7.2 Use Similar Triangles to Solve Problems

Student Text Pages
342–351

Suggested Timing
80–160 min

Tools
- ruler
- metre stick

Related Resources
- BLM 7–1 Great North American Trigonometry Race Map
- BLM 7–7 Making a Clinometer
- BLM 7–8 Section 7.2 Practice Master
- BLM 7–9 Section 7.2 Achievement Check Rubric
- A–6 Knowledge/Understanding General Scoring Rubric
- A–22 Report Checklist

TI-Navigator™

Go to **www.mcgrawhill.ca/books/principles10** and follow the links to the file for this section.

Teaching Suggestions

- In this section, the properties of similar triangles learned in the previous section are applied to solve a variety of problems. In addition to side and angle relationships, students discover that the areas of similar figures are related by the square of the scale factor (ratio of corresponding sides).
- Discuss the photo and how similar triangles can be used to find the height of the tree. (5 min)

Investigate

- The **Investigate** provides prompts for groups of students to brainstorm their own method of finding the height of the tree, using the properties of similar triangles. By the end of the investigation groups should have a method that they can feel fairly confident will work. In **question 15**, students have the opportunity to actually test out their method on a real tree. (10 min)
- This activity could be given to the class as a challenge. Select a tree (or a different object, such as a flag pole), and carry out accurate measurements beforehand. Each group can make their measurements and present their solution and result to the class. Then groups can be awarded prizes for
  – most efficient method
  – most creative method
  – most accurate result
  – most clearly communicated solution, etc.
  Winners can be declared by a combination of class and teacher votes.
- Some students could make/use a tool called a clinometer. **BLM 7–7 Making a Clinometer** shows a technique for how to make one, adapted from the Ontario Grade 10 Academic Mathematics Public Course Profiles. This tool can be used here, but may be more useful in the trigonometry sections that follow.

Examples

- In the previous section students learned that a property of similar triangles is that corresponding side lengths are in the same ratio. This ratio is called the scale factor. Introducing this terminology will be helpful as students work through the **Examples**. (25–30 min)
- **Example 1** illustrates how to apply similar triangles to solve for an inaccessible length or distance, in this case the width of a river. It is important for students to recognize that they must first establish similarity of two triangles by applying known or inferred geometric principles (step 1). Once two triangles have been proven similar, then they can find the scale factor by making connections between two known sides (step 2). Finally, they can apply the scale factor to solve for an unknown length (step 3).
- **Example 2** shows how the areas of similar figures are related. This part can be taught as an investigation. Students may already be familiar with the side length–area relationship for squares. Challenge students to explore the same relationship for similar rectangles and triangles. This can be done using *The Geometer's Sketchpad*®, or by drawing on grid paper. The main idea is that the areas of similar figures are related by the square of the scale factor.

- In **Example 3**, the property of related areas of similar triangles is applied to solve a contextual problem. The approach parallels the one modelled in the first example:
  Step 1: Show that two triangles are similar using known or inferred geometric information.
  Step 2: Use two known corresponding sides to find the scale factor.
  Step 3: Apply the square of the scale factor to find the unknown area.

Communicate Your Understanding

- Review the vocabulary in this section (scale factor, k) before discussing the **Communicate Your Understanding** questions as a class or in small groups. (5 min)
- These questions can be used to help students consolidate their understanding of scale factor and how it relates corresponding sides of similar triangles. Students should also be able to explain how the areas of similar figures are related and how the square of the scale factor can be used to link these. Use **A–6 Knowledge/Understanding General Scoring Rubric** when assessing students for this section.
- Use **BLM 7–8 Section 7.2 Practice Master** for remediation or extra practice.

Investigate Answers (page 342)

1. Answers may vary. For example: The students could use the properties of similar triangles and the ratio of the measures of the side lengths along the ground of each triangle.

2. Diagrams will vary. Answers may vary. For example: Properties of similar triangles will allow students to determine the height of the tree.

3. Answers will vary.

4. Answers will vary.

Communicate Your Understanding Responses (page 347)

C1. a) The scale factor, k, relates the lengths of corresponding sides of similar triangles. Each side of the triangle is multiplied by or divided by the same scale factor, k, to get the sides of the similar triangle.

b) The ratio of corresponding sides is equal to the scale factor, so you can set up an equation to solve for an unknown side length.

C2. a) The square of the scale factor relates the areas of two similar figures.

b) The area of a triangle is the product of the square of the scale factor and the area of the similar triangle.

C3. You can set up an equation where the metre stick corresponds to the height of the inaccessible object, and the shadow of the metre stick corresponds to the shadow of the object. The triangles formed will be similar and the ratios of the corresponding sides will be equal. You can solve the proportion for the unknown length if three of the terms are known.

Common Errors

- Some students relate incorrect pairs of sides of similar figures.

R_x Encourage students to take care in identifying corresponding angles and sides when establishing similarity. Emphasize the importance of writing the letters of each triangle in corresponding order.

- Some students relate areas of similar figures by the scale factor instead of its square.

R_x Have students think of the length-area relationships of squares, and then extend this concept to triangles. Remind them that the scale factor relates one-dimensional (linear) measures, whereas comparison of areas involves two-dimensional measures. Thus, the scale factor is applied twice.

Accommodations

Gifted and Enrichment—Challenge students to use the Internet to research and learn more about clinometers.

Perceptual—Encourage students to colour-code the corresponding sides of similar triangles.

Motor—Let students complete fewer of the same type of questions in the Practise.

Language—Allow students to complete the Connect and Apply questions in the Language Lab.

Practise

- **Questions 1** through **4** further consolidate student understanding of scale factor and how it connects side lengths and areas of similar figures. Have students carry out these investigative activities in pairs.
- For **questions 5** through **7**, students can trace parts of the figure to create cutouts. This allows them to more easily explore transformations and to compare angles.
- **Question 8** involves the areas of similar triangles, which are related by the square of the scale factor.
- **Question 9** prepares students for the performance task in **question 15**.
- For **question 12**, the scale factor relating the entire crest to the smaller triangles is 2. (Hint: Look at the base of the large triangle.)
- For **question 13**, tracing paper and scissors may be helpful.
- **Question 15** can be assigned as a performance task.
- **Question 17** assesses students' ability to reason and prove, and to communicate higher order mathematical thinking. One approach is to draw one triangle and label its base and height, b and h, then draw a similar triangle and label its base and height, kb and kh.
- For **questions 19** and **20**, encourage students to begin with a diagram.
- Use **A–22 Report Checklist** when assessing students for **question 21**.
- **Question 22** can be assigned as a performance task or assignment. Use **BLM 7–1 Great North American Trigonometry Race Map** to support this question.
- For **question 23**, students need to make connections to the Pythagorean theorem, geometric reasoning, and area of a triangle, as well as apply the ratio of sides and areas of similar triangles.
- **Question 24** encourages students to construct/identify similar triangles (i.e., relating one step to the entire staircase).
- For **question 25**, the number of centimetres in one kilometre is needed. Students must then apply the square of this scale factor.
- For **question 26**, encourage students to begin with a diagram. A globe may be helpful to illustrate how the curvature of Earth distorts the line segments of the triangle into curves.

Achievement Check Sample Solution, question 23, page 350

Provide students with **BLM 7–9 Section 7.2 Achievement Check Rubric** to help them understand what is expected.

23. a) $\angle ZXY = 90° - 23°$
$= 67°$

Use the Pythagorean theorem to find ZX.

$ZX^2 = 5^2 + 12^2$
$ZX^2 = 169$
$ZX = 13$

The length of ZX is 13 cm.

b) The hypotenuse of the actual garden is 6.5 m, or 650 cm. The hypotenuse in the scale drawing is 13 cm. The scale of the drawing is 13:650, or 1:50.

Then, the other sides of the actual garden are 5×50, or 250 cm, and 12×50, or 600 cm.

Therefore, the perimeter of the actual garden is $650 + 250 + 600$, or 1500 cm (15 m).

c) $k = 0.02$

d) The area of the scale triangle is $\frac{1}{2}(12)(5)$, or 30 cm^2. The area of the actual garden is $\frac{1}{2}(600)(250)$, or 75 000 cm^2. Then, the ratio of the areas is $\frac{30}{75\,000}$, or 1:2500.

Student Success

Have students complete a journal entry explaining how to use similar triangles to solve problems.

Have groups of students solve different problems, copy their solutions onto large paper, and then have a gallery walk to showcase the different solutions.

Refer to the introduction of this Teacher's Resource for more information about how to use a gallery walk strategy.

Literacy Connections

Draw attention to the marginal item on page 343, which defines "scale factor" and identifies the letter k as the constant. Ask students to recall other letters that represent constant values that are always used in the same manner. Answers might include such letters as m and b as used in $y = mx + b$ or a, b, and c as used in $ax^2 + bx + c = 0$, and so on.

Add "scale factor" to the Word Wall.

Mathematical Processes Integration

The table shows questions that provide good opportunities for students to use the mathematical processes.

Process Expectations	Selected Questions
Problem Solving	13, 22, 26–30
Reasoning and Proving	2, 5, 17, 20, 23, 26
Reflecting	4, 21
Selecting Tools and Computational Strategies	18, 21–23, 30
Connecting	2, 4, 9–16, 19, 21–26, 28
Representing	1, 3, 13, 17, 18, 21, 22, 29
Communicating	2, 4, 15, 20, 21, 26

Ongoing Assessment

- Use Achievement Check question 23 to monitor student success. See Achievement Check Answers and **BLM 7–9 Section 7.2 Achievement Check Rubric**.
- Chapter Problem question 22 can also be used as an assessment tool.
- Communicate Your Understanding questions can be used as quizzes to assess students' communication skills.