# 6.4 The Quadratic Formula

**Student Text Pages**
292–303

**Suggested Timing**
70 min

**Tools**
- grid paper

**Technology Tools**
- graphing calculator

**Related Resources**
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 6–8 Section 6.4 Practice Master
- BLM 6–9 Section 6.4 Achievement Check Rubric

**TI-Navigator™**

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

## Teaching Suggestions

### Investigate

- The curriculum expectations state that students are only expected to understand the development of the quadratic formula, not reproduce its development. **Investigate A** supports this understanding by using completing the square to solve quadratic equations.
- This leads into **Investigate B**, which transfers the process to the development of the quadratic formula. By using the side-by-side method, students will experience fewer difficulties with the algebraic development of the formula. (20 min)

### Examples

- Solve a few examples similar to **Example 1**. Have students list what is known before substituting into the formula. (10 min)
- **Examples 2** and **3** relate the quadratic formula to finding the *x*-intercepts of a parabola. Include examples with one intercept and no intercepts. (10 min)
- Include contextual examples, such as **Example 4**. The use of the quadratic formula allows the use of decimals and fractions, and less "forced" situations. (5 min)
- The solutions in **Example 1** part b) and **Example 2** indicate that the equations could have been solved by factoring. Encourage students to select the method most efficient for the problem at hand.
- Include a discussion of when it is appropriate to leave answers in exact versus approximate form. At this point, it is acceptable to have students leave their answers in exact form (e.g., $x = \frac{-2 \pm \sqrt{7}}{3}$), as long as the denominator is positive. Encourage students to convert to decimal form only when needed for a specific context, such as distance measurement or graphing. Also, they should not round off until the final answer, and then only to the precision indicated.

### Communicate Your Understanding

- Have students complete the **Making Connections** mind map at the end of this section, on page 303.
- Review the vocabulary in this section (quadratic formula).
- Have students discuss the questions with a partner and write down their answers before discussing as a class.
- In question C1, students may discover the properties of the discriminant for two (discriminant is positive), one (discriminant is zero), and no intercepts (discriminant is negative). Although it is not developed as a concept until grade 11, encourage these students to develop their ideas. (10 min)
- Use **BLM 6–8 Section 6.4 Practice Master** for remediation or extra practice.

**Investigate Answers (pages 292–293)**

**A**

**1.** The graph of $y = x^2 - 9$ has two $x$-intercepts.

**2. a)**
$$\begin{aligned} x^2 - 9 &= 0 \\ x^2 &= 9 \\ x &= \pm 3 \end{aligned}$$

**b)** 2

**c)** There are two roots and two $x$-intercepts. The roots of $x^2 - 9 = 0$ are the $x$-intercepts of the graph of $y = x^2 - 9$.

**3. a)** The graph of $y = (x + 2)^2 - 9$ has two $x$-intercepts.

**b)**
$$\begin{aligned} (x + 2)^2 - 9 &= 0 \\ (x + 2)^2 &= 9 \\ x + 2 &= \pm 3 \end{aligned}$$
$$\begin{aligned} x + 2 &= -3 \quad \text{or} \quad & x + 2 &= 3 \\ x &= -3 - 2 & x &= 3 - 2 \\ x &= -5 & x &= 1 \end{aligned}$$

**c)** There are two roots and two $x$-intercepts. The roots of $(x + 2)^2 - 9 = 0$ are the $x$-intercepts of the graph of $y = (x + 2)^2 - 9$.

**4. a)**
$$\begin{aligned} x^2 + 10x + 16 &= 0 \\ (x^2 + 10x) + 16 &= 0 \\ (x^2 + 10x + 5^2 - 5^2) + 16 &= 0 \\ (x^2 + 10x + 5^2) - 5^2 + 16 &= 0 \\ (x + 5)^2 - 9 &= 0 \\ (x + 5)^2 &= 9 \\ x + 5 &= \pm 3 \end{aligned}$$
$$\begin{aligned} x + 5 &= -3 \quad \text{or} \quad & x + 5 &= 3 \\ x &= -3 - 5 & x &= 3 - 5 \\ x &= -8 & x &= -2 \end{aligned}$$

**b)** Solve the equation $x^2 + 10x + 16 = 0$ by completing the square for the expression $x^2 + 10x$; isolating the perfect square, $(x + 5)^2$; taking the square root of both sides; and finally solving for $x$.

**5.** Answers may vary. For example: Yes, if the quadratic relation has one or more $x$-intercepts.

**6. a)**
$$\begin{aligned} 3x^2 + 30x + 48 &= 0 \\ x^2 + 10x + 16 &= 0 \\ (x^2 + 10x) + 16 &= 0 \\ (x^2 + 10x + 5^2 - 5^2) + 16 &= 0 \\ (x^2 + 10x + 5^2) - 5^2 + 16 &= 0 \\ (x + 5)^2 - 9 &= 0 \\ (x + 5)^2 &= 9 \\ x + 5 &= \pm 3 \end{aligned}$$
$$\begin{aligned} x + 5 &= -3 \quad \text{or} \quad & x + 5 &= 3 \\ x &= -3 - 5 & x &= 3 - 5 \\ x &= -8 & x &= -2 \end{aligned}$$

**b)**
$$\begin{aligned} 5x^2 - 20x - 52 &= 0 \\ 5(x^2 - 4x) - 52 &= 0 \\ 5(x^2 - 4x + (-2)^2 - (-2)^2) - 52 &= 0 \\ 5(x^2 - 4x + (-2)^2) - 5(-2)^2 - 52 &= 0 \\ 5(x - 2)^2 - 72 &= 0 \\ (x - 2)^2 &= \frac{72}{5} \\ x - 2 &= \pm\sqrt{\frac{72}{5}} \end{aligned}$$
$$\begin{aligned} x - 2 &= -\sqrt{\frac{72}{5}} \quad \text{or} \quad & x - 2 &= \sqrt{\frac{72}{5}} \\ x &= 2 - \sqrt{\frac{72}{5}} & x &= 2 + \sqrt{\frac{72}{5}} \end{aligned}$$

**B**

Solve the equation $2x^2 + 5x + 1 = 0$ by dividing each term in the equation by the coefficient of the $x^2$-term, 2; completing the square for the expression $x^2 + \frac{5}{2}x$; isolating the perfect square expression, $\left(x+\frac{5}{4}\right)^2$; taking the square roots of both sides; and finally solving for $x$. The same steps can be completed for the quadratic equation of the form $ax^2 + bx + c = 0$, $a \neq 0$, to obtain the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Communicate Your Understanding Responses (page 300)**

**C1. a)** $x = \frac{-2 + \sqrt{5}}{8}$, $x = \frac{-2 - \sqrt{5}}{8}$

**b)** Find the midpoint of the $x$-intercepts in part a). The equation of the axis of symmetry is $x = -\frac{1}{4}$.

**C2. a)** There will be two real roots if the discriminant (value under the square root) is greater than zero.

**b)** There will be one real root if the discriminant is equal to zero.

**c)** There will be no real roots if the discriminant is less than zero.

## Practise

- Discuss some of the solutions to **questions 1** through **5** in class before assigning questions from Connect and Apply.
- In **question 2**, students use the Zero operation of a graphing calculator. See Investigate B in Section 6.2.
- Include a discussion on the use of variables other than $x$ and $y$. Students often get confused with these variables when using the quadratic formula.
- **Question 6** can be checked using a graphing calculator. Graph both the relations $y = -0.1x^2 + x + 0.5$ and $y = 2.6$. Use the Intersect operation to determine the points of intersection of the two relations. Then subtract the $x$-coordinates.
- **Question 12** refers to the Chapter Problem. Students will need to determine the $y$-intercept and its partner on the parabola. Assign this question now or save it until the Chapter Problem Wrap-Up.
- In **question 16**, the quadratic should be set up in the form $\left(x - \frac{14 + \sqrt{140}}{4}\right)\left(x - \frac{14 - \sqrt{140}}{4}\right) = 0$, and then simplified.
- In **question 17**, students need to develop a list of ordered pairs and use finite differences to prove that the relationship is quadratic. They can then use any tool in their arsenal to determine the equation.

**Common Errors**

- Some students may assign incorrect signs to $a$, $b$, and $c$.

$R_x$ Have students list the specific values of $a$, $b$, and $c$ before substituting. Discuss that $-b$ simply means to change the sign of $b$, and that the result is often a positive value.

- Some students may assign incorrect values to $\sqrt{b^2 - 4ac}$. They will place the coefficient of $x$ into $b$, of $y$ into $a$, and the constant term into $c$.

$R_x$ Have students colour-code their examples to see where each coefficient is substituted.

- Some students may be confused about the use of 4 in $\sqrt{b^2 - 4ac}$.

$R_x$ Explain that formulas often include numbers. Provide examples such as $C = 2\pi r$ and $P = 2(l + w)$.

## Achievement Check Sample Solution, question 15, page 302

Provide students with **BLM 6–9 Section 6.4 Achievement Check Rubric** to help them understand what is expected.

**15. a)** The area of the flower bed will be 54 $m^2$.
Let $x$ represent the width of the path.
Then, the outer dimensions of the path will be $(2x + 9)$ by $(2x + 6)$.

The total area of the path and the flower bed is
$(2x + 9)(2x + 6)$
$= 4x^2 + 12x + 18x + 54$
$= 4x^2 + 30x + 54$

Since the area of the path is equal to that of the flower bed,
$4x^2 + 30x + 54 = 108$
$4x^2 + 30x - 54 = 0$

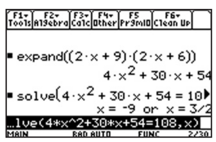
**b)** For $4x^2 + 30x - 54 = 0$, $a = 4$, $b = 30$, and $c = -54$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-30 \pm \sqrt{30^2 - 4(4)(-54)}}{2(4)}$$
$$= \frac{-30 \pm \sqrt{900 + 864}}{8}$$
$$= \frac{-30 \pm \sqrt{1764}}{8}$$
$$= \frac{-30 \pm 42}{8}$$

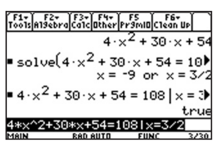
Therefore, $x = 1.5$ or $x = -9$.
Since $x$ represents a length, the answer is 1.5 m.
Therefore, the outer dimensions of the path are $2(1.5) + 9$ by $2(1.5) + 6$ m, or 12 m by 9 m. So, the perimeter is $2(12 + 9)$, or 42 m.

**c) CAS**

Step 1: Expanding the expression.
Step 2: Solve the equation.



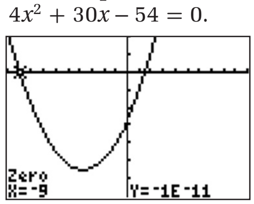

Step 3: Check only the solution $x = \frac{3}{2}$, as $x$ must be greater than 0.

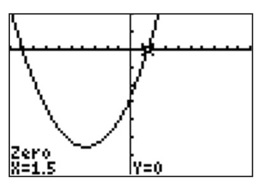



Since $x = \frac{3}{2}$ is correct, it follows that the perimeter is 42 m, as in part b).

**Graphing Calculator**

Graph the relation $y = 4x^2 + 30x - 54$ and use the Zero operation to find the $x$-intercepts, which are the roots to the corresponding quadratic equation $4x^2 + 30x - 54 = 0$.




Since $x = 1.5$ is the only possible solution, it follows that the perimeter is 42 m, as in part b).

## Accommodations

**Gifted and Enrichment**—Challenge students to learn more about quadratic equations in which there are no real roots, and to investigate imaginary numbers and complex numbers.

**Visual**—Let students use colour-coding when substituting the values of the coefficients of a quadratic equation in the quadratic formula. For example: For the quadratic equation $2x^2 + 6x + 5 = 0$, the roots are $x = \frac{-6 \pm \sqrt{6^2 - 4(2)(5)}}{2(2)}$

**Perceptual**—Encourage students to write an extra step when solving quadratic equations to determine the $x$-values. For example: The answer $x = \frac{-3 \pm \sqrt{5}}{2}$ should be followed with the step $x = \frac{-3 - \sqrt{5}}{2}$ or $x = \frac{-3 + \sqrt{5}}{2}$.

**Spatial**—Allow students to use a graphing calculator to graph the quadratic relations that model real-life situations. This will allow them to see the relationship between the $x$-intercepts of the quadratic relations and the solutions to the problems.

## Student Success

Ask students to research the history of the quadratic formula or other formulas (e.g., cubic) and write a journal entry on their findings.

Use a graffiti plus gallery walk to demonstrate multiple ways of solving a quadratic equation.

Refer to the introduction of this Teacher's Resource for more information about how to use a graffiti plus gallery walk strategy.

## Literacy Connections

Draw attention to the marginal item on page 293, the definition of "quadratic formula." As a fun follow-up, have the students listen to the Dave Mitchell CD of a marching band singing the quadratic formula. Ask the students to choose a song for the formula that will be their theme song. Go to **www.mcgrawhill.ca/books/principles10** and follow the links for information on where to purchase this CD.

Question 21 on page 302 uses the "golden ratio." Have a few students research this and make a presentation to the class.

Add "quadratic formula" and "golden ratio" to the Word Wall.

## Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

| Process Expectations | Selected Questions |
|---|---|
| Problem Solving | 16–18, 21–24 |
| Reasoning and Proving | 3, 4, 6–8, 11, 12, 14–18, 20–23 |
| Reflecting | 2, 5, 8, 15 |
| Selecting Tools and Computational Strategies | 13, 21, 23, 24 |
| Connecting | 6, 7, 10–15, 17, 19, 21–24 |
| Representing | 2, 3, 8, 15, 18–20 |
| Communicating | n/a |

## Ongoing Assessment

- Use Achievement Check question 15 to monitor student success. See Achievement Check Answers and **BLM 6–9 Section 6.4 Achievement Check Rubric**.
- Chapter Problem question 12 can be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.