6.1 Maxima and Minima

Student Text Pages
264–273

Suggested Timing
70–140 min, depending on whether you wish to cover factoring of the a coefficient during a different class period

Tools
- algebra tiles
- grid paper

Technology Tools
- graphing calculator
- computer
- *The Geometer's Sketchpad®*
- Algebra Tiles.gsp

Related Resources
- G–1 Grid Paper
- G–2 Placemat
- G–3 Coordinate Grids
- T–4 *The Geometer's Sketchpad®* 3
- T–5 *The Geometer's Sketchpad®* 4
- BLM 6–4 Section 6.1 Practice Master

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the files for this section.

Teaching Suggestions

- It is important for students to see that completing the square is the process of forming a perfect square geometrically. Students best learn the process of completing the square through a hands-on approach or by seeing a demonstration using algebra tiles, rather than by simply being taught the algorithm.

Investigate

- Have students work through the **Investigate** using algebra tiles or virtual algebra tiles in *The Geometer's Sketchpad®*.
- If needed, use **T–4 *The Geometer's Sketchpad®* 3** or **T–5 *The Geometer's Sketchpad®* 4** to support this activity.
- Stress the importance of building a square by adding the appropriate number of unit tiles. It is also important for the students to communicate their understanding before moving on to an algebraic method. (20 min)

Examples

- **Examples 1** and **2** consolidate learning by going through the steps using algebra tiles and relating the steps to an algebraic algorithm.
- Use examples of the form $y = x^2 + bx + c$, where b is a positive even number. Negatives are much more difficult to illustrate with algebra tiles, so introduce them after the students demonstrate an understanding of the procedure. Provide sufficient examples where factoring first is necessary. (15 min)
- Stress contextual examples so that quadratics have meaning to the students. Paths of projectiles, such as the one used in **Example 3**, and parabolic arches under bridges make good examples. Keep to examples where there are no fractions when completing the square. Also, explain the importance of multiple representations—words, numbers, graphs, and algebraic expressions. (5 min)
- **Example 4** may give students some difficulties. Have them build a table such as the one shown here and investigate the pattern of price changes and number of rentals. (5 min)

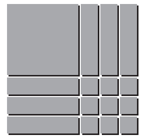
Number of Decreases	Price ($)	Number of Rentals	Revenue ($)
0	12.00	36	$12 \times 36 = 432$
1	11.50	38	$11.5 \times 38 = 437$
2	11.00	40	$11 \times 40 = 440$
⋮	⋮	⋮	⋮
x	$12 - 0.5x$	$36 + 2x$	$(12 - 0.5x)(36 + 2x)$

Communicate Your Understanding

- Review the vocabulary in this section (completing the square).
- Have students discuss the questions with a partner. Take them up as a class and discuss any misunderstandings or errors. (10 min)
- Use **BLM 6–4 Section 6.1 Practice Master** for remediation or extra practice.

Investigate Answers (pages 264–265)

1. a)



b) $(x^2 + 6x + 9) = (x + 3)(x + 3)$

2. a)

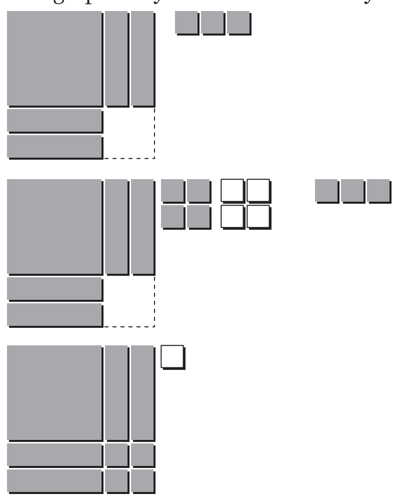
b) $(x^2 + 4x + 4) = (x + 2)(x + 2)$

3. a) Algebra tiles representing $x^2 + 6x$ are arranged to partially form a square, with half of the x-tiles placed to the right of the x^2-tile and half of the x-tiles placed below the x^2-tile. The five unit tiles are off to the side. To complete the perfect square, 3×3, or 9 unit tiles are added. To preserve the original expression, 9 negative unit tiles are also added. With the square completed, the unit tiles are simplified.

b)
$$\begin{aligned} y &= x^2 + 6x + 5 \\ &= (x^2 + 6x) + 5 \\ &= (x^2 + 6x + 9 - 9) + 5 \\ &= (x^2 + 6x + 9) - 4 \\ &= (x + 3)^2 - 4 \end{aligned}$$

c) The graphs of $y = x^2 + 6x + 5$ and $y = (x + 3)^2 - 4$ are the same.

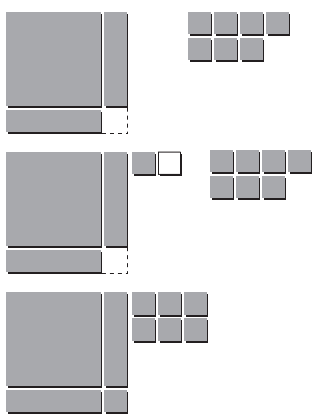
4. a)



b)
$$\begin{aligned} y &= x^2 + 4x + 3 \\ &= (x^2 + 4x) + 3 \\ &= (x^2 + 4x + 4 - 4) + 3 \\ &= (x^2 + 4x + 4) - 1 \\ &= (x + 2)^2 - 1 \end{aligned}$$

c) The graphs of $y = x^2 + 4x + 3$ and $y = (x + 2)^2 - 1$ are the same.

5.



$$\begin{aligned} y &= x^2 + 2x + 7 \\ &= (x^2 + 2x) + 7 \\ &= (x^2 + 2x + 1 - 1) + 7 \\ &= (x^2 + 2x + 1) + 6 \\ &= (x + 1)^2 + 6 \end{aligned}$$

Common Errors

- Some students may not factor the coefficient of x^2 properly.

R_x Have students complete the square using algebra tiles. Have them investigate $x^2 + 2x + 3$ and $2x^2 + 4x + 5$. Make the connection to the physical re-arrangement from $4x$ to $2x$ by $2x$.

Accommodations

Visual—Allow students to use graphing calculators to determine the relationship between the vertex (h, k) of a quadratic relation expressed in the form $y = a(x - h)^2 + k$, and the vertex (h, k) of the quadratic relation on the graphing calculator screen.

Perceptual—Provide students with algebra tiles to use when writing quadratic equations in the form $y = a(x - h)^2 + k$ when completing the square.

Motor—Let students work with a partner who will write the steps for completing the square for a quadratic relation.

Memory—Encourage students to show extra steps when factoring perfect square trinomials. For example:

$$\begin{aligned} & x^2 + 10x + 25 \\ &= (x + 5)(x + 5) \\ &= (x + 5)^2 \end{aligned}$$

Communicate Your Understanding Responses (page 270)

C1. a) Group the first two terms, $x^2 + 10x$. Add and subtract the square of half of the linear coefficient. Factor $x^2 + 10x + 25$ and simplify $15 - 25$. The result is $y = (x + 5)^2 - 10$.

b) Group the first two terms, $-2x^2 - 4x$. Remove the common factor -2 from the first two terms. Add and subtract the square of half the linear coefficient. Factor $x^2 + 2x + 1$ and distribute the -2 to -1 before simplifying with -5. The result is $y = -2(x + 1)^2 - 3$.

C2. a) The vertex $(-5, -10)$ is a minimum point since a is positive.

b) The vertex $(-1, -3)$ is a maximum point since a is negative.

C3. The graph of $y = a(x - h)^2 + k$ is the graph of $y = x^2$ vertically stretched by a factor of a and translated h units to the right and k units upward.

Practise

- Each of **questions 1** through **11** targets specific basic skills. Assign all questions or part of each question. If there is time in class, take up some of these questions before moving on to Connect and Apply.
- **Question 13** shows that the distance-time and distance-distance relationships are both quadratic.
- **Questions 12**, **14**, and **15** consolidate understanding of basic applications of completing the square.
- **Question 16** relates to **Example 4**. Stress multiple representations of the relation. Students will need to multiply price and number expressions $(4 - 0.5x)(120 + 20x)$.
- **Question 18** refers to the Chapter Problem. Assign it now or save it until the Chapter Problem Wrap-Up at the end of the chapter.
- **Question 22** includes a restriction on the values of t. Explain to the students that mathematical models are generally only valid for a specific set of values. (The term “domain” is not introduced until grade 11.)
- For **question 23**, have students draw a diagram and write the length in terms of the width: $2l = 20 - 2w$.
- For **question 24**, have students draw a diagram and write an expression for the length in terms of the width: $l = 200 - 2w$.
- **Question 25** relates to **question 13**. Students should compare the graphs and relate the angle of elevation to the maximum height. For those students who have some background in trigonometry, the coefficient of t will be $v_0\sin\theta$.
- **Question 26** requires students to substitute twice in order to determine the values of all the coefficients.
- **Question 27** is best done through algebraic manipulation.
- In grade 11, students will extend completing the square to quadratics involving fractions and decimals. Provide a few of the following questions to those students needing a challenge.

a) $y = x^2 + 5x + 7$
b) $y = x^2 - x + 3$
c) $y = x^2 - 9x - 4$
d) $y = 5x^2 + 5x + 2$
e) $y = 2x^2 - 2x + 5$
f) $y = 1.5x^2 + 7.5x - 3.5$

Student Success

One suggestion for a journal entry is to research real-life applications and describe one of them. Another is to provide an equation, for example, $y = -2(x - 3)^2 + 4$, and ask students to describe everything they know about it.

Use a placemat activity to summarize critical information from this section. Use **G–2 Placemat** to support this activity.

Refer to the introduction of this Teacher's Resource for more information about how to use a placemat strategy.

Literacy Connections

Draw attention to the marginal item on page 265, which explains the meaning of completing the square. Once students have been through the process with algebra tiles, show them the physical square created and remind them that to complete something means to fill it. Therefore, completing the square is a process of adding a piece to create a square.

Question 30 offers an opportunity for enrichment. Students can look up the full story behind the word "googol" in a source on the history of mathematics.

Create a Word Wall for this chapter. Include "symmetry," "maxima," and "minima," in addition to the vocabulary term for this section.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	16, 21, 23, 24, 26, 28–30
Reasoning and Proving	11, 19, 21, 27
Reflecting	16, 27
Selecting Tools and Computational Strategies	n/a
Connecting	12–16, 18, 20–26, 28
Representing	1, 5, 6, 9–11, 13, 16, 22–26, 28, 29
Communicating	n/a

Ongoing Assessment

- Chapter Problem question 18 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.