Task

Cari Sports Centre

Student Text Pages
322

Suggested Timing
25–35 min

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 6–16 Task: Cari Sports Centre Rubric

Specific Expectations

Quadratic Relations of the Form $y = ax^2 + bx + c$

Investigating the Basic Properties of Quadratic Relations

QR1.01 collect data that can be represented as a quadratic relation, from experiments using appropriate equipment and technology (e.g., concrete materials, scientific probes, graphing calculators), or from secondary sources (e.g., the Internet, Statistics Canada); graph the data and draw a curve of best fit, if appropriate, with or without the use of technology;

QR1.02 determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ $(a \neq 0)$ can be graphically represented as a parabola, and that the table of values yields a constant second difference;

QR1.03 identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the y-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.

Solving Quadratic Equations

QR3.04 interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the x-intercepts of the corresponding relations;

QR3.08 solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing). (***Sample problems:*** Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x - 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $-4.9t^2 + 50t + 1.5 = 0$ by graphing $h = -4.9t^2 + 50t + 1.5$ using technology.)

Solving Problems Involving Quadratic Relations

QR4.01 determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques);

QR4.02 solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Teaching Suggestions

- Have students discuss the Task in pairs or small groups and start to prepare strategies for solving it. Some questions to consider include:
  - What sort of relationship could the data represent?
  - What will the curve of best fit look like?
  - How are the questions in parts c) and d) related to the curve of best fit?
  - Are there any tools that will help in solving the problems?
- Have a short class discussion to clarify the requirements of the Task, vocabulary, etc.
- Assign the Task to individual students or to pairs of students. The assignment may be completed at home or in class.

Accommodations

Gifted and Enrichment—Challenge students to use the Internet to research the costs of running a sports facility.

Motor—Allow students to work in groups to complete this Task.

Language—Let students work with a reading buddy who will read aloud the information in this Task.

Memory—Provide students with opportunities to review the steps to create a scatter plot using a graphing calculator.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they demonstrate. When evaluating a response, look for the following:

- Knowledge of properties of quadratic relations and the solution of quadratic equations
- Ability to draw a scatter plot; find a line of best fit; and find first and second differences
- Ability to find the coordinates of the vertex of a quadratic relation and the equation of the line of symmetry
- Ability to solve quadratic equations
- Accuracy of calculations in substitutions and algebraic manipulation
- Ability to choose and use appropriate tools/technology
- Evidence of planning, reasoning, reflecting, and justification
- Evidence of ability to connect the problem context to the underlying mathematical ideas
- Evidence of communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

- Use **BLM 6–16 Task: Cari Sports Centre Rubric** to assess student achievement.

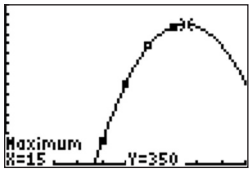
Level 3 Sample Response

a)

x	y	First Differences	Second Differences
8	56		
		200 − 56 = 144	
10	200		96 − 144 = −48
		296 − 200 = 96	
12	296		48 − 96 = −48
		344 − 296 = 48	
14	344		

Since the second differences are constant, the data represent a quadratic relation.

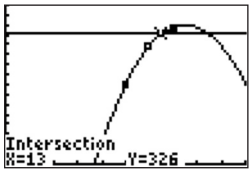
b)

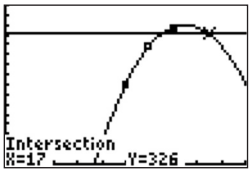


The vertex of the quadratic relation is (15, 350). The ticket price that will give the greatest daily profit is $15.

c) The constant daily expense is the value of the y-intercept, or –$1000. This is negative since these expenses are incurred before any income is made.

d)





This shows that for the range $13 to $17 daily profit will be at least $326.

Note: Some students may solve the quadratic equation $326 = -6x^2 + 180x - 1000$ to get the solution.

Level 3 Notes

Look for the following:

- Knowledge and understanding of the properties of quadratic relations
- Ability to draw a scatter plot; find a line of best fit; and find first and second differences

- Ability to find the coordinates of the vertex of a quadratic relation and the equation of the line of symmetry
- Ability to solve quadratic equations
- Ability to select and use technology in solving the problem
- Evidence of planning and reasoning in analysing the problem
- Mostly accurate calculations
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some knowledge and understanding of the properties of quadratic relations
- Some ability to draw a scatter plot; find a line of best fit; and find first and second differences
- Some ability to find the coordinates of the vertex of a quadratic relation and the equation of the line of symmetry
- Some ability to solve quadratic equations
- Some ability to select and use technology in solving the problem
- Some evidence of planning and reasoning in analysing the problem
- Some accurate calculations
- Some well-constructed mathematical arguments
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Detailed knowledge and understanding of the properties of quadratic relations
- Detailed ability to draw a scatter plot; find a line of best fit; and find first and second differences
- Detailed ability to find the coordinates of the vertex of a quadratic relation and find the equation of the line of symmetry
- Detailed ability to solve quadratic equations
- Detailed ability to select and use technology in solving the problem
- Detailed planning and reasoning in analysing the problem
- Accurate calculations
- Very well constructed mathematical arguments
- Use of very good form and correct mathematical notation

Task — York Leisure Centre

Student Text Pages
323

Suggested Timing
30–40 min

Tools
- grid paper

Technology Tools
- graphing calculator

Related Resources
- G–1 Grid Paper
- G–3 Coordinate Grids
- BLM 6–17 Task: York Leisure Centre Rubric

Specific Expectations

Quadratic Relations of the Form $y = ax^2 + bx + c$

Investigating the Basic Properties of Quadratic Relations

QR1.02 determine, through investigation with and without the use of technology, that a quadratic relation of the form $y = ax^2 + bx + c$ ($a \neq 0$) can be graphically represented as a parabola, and that the table of values yields a constant second difference;

QR1.03 identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.

Relating the Graph of $y = x^2$ and Its Transformations

QR2.02 explain the roles of a, h, and k in $y = a(x - h)^2 + k$, using the appropriate terminology to describe the transformations, and identify the vertex and the equation of the axis of symmetry.

Solving Quadratic Equations

QR3.04 interpret real and non-real roots of quadratic equations, through investigation using graphing technology, and relate the roots to the *x*-intercepts of the corresponding relations;

QR3.06 sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations; graphing $h = -4.9t^2 + 50t + 1.5$ using technology);

QR3.08 solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing). (***Sample problems:*** Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x - 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $-4.9t^2 + 50t + 1.5 = 0$ by graphing $h = -4.9t^2 + 50t + 1.5$ using technology.)

Solving Problems Involving Quadratic Relations

QR4.01 determine the zeros and the maximum or minimum value of a quadratic relation from its graph (i.e., using graphing calculators or graphing software) or from its defining equation (i.e., by applying algebraic techniques);

QR4.02 solve problems arising from a realistic situation represented by a graph or an equation of a quadratic relation, with and without the use of technology (e.g., given the graph or the equation of a quadratic relation representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball hit the ground? Over what time interval is the height of the ball greater than 3 m?).

Teaching Suggestions

- Have students discuss the Task in pairs or small groups and start to prepare strategies for solving it. Some questions to consider include:
  - Can you sketch the graph of the relation? What kind of curve is it?
  - What needs to be done to find the coordinates of the vertex and the equation of the line of symmetry?
  - What technique is used to change a quadratic relation from the form $y = ax^2 + bx + c$ to $y = a(x - h)^2 + k$?

Accommodations

Gifted and Enrichment—Challenge students to create a plan to maximize the profit for the York Leisure Centre.

Visual—Let the students find the coordinates of the vertex of the quadratic relation in more than one way.

Motor—Provide students with large sheets of grid paper to graph the quadratic relations modelling the daily profit for the York Leisure Centre.

Memory—Encourage students to review the steps required to complete the square for a quadratic relation.

  - What strategies can you think of to help you solve parts c) and d)?
  - Are there any tools that will help in solving the problems?
- You may also wish to have a short class discussion to clarify the requirements of the Task, vocabulary, etc.
- Assign the Task to individual students or to pairs of students. The assignment may be completed at home or in class.

Hints for Evaluating a Response

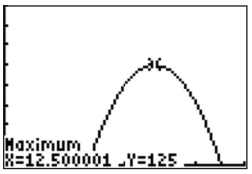
Student responses are being assessed for the level of mathematical understanding they demonstrate. When evaluating a response, look for the following:

- Knowledge of properties of quadratic relations and the solution of quadratic equations
- Ability to graph a quadratic relation and to find the coordinates of the vertex of a quadratic relation
- Ability to write a quadratic relation in the form $y = a(x - h)^2 + k$
- Accuracy of calculations in substitutions and algebraic manipulation
- Evidence of planning, reasoning, reflecting, and justification
- Ability to connect the problem context to the underlying mathematical ideas
- Communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

- Use **BLM 6–17 Task: York Leisure Centre Rubric** to assess student achievement.

Level 3 Sample Response

a) 

From the graph, the vertex is (12.5, 125). Therefore, the equation of the line of symmetry is $x = 12.5$.

b) It follows from the graph in part a) that the $y = a(x - h)^2 + k$ form of the equation is $y = -4(x - 12.5)^2 + 125$.

c) $0 = -4x^2 + 100x - 500$

$0 = x^2 - 25x + 125$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{25 \pm \sqrt{(-25)^2 - 4(1)(125)}}{2(1)}$$

$$= \frac{25 \pm \sqrt{625 - 500}}{2}$$

$$= \frac{25 \pm \sqrt{125}}{2}$$

The appropriate roots are 18.09 and 6.91. This means that the daily profit, y, will be greater than \$0 when the cost of admission is between \$6.91 and \$18.09, with a daily maximum profit of \$125 when admission is \$12.50.

d) Add \$2.50 to the cost of admission, so that (admission + refreshments) becomes $\$(x + 2.50)$. Substitute this expression for x into $0 = -4x^2 + 100x - 500$.

$0 = -4(x + 2.5)^2 + 100(x + 2.5) - 500$

$0 = -4(x^2 + 5x + 6.25) + 100x + 250 - 500$

$0 = -4x^2 - 20x + 25 + 100x + 250 - 500$

$0 = -4x^2 + 80x - 225$

Use the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2b}$$
$$= \frac{-80 \pm \sqrt{80^2 - 4(-4)(-225)}}{2(-4)}$$
$$= \frac{-80 \pm \sqrt{6400 - 3600}}{-8}$$
$$= \frac{-80 \pm \sqrt{2800}}{-8}$$

The appropriate roots are 3.39 and 16.61. So the range \$4.10 < x < \$15.59 would ensure a daily profit greater than \$0.

Note: Students will give answers for the price of admission based on several possible viewpoints:

- For maximum profit, the cost would be \$10.00.
- To ensure the least possible charge to make a profit, some students will choose an admission between \$4.10 and \$10.00.
- Some students may argue that the costs of running the coffee shop will make the amount of profit added less than \$2.50 and adjust the above numbers to somewhere between (\$4.10 to \$6.91) and (\$10.00 to \$12.50).

Students should realize that any admission charge greater than the x-value of the vertex of the graph would be completely unreasonable.

Level 3 Notes

Look for the following:

- Knowledge and understanding of the properties of quadratic relations
- Ability to graph a quadratic relation and to find the coordinates of the vertex of a quadratic relation
- Ability to write a quadratic relation in the form $y = a(x - h)^2 + k$
- Ability to solve quadratic equations
- Ability to select and use technology in solving the problem
- Evidence of planning and reasoning in analysing the problem
- Mostly accurate calculations
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some knowledge and understanding of the properties of quadratic relations
- Some ability to graph a quadratic relation and to find the coordinates of the vertex of a quadratic relation
- Some ability to write a quadratic relation in the form $y = a(x - h)^2 + k$
- Some ability to solve quadratic equations
- Some ability to select and use technology in solving the problem
- Some planning and reasoning in analysing the problem
- Some accurate calculations
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- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Detailed knowledge and understanding of the properties of quadratic relations
- Detailed ability to graph a quadratic relation and to find the coordinates of the vertex of a quadratic relation
- Detailed ability to write a quadratic relation in the form $y = a(x - h)^2 + k$
- Detailed ability to solve quadratic equations
- Detailed ability to select and use technology in solving the problem
- Detailed planning and reasoning in analysing the problem
- Accurate calculations
- Very well constructed mathematical arguments
- Use of very good form and correct mathematical notation

Task — Abbey Leisure Centre

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Related Resources
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Specific Expectations

Quadratic Relations of the Form $y = ax^2 + bx + c$

Investigating the Basic Properties of Quadratic Relations

QR1.03 identify the key features of a graph of a parabola (i.e., the equation of the axis of symmetry, the coordinates of the vertex, the *y*-intercept, the zeros, and the maximum or minimum value), and use the appropriate terminology to describe them.

Solving Quadratic Equations

QR3.01 expand and simplify second-degree polynomial expressions [e.g., $(2x + 5)^2$, $(2x - y)(x + 3y)$], using a variety of tools (e.g., algebra tiles, diagrams, computer algebra systems, paper and pencil) and strategies (e.g., patterning);

QR3.05 express $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$ by completing the square in situations involving no fractions, using a variety of tools (e.g. concrete materials, diagrams, paper and pencil);

QR3.06 sketch or graph a quadratic relation whose equation is given in the form $y = ax^2 + bx + c$, using a variety of methods (e.g., sketching $y = x^2 - 2x - 8$ using intercepts and symmetry; sketching $y = 3x^2 - 12x + 1$ by completing the square and applying transformations; graphing $h = -4.9t^2 + 50t + 1.5$ using technology);

QR3.08 solve quadratic equations that have real roots, using a variety of methods (i.e., factoring, using the quadratic formula, graphing). (***Sample problems:*** Solve $x^2 + 10x + 16 = 0$ by factoring, and verify algebraically. Solve $x^2 + x - 4 = 0$ using the quadratic formula, and verify graphically using technology. Solve $-4.9t^2 + 50t + 1.5 = 0$ by graphing $h = -4.9t^2 + 50t + 1.5$ using technology.)

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Teaching Suggestions

- Have the students discuss the Task in pairs or small groups and start to prepare strategies for solving it. Some questions to consider include:
  – What strategies can you think of to help you solve the problem?
  – Can you make a scatter plot of the data in the table?
  – How might you find the equation of the relation given by the data?
  – Are there any tools that will help in solving the problems?
- If students are unable to set up an algebraic method, have them use a data table and graphing technology. Hint: Make a table for increases by $20 versus the annual club revenue.
- Have a short class discussion to clarify the requirements of the Task, vocabulary, etc.

Accommodations

Gifted and Enrichment—Challenge students to create a plan to increase the membership at the Abbey Leisure Centre.

Spatial—Encourage students to use the information in the Task to create a quadratic relation that models the situation and to graph the quadratic relation using a graphing calculator.

Motor—Let students dictate their report to a scribe who will record their answer, or use the school's language lab to prepare their report.

Memory—Provide students with opportunities to review the steps to determine the maximum or minimum value of a quadratic relation.

- Assign the Task to individual students or to pairs of students. The assignment may be completed at home or in class.

Hints for Evaluating a Response

Student responses are being assessed for the level of mathematical understanding they demonstrate. When evaluating a response, look for the following:

- Knowledge of properties of quadratic relations and the solution of quadratic equations
- Ability to find the coordinates of the vertex of a quadratic relation
- Ability to represent a quadratic relation in the forms $y = a(x - r)(x - s)$ and $y = a(x - h)^2 + k$
- Accuracy of calculations in substitutions and algebraic manipulation
- Ability to use technology as a tool to help solve the problem
- Evidence of planning, reasoning, reflecting, and justification
- Ability to connect the problem context to the underlying mathematical ideas
- Communication of mathematical thinking and mathematical vocabulary

Ongoing Assessment

- Use **BLM 6–18 Task: Abbey Leisure Centre Rubric** to assess student achievement.

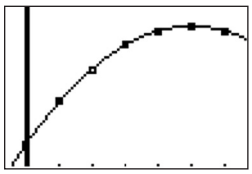
Level 3 Sample Response

Non-Algebraic Method

Use a table to track what happens to the annual revenue for increases of $20 in the membership fee.

Increases of $20	Annual Membership Fee ($)	Number of Members	Annual Revenue ($)
0	400	300	400 × 300 = 120 000
1	420	290	420 × 290 = 121 800
2	440	280	440 × 280 = 123 200
3	460	270	460 × 270 = 124 200
4	480	260	480 × 260 = 124 800
5	500	250	500 × 250 = 125 000
6	520	240	520 × 240 = 124 800

Use a graphing calculator to create a scatter plot of the data and add a curve of best fit.



Use the Maximum operation of the graphing calculator to find the vertex, (5, 125 000).

So, 5 increases of $20 will maximize the revenue. The board should charge $500 for an annual membership fee to maximize the revenue.

Algebraic Method

Let R represent the revenue, in dollars.
Let n represents the number of \$20 increases.
Then, the cost of annual membership can be calculated as $400 + 20n$ and the number of members as $300 - 10n$.

$$\begin{aligned} R &= (400 + 20n)(300 - 10n) \\ &= 120\,000 - 4000n + 6000n - 200n^2 \\ &= 120\,000 + 2000n - 200n^2 \\ &= -200(n^2 - 10n) + 120\,000 \\ &= -200(n^2 - 10n + (-5)^2 - (-5)^2) + 120\,000 \\ &= -200(n^2 - 10n + (-5)^2) + 200(25) + 120\,000 \\ &= -200(n - 5)^2 + 125\,000 \end{aligned}$$

The relation has a maximum of 125 000 when $n = 5$.
There should be 5 increases of \$20.
$400 + 20(5) = 500$
The board should charge \$500 for an annual membership fee to maximize the revenue.

Level 3 Notes

Look for the following:

- Knowledge and understanding of the properties of quadratic relations
- Ability to represent a quadratic relation in the forms $y = a(x - r)(x - s)$ and $y = a(x - h)^2 + k$
- Ability to find the coordinates of the vertex of a quadratic relation
- Ability to select and use technology in solving the problem
- Evidence of planning and reasoning in analysing the problem
- Mostly accurate calculations
- Well-constructed mathematical arguments
- Use of good form and correct mathematical notation

What Distinguishes Level 2

At this level, look for the following:

- Some knowledge and understanding of the properties of quadratic relations
- Some ability to represent a quadratic relation in the forms $y = a(x - r)(x - s)$ and $y = a(x - h)^2 + k$
- Some ability to find the coordinates of the vertex of a quadratic relation
- Some ability to select and use technology in solving the problem
- Some planning and reasoning in analysing the problem
- Some accurate calculations
- Some well-constructed mathematical arguments
- Some use of good form and correct mathematical notation

What Distinguishes Level 4

At this level, look for the following:

- Detailed knowledge and understanding of the properties of quadratic relations
- Detailed ability to represent a quadratic relation in the forms $y = a(x - r)(x - s)$ and $y = a(x - h)^2 + k$
- Detailed ability to find the coordinates of the vertex of a quadratic relation
- Detailed ability to select and use technology in solving the problem
- Detailed planning and reasoning in analysing the problem
- Accurate calculations
- Very well constructed mathematical arguments
- Use of very good form and correct mathematical notation