5.6 Factor a Perfect Square Trinomial and a Difference of Squares

Student Text Pages
248–255

Suggested Timing
70 min

Technology Tools
- TI-89 calculator

Related Resources
- BLM 5–10 Section 5.6 Practice Master
- T–7 The Computer Algebra System (CAS) on the TI-89 Calculator
- A–9 Communication General Scoring Rubric

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

Investigate

- **Investigates A** and **B** both offer two methods: pencil and paper and CAS. Students learn best how to factor these special examples when exploring patterns. Suggest that students relate these examples to those from Section 5.2.
- If you have CAS available, use **T–7 The Computer Algebra System (CAS) on the TI-89 Calculator** to support these activities. (30 min)

Examples

- Present **Examples 1**, **2**, **3**, and **4**. Ensure students understand the special relationships between the sum and product.
- In **Example 4**, illustrate the final factored form using rectangles or algebra tiles. (10 min)
- Students may ask about how to factor $x^2 + y^2$. Compare $x^2 - 1$ to $x^2 + 1$. For $x^2 - 1$, the product is -1 and the sum is 0, so the numbers are 1 and -1. For $x^2 + 1$, the product is $+1$ and the sum is 0. No pair of numbers satisfies these conditions. This is always the case for $x^2 + y^2$.

Communicate Your Understanding

- Assign the questions in this section. For question C1, see the above comment. Question C2 is an excellent question that assesses students' understanding of factoring perfect squares. (10 min)
- Use **BLM 5–10 Section 5.6 Practice Master** for remediation or extra practice.

Investigate Answers (pages 248–250)

A

Method 1

1. a) $(x + 1)(x - 1) = x^2 - 1$ **b)** $(y + 2)(y - 2) = y^2 - 4$
c) $(3c - 10)(3x + 10) = 9c^2 - 100$ **d)** $(2m - 4)(2m + 4) = 4m^2 - 16$

2. Answers may vary. The terms in the binomials are the same, but the signs between them are different.

3. a) The first term in the simplified expansion is the square of the first terms of the two binomials.

b) The last term in the simplified expansion is the square of the last terms of the two binomials, preceded by a minus sign.

4. Answers may vary. A difference of squares is an expression of the form $a^2 - b^2$ that involves the subtraction of two squares.

5. a) $x^2 - 25 = (x + 5)(x - 5)$ **b)** $y^2 - 36 = (y + 6)(y - 6)$
c) $16k^2 - 49 = (4k + 7)(4k - 7)$ **d)** $25n^2 - 144 = (5n + 12)(5n - 12)$

6. $a^2 - b^2 = (a + b)(a - b)$

7. $100y^2 - 49x^2 = (10y)^2 - (7x)^2$
$= (10y + 7x)(10y - 7x)$

Method 2

2. a) $x^2 - 81 = (x - 9)(x + 9)$ **b)** $y^2 - 64 = (y - 8)(x + 8)$
c) $25d^2 - 36 = (5d - 6)(5d + 6)$ **d)** $16k^2 - 121 = (4k - 11)(4k + 11)$
e) $144b^2 - 25k^2 = (12b - 5k)(12b + 5k)$ **f)** $4n^2 - 49p^2 = (2n - 7p)(2n + 7p)$

3. Answers may vary. For example: A difference of squares is an expression of the form $a^2 - b^2$ that involves the subtraction of two squares.

4. a) Answers may vary. The terms in the binomials are the same, but the signs between them are different.
b) The first term of the polynomial is the square of the first terms of the factors.
c) The last term of the polynomial is the square of the last terms of the factors, preceded by a minus sign.

5. $a^2 - b^2 = (a + b)(a - b)$

B

Method 1

1. a) $(x + 3)^2 = x^2 + 6x + 9$ **b)** $(y - 5)^2 = y^2 - 10y + 25$
c) $(k + 7)^2 = k^2 + 14k + 49$ **d)** $(2h + 3)^2 = 4h^2 + 12h + 9$
e) $(3b - 5)^2 = 9b^2 - 30b + 25$

2. a) The first term of each trinomial is the square of the first term in each binomial.
b) The last term of each trinomial is the square of the last term in each binomial.
c) The middle term of each trinomial is 2 × first term of binomial × last term of binomial. The middle term is preceded by a minus sign if the binomial being squared is a difference.

3. Answers may vary. For example: A perfect square trinomial is a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

4. a) $x^2 + 12x + 36 = (x + 6)^2$ **b)** $y^2 - 6y + 9 = (y - 3)^2$
c) $4k^2 + 20k + 25 = (2k + 5)^2$ **d)** $9k^2 - 24k + 16 = (3k - 4)^2$

5. $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

6. $x^2 + 14x + 49 = (x)^2 + 2(x)(7) + (7)^2$
$= (x + 7)^2$

Method 2

2. a) $x^2 + 8x + 16 = (x + 4)^2$ **b)** $y^2 - 10y + 25 = (y - 5)^2$
c) $4k^2 - 20k + 25 = (2k - 5)^2$ **d)** $9k^2 + 24k + 16 = (3k + 4)^2$
e) $25t^2 + 30t + 9 = (5t + 3)^2$ **f)** $16z^2 - 8z + 1 = (4z - 1)^2$

3. a) The first term of each trinomial is the square of the first term in each binomial.
b) The last term of each trinomial is the square of the last term in each binomial.
c) The middle term of each trinomial is 2 × first term of binomial × last term of binomial. The middle term is preceded by a minus sign if the binomial being squared is a difference.

4. Answers may vary. For example: A perfect square trinomial is a trinomial of the form $a^2 + 2ab + b^2$ or $a^2 - 2ab + b^2$.

5. $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$

Communicate Your Understanding Responses (page 253)

C1. Answers will vary. For example: There are no two integers that have a product of 9 and a sum of 0.

C2. Answers will vary. For example: When squaring a binomial, the O and I steps of FOIL will create two terms that are the same. When the two terms from this step are added together, this will lead to the coefficient 2 in front of the middle term.

Common Errors

- Some students may still attempt to factor perfect squares using a longer method.

R_x Have students break down the first and last terms into their square roots. Then, check to see if the middle term is two times the product of the square roots.

Accommodations

Perceptual—Provide students with a series of index cards with the steps to simplifying and factoring a more complicated expression, such as $(4x + 3)^2 - (x - 3)^2$. Have the students arrange the index cards in the correct order to simplify and factor the expression.

Spatial—Let students use CAS to check their answers.

Memory—Encourage students to create cue cards to remember the rules for factoring a difference of squares and a perfect square trinomial.

Student Success

Construct a decision tree for students to identify and sequence the steps in factoring general algebraic expressions.

Refer to the introduction of this Teacher's Resource for more information about how to use a decision tree strategy.

Practise

- Assign all parts of **questions 1** through **4**, which will provide a good variety of practice.
- **Question 5** is a good communication question. Use **A–9 Communication General Scoring Rubric** when assessing students.
- In **question 11**, have students draw a rectangle and decide whether it is a square.
- **Question 13** refers to the Chapter Problem. Suggest that a top-view diagram would help, and that students refer to **question 7** for a hint.
- In **question 14**, remind students of the formula for the area of a circle, and to look for a perfect square or difference of squares.
- **Question 16** refers to a parabola, an important connection to Chapter 4.
- Students enjoy exploring patterns, and **question 18** encourages a concrete connection to the algebra. Suggest they make a table of values to assist them in recognizing patterns.
- For **question 19**, have students refer to **question 11**.
- For **questions 20** parts c) to f), **21**, and **22**, suggest to students that they replace x^2 or x^3 with y.

Literacy Connections

Discuss question 19, which refers to a polyhedron and the shapes that it might take. This will help to solidify the idea that "poly" refers to many. It can represent a number of different cases in the same way that a polygon is a many-sided figure and a polynomial is a many-termed algebraic expression. This may provide you with an opportunity for enrichment.

Add "polyhedron" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	n/a
Reasoning and Proving	5, 8, 9, 14, 15, 19–25
Reflecting	13, 18, 22
Selecting Tools and Computational Strategies	20
Connecting	7, 12–14, 17, 19, 23–25
Representing	13, 15, 18
Communicating	5, 12, 15, 18, 22

Ongoing Assessment

- Chapter Problem question 13 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.