $3x^2 + 6x = 3x(x + 2)$

**b)** $2x^2 + 8x = 2x(x + 4)$

**c)** $4y + 10y^2 = 2y(2 + 5y)$

**d)** $7y^3 + 14y^2 = 7y^2(y + 2)$

**7.** Answers will vary. For example: Find the GCF. Write it as the first factor. Divide each term by the GCF, writing the result inside brackets as the second factor.

**Method 2**

**2. a)** $2x + 2 = 2(x + 1)$ **b)** $2x + 4 = 2(x + 2)$
**c)** $2x + 6 = 2(x + 3)$ **d)** $2x + 8 = 2(x + 4)$
**e)** $2x + 10 = 2(x + 5)$

**3.** Answers may vary. For example: Each pair of factors has the same common factor, 2. Each pair of factors has a different expression for the second factor. Answers will vary.

**4. a)** $3x^2 + 6x + 9 = 3(x^2 + 2x + 3)$ **b)** $3x^2 + 12x + 15 = 3(x^2 + 4x + 5)$

**5. a)** The GCF is 3.
**b)** The operation of division provides the coefficients of the terms in the second factor.

**6. a)** $6x^2 + 9x + 24 = 3(2x^2 + 3x + 8)$ **b)** The GCF is 3.
**c)** Yes.

**7. a)** $2x^2 + 4x = 2x(x + 2)$ **b)** $2x^2 + 8x = 2x(x + 4)$

**8. a)** The GCF of the coefficients is 2. The GCF of the variables is $x$.
**b)** The GCF for both polynomials in step 7 is the product of the GCF of the coefficients, 2, multiplied by the GCF of the variables, $x$.
**c)** The operation of division provides the terms in the second factor.

**9. a)** $6x^2 + 15x = 3x(2x + 5)$ **b)** The GCF is $3x$.
**c)** Yes.

**10.** Answers will vary. For example: Find the GCF. Write it as the first factor. Divide each term by the GCF, writing the result inside brackets as the second factor.

**Communicate Your Understanding Responses (page 234)**

**C1.** The expression represented by the algebra tiles to the left is $2x$. The expression represented by the algebra tiles at the top is $x + 2$. The expression represented by the algebra tiles in the centre is $2x^2 + 4x$. In factored form, $2x^2 + 4x = 2x(x + 2)$.

**C2. a)** $5x$ was subtracted from the second term rather than divided into the second term.
$35x^2 - 5x = 5x(7x - 1)$
**b)** 4 is not a common factor of the second term. The greatest common factor is $y^2$.
$4y^3 + 7y^2 = y^2(4y + 7)$
**c)** The greatest common factor was not used.
$16k^3m^2 - 8k^2m = 8k^2m(2km - 1)$.
**d)** The greatest common factor was not used.
$9a^3b^5 + 6a^2b^4 = 3a^2b^4(3ab + 2)$.

**C3. a)** Factor out the common factor $(d - 5)$. $3c(d - 5) - 8(d - 5) = (d - 5)(3c - 8)$.
**b)** Factor out the common factor $2x$ from the first two expressions. Factor out the common factor $-3$ from the last two expressions. Then, factor out the common factor $5x - 7y$ from each resulting expression.
$10x^2 - 14xy - 15x + 21y$
$= 2x(5x - 7y) - 3(5x - 7y)$
$= (5x - 7y)(2x - 3)$

## Practise

**Common Errors**

- Some students may not find the greatest common factor.

$R_x$ Have students look at the resulting polynomial after factoring to see if there is anything else that will divide evenly. You may also illustrate using algebra tiles.

- For **questions 1** through **6**, assign a variety to provide practice in the basic procedures. Ensure that students find the greatest common factor, especially for polynomials with degree greater than 2.
- **Questions 8** and **9** provide good, simple applications of factoring.
- In **question 10**, have students draw a diagram first, then make a chart to organize their thinking.
- **Question 12** refers to the Chapter Problem. Assign the question now, or save it until the Chapter Problem Wrap-Up. It is a good contextual application of the concepts being taught.
- **Question 13** is fairly abstract, since the differences are all algebraic. Suggest to students that they think of how to write the differences if numbers were involved.

## Accommodations

**Gifted and Enrichment**—Challenge students to research scientific formulas that can be expressed in a factored form using common factoring.

**Visual**—Let students use colour-coding when factoring expressions by grouping. For example:

$$\begin{aligned} & ax + ay + bx + by \\ = \ & a(x + y) + b(x + y) \\ = \ & (x + y)(a + b) \end{aligned}$$

**Spatial**—Allow students to use algebra tiles when completing questions in this section.

**Memory**—Encourage students to show small sequential steps when factoring expressions where the common factor is a monomial algebraic expression. For example:

$$\begin{aligned} & x^2y^3 + x^3y^2 \\ = \ & x \times x \times y \times y \times y \\ & + x \times x \times x \times y \times y \\ = \ & x \times x \times y \times y \times (y + x) \\ = \ & x^2y^2(y + x) \end{aligned}$$

## Student Success

Use a timed retell strategy to have students explain how to identify common factors.

Refer to the introduction of this Teacher's Resource for more information about how to use a timed retell strategy.

- **Question 14** refers to Chapter 4, where students let $y = 0$ and found $x$ by inspecting the graph.
- **Question 15** is a good communication question that invites level 3 and 4 responses. Use **A–9 Communication General Scoring Rubric** when assessing students.

## Literacy Connections

Add "factoring" to the Word Wall.

## Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectations | Selected Questions |
|---|---|
| Problem Solving | 17 |
| Reasoning and Proving | 7–10, 15, 17 |
| Reflecting | 8–10 |
| Selecting Tools and Computational Strategies | 17 |
| Connecting | 10, 12–16 |
| Representing | 2, 7, 12, 13, 17 |
| Communicating | 8, 9, 15 |

## Ongoing Assessment

- Chapter Problem question 12 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.